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The Stress Distribution in the Composite Materials with Locally Curved Fibers

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Abstract. Nowadays composite materials are widely used in industry as they can withstand high loads. In this paper, in the case of the piecewise-homogeneous body model with the use of the three -dimensional linearized theory of elastic stability the problem of stress distribution in the composite materials with curved layers is investigated. The case in which layers are antiphased locally curved is considered. It is also assumed that external compressive forces act at infinity in the direction along the fiber.
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Investigations of the stability, stress strain state of composite materials have been carried out in many works [1-5]. Due to wide usage of composite materials in industry, stress strain distribution, stability of composite materials are very important problems. Guz, Ilyushin, Akbarov made great contribution to the theory of composite materials.

In this paper, composite material with infinite number of non-intersecting fibers is considered. It is assumed that matrix and filler are anisotropic. This paper also investigates the influence of antiphased piecewise-homogeneous body model using the three-dimensional linearized theory of elastic stability problem of stress distribution in composite materials with curved fibers.

Elements of matrix and filler will be denoted by (1) and (2) respectively. To each fiber the Cartesian coordinate system $O_m^{(k)} x_{1m}^{(k)} x_{2m}^{(k)} x_{3m}^{(k)}$ $(k = 1, 2;\;\; m = 1, 2, 3, ...)$is assigned. Suppose that fibers lie in $x_{1m}^{(2)} x_{2m}^{(2)} x_{3m}^{(2)}$, and width of each filler is a constant. It is assumed that the external compressive forces act at infinity in the direction along the fiber.
For each fiber equation of equilibrium, generalized Hook's law and Cauchy relations are given in the form

$$\frac{\partial \sigma_{ij}^{(k)m}}{\partial x_{jm}^{(k)}} = 0' \sigma_{ij}^{(k)m} = C_{ijrs}^{(k)m} \cdot \varepsilon_{rs}^{(k)m};$$

$$\varepsilon_{rs}^{(k)m} = \frac{1}{2}\left(\frac{\partial u_r^{(k)m}}{\partial x_{sm}^{(k)}} + \frac{\partial u_s^{(k)m}}{\partial x_{rm}^{(k)}}\right); i, j, r, s = 1, 2 \tag{1}$$

In (1) generally accepted notations are used. If we denote upper limit of $m^{(k)}-$th fiber by S_m^+, the lower limit by S_m^-, condition of full contact can be written as

$$\sigma_{ij}^{(1)m}\Big|_{S_m^+}\cdot n_j^{m,+}=\sigma_{ij}^{(2)m}\Big|_{S_m^+}\cdot n_j^{m,+};$$

$$\sigma_{ij}^{(1)m}\Big|_{S_m^-}\cdot n_j^{m,-}=\sigma_{ij}^{(2)m}\Big|_{S_m^-}\cdot n_j^{m,-};\tag{2}$$

$$u_i^{(1)m}\Big|_{S_m^+}=u_1^{(2)m}\Big|_{S_m^+};$$

$$u_i^{(1)m}\Big|_{S_m^-}=u_1^{(2)m}\Big|_{S_m^-};i,j,r,s=1,2,3$$

Here $n_j^{m,\pm}-$normal vectors to the surface$S_m^{\pm}$.
Assume that equation of the middle line of the $m^{(2)}$ filler is given in the form

$$x_{2m}^{(2)}=F_m(x_{1m}^{(2)})=\varepsilon\cdot f_m(x_{1m}^{(2)})\tag{3}$$

in (3) $\varepsilon\in[1,0)-$is a small dimensionless constant.
Using assumption that width of fillers are constant and (3) we obtain the equation for $S_m^{\pm}$:

$$x_{1m}^{(2)\pm}=t_{1m}\mp H_m^{(2)}\cdot\frac{dF_m(t_{1m})}{dt_{1m}}\cdot T(t_{1m})$$

$$x_{2m}^{(2)\pm}=F_m(t_{1m})\pm H_m^{(2)}\cdot T(t_{1m})\tag{4}$$

where $T(t_{1m})=\left[1+\left(\frac{dF_m(t_{1m})}{dt_{1m}}\right)^2\right]^{-\frac{1}{2}}$ (**??**)
From (4) after some algebra we derive equations for the normal vectors to the surfaces in the form

$$n_1^{m,\pm}=-\frac{dx_{2m}^{(2)\pm}}{dt_{1m}}\cdot V^{\pm}(t_{1m});n_2^{m,\pm}=\frac{dx_{1m}^{(2)\pm}}{dt_{1m}}\cdot V^{\pm}(t_{1m});\tag{5}$$

where

$$V^{\pm}(t_{1m})=\left[\left(\frac{dx_{1m}^{(2)\pm}}{dt_{1m}}\right)^2+\left(\frac{dx_{2m}^{(2)\pm}(t_{1m})}{dt_{1m}}\right)^2\right]^{\frac{1}{2}}\tag{6}$$

In (4)-(6) $t_{1m}-$ is the parameter and $-\infty<t_{1m}<\infty$; $x_{1m}^{(2)\pm}$,$x_{2m}^{(2)\pm}$-coordinates of the $S_m^{\pm}$; $H_m^{(2)}-$half of the width of m^{th} fiber.
Quantities that express stress strain state of any *m-th* fiber will be searched in the form

$$\sigma_{ij}^{(k)m}=\sum_{q=0}^{\infty}\varepsilon^{q}\sigma_{ij}^{(k)m,q};\varepsilon_{ij}^{(k)m}=\sum_{q=0}^{\infty}\varepsilon^{q}\varepsilon_{ij}^{(k)m,q};$$

$$u_{ij}^{(k)m}=\sum_{q=0}^{\infty}\varepsilon^{q}u_{ij}^{(k)m,q} \tag{7}$$

Expression for $x_{im}^{(2)\pm}$and $n_i^{(m)\pm}$also written as a series in term of $\varepsilon$and expression of each approximation of (7) expanded in placeCityTaylor series, and from (2) we obtain a necessary relation for each approximation.
Generalized Hook's law for this three dimensional case produces

$$\varepsilon_{11}^{(k)}=\frac{1}{E_1^{(k)}}\left[\left(1-(\nu_{13}^{(k)})^2\cdot\frac{E_3^{(k)}}{E_1^{(k)}}\right)\cdot\sigma_{11}^{(k)}{}_{-}\left(\nu_{12}^{(k)}+\nu_{13}^{(k)}\cdot\nu_{32}^{(k)}\right)\cdot\sigma_{22}^{(k)}\right];$$

$$\varepsilon_{12}^{(k)}=\frac{1}{2G_{12}^{(k)}}\sigma_{12}^{(k)} \tag{8}$$

$$\varepsilon_{22}^{(k)}=\frac{1}{E_1^{(k)}}\left[\left(-\nu_{12}^{(k)}-\nu_{13}^{(k)}\cdot\nu_{32}^{(k)}\right)\cdot\sigma_{11}^{(k)}+\frac{E_1^{(k)}}{E_2^{(k)}}\left(1-\frac{E_2^{(k)}}{E_3^{(k)}}(\nu_{32}^{(k)})^2\right)\cdot\sigma_{22}^{(k)}\right)\Bigg]$$

in (8) used notation from [6].

The values of zero's approximation correspond to the stress strain distribution in the composite material with horizontal (non curved) fibers and balances with force$\langle p\rangle$.

$$\sigma_{11}^{(1),0}=\langle p\rangle\cdot\left[\gamma^{(1)}+\gamma^{(2)}\cdot\frac{E_1^{(2)}\left(1-\left(\nu_{13}^{(1)}\right)^2\cdot\frac{E_3^{(1)}}{E_1^{(1)}}\right)}{E_1^{(1)}\left(1-\left(\nu_{13}^{(2)}\right)^2\cdot\frac{E_3^{(2)}}{E_1^{(2)}}\right)}\right]^{-1};$$

$$\sigma_{11}^{(2),0}=\frac{E_1^{(2)}\left(1-\left(\nu_{13}^{(1)}\right)^2\cdot\frac{E_3^{(1)}}{E_1^{(1)}}\right)}{E_1^{(1)}\left(1-\left(\nu_{13}^{(2)}\right)^2\cdot\frac{E_3^{(2)}}{E_1^{(2)}}\right)}\cdot\sigma_{11}^{(1),0}; \tag{9}$$

$$u_1^{(k),0}=\frac{1}{E_1^{(k)}}\left(1-\left(\nu_{13}^{(k)}\right)^2\cdot\frac{E_3^{(k)}}{E_1^{(k)}}\right)\cdot\sigma_{11}^{(k),0}\cdot x_1^{(k)};$$

$$u_2^{(k),0}=-\frac{1}{E_1^{(k)}}\left(\nu_{12}^{(k)}+\nu_{13}^{(k)}\cdot\nu_{32}^{(k)}\right)\cdot\sigma_{11}^{(k),0}\cdot x_2^{(k)}+C^{(k)};$$

$$C^{(k)}=const;\gamma^{(k)}=\frac{H^{(k)}}{H^{(1)}+H^{(2)}}$$

The value of the first, second and the following approximations correspond to the stress distribution with curved fibers. Using (9) and (1) we expand values of each approximations to the placeCityTaylor series around $\left(t_{1m}, \pm H_m^{(k)}\right)$, after some algebra, from (2) we obtain necessary relations for each approximation. Then, applying Fourier transform to the system of differential equations and evaluations we obtain non-homogenous system of linear equations with respect to unknown coefficients.

In this article, stress distribution in composite materials with infinite number of antiphased and locally curved fibers is investigated. Since fibers are located periodically with the period of 4 $\left(\mathrm{H}^{(2)} + H^{(1)}\right)$, from the composite materials we select only four fibers, denoted as $1^{(1)}, 1^{(2)}, 2^{(1)}$ and $2^{(2)}$. Equation of the middle of the surface of $1^{(2)}$is taken in the form

$$x_{21}^{(2)} = F_1(x_{11}^{(2)}) = \varepsilon \cdot f_1(x_{11}^{(2)}) = A \cdot \exp(-(x_{11}^{(2)}/L)^2) \tag{10}$$

and equation of the middle of the surface $2^{(2)}$in the form

$$x_{22}^{(2)} = F_2(x_{12}^{(2)}) = \varepsilon \cdot f_2(x_{12}^{(2)}) = -A \cdot \exp(-(x_{12}^{(2)}/L)^2) \tag{11}$$

In (10) and (11) A and L introduced to describe local characteristics of the fibers. And new dimensionless parameter $\varepsilon = A/L$is introduced.

Numerical results and discussions

Let us introduce $\gamma = H^{(2)}/L$, that represents the concentration of layers.

We will use commonly accepted notation:

$\sigma_{nn}^{+}(\sigma_{nn}^{-})$-is the stress in the direction of the normal vector$\vec{n}$to the surface $S^{+}(S^{-})$

$\sigma_{\tau\tau}^{(1)+}(\sigma_{\tau\tau}^{(1)-})$-is the stress in the direction of the tangent vector$\vec{\tau}$to the surface $S^{+}(S^{-})$

$\sigma_{n\tau}^{+}$–is the stress in tangent direction to the surfaceS^{+}

$\eta^{(2)}$–is the concentration of the filler in composite material

The concrete numerical investigations were carried out in the case, when the materials of the matrix and filler are homogeneous and anisotropic with elastic characteristics E (Young's modulus) and ν(Poisson coefficient). Before numerical analysis it is necessary to note that, the values related to the S^{+}will be denoted by the upper indices (+) and the values related to S^{-}-by the upper indices (-).

For numerical analysis it is assumed that $\nu_1^{(2)} = \nu^{(2)} = \nu^{(1)} = 0.3$; $E_1^{(2)}/E^{(1)} = 50$and $\varepsilon = 0.05$

$\sigma_{nn}^{+}\left(\sigma_{\tau\tau}^{(1)+}\right)$, $\sigma_{nn}^{-}\left(\sigma_{\tau\tau}^{(1)-}\right)$-are stress components in the direction of $\vec{n}\left(\vec{\tau}\right)$on the surfaces $S^{+}$and S^{-}respectively, $\sigma_{n\tau}^{+}$-stress component in tangent direction of S^{+}.

Let us analyze the impact of $\frac{E^{(2)}}{E^{(1)}}$, $\frac{E^{(2)}}{G_{12}^{2}}$to the stress distribution of the stresses listed above, Table 1, 2, and 3, are calculated for the case $\gamma = 0.1$ and $\eta^{(2)} = 0.5;\ 0.2;\ 0.1$respectively.

In these tables values of $\sigma_{nn}^{+}/\sigma_{11}^{(1),0}$, $\sigma_{nn}^{-}/\sigma_{11}^{(1),0}$, $\sigma_{\tau\tau}^{-}/\sigma_{11}^{(1),0}$are calculated at $t_1/L = 0.8$, the value of $\sigma_{n\tau}^{+}/\sigma_{11}^{(1),0}$ at $t_1/L = 1.6$.

$\frac{E^{(??)}}{G_{12}^2}$	$\frac{E^{(??)}}{E^{(??)}}$	ε	$\sigma_{nn}^+/\sigma_{11}^{(??),0}$	$\sigma_{nn}^-/\sigma_{11}^{(??),0}$	$\sigma_{n\tau}^+/\sigma_{11}^{(??),0}$
2	0.9	0.01	0.092	-0.074	0.010
		0.04	0.476	-0.197	0.050
		0.05	0.641	-0.205	0.066
	0.01	0.01	0.073	-0.072	0.009
		0.04	0.306	-0.277	0.042
		0.05	0.387	-0.341	0.055
100	0.9	0.01	0.055	-0.021	-0.005
		0.04	0.419	-0.014	-0.018
		0.05	0.606	-0.026	-0.021
	0.01	0.01	0.057	-0.034	-0.005
		0.04	0.169	-0.118	-0.025
		0.05	0.218	-0.139	-0.032

Table 1

$\frac{E^{(??)}}{G_{12}^2}$	$\frac{E^{(??)}}{E^{(??)}}$	ε	$\sigma_{nn}^+/\sigma_{11}^{(??),0}$	$\sigma_{nn}^-/\sigma_{11}^{(??),0}$	$\sigma_{n\tau}^+/\sigma_{11}^{(??),0}$
2	0.9	0.01	0.078	-0.073	0.013
		0.04	0.339	-0.267	0.061
		0.05	0.436	-0.323	0.081
	0.01	0.01	0.068	-0.072	0.011
		0.04	0.280	-0.261	0.052
		0.05	0.353	-0.323	0.063
100	0.9	0.01	0.039	-0.030	-0.028
		0.04	0.206	-0.071	-0.111
		0.05	0.273	-0.066	-0.137
	0.01	0.01	0.034	-0.031	-0.026
		0.04	0.148	-0.112	-0.108
		0.05	0.189	-0.134	-0.135

Table 2

$\frac{E^{(??)}}{G_{12}^2}$	$\frac{E^{(??)}}{E^{(??)}}$	ε	$\sigma_{nn}^+/\sigma_{11}^{(??),0}$	$\sigma_{nn}^-/\sigma_{11}^{(??),0}$	$\sigma_{n\tau}^+/\sigma_{11}^{(??),0}$	$\sigma_{\tau\tau}^-/\sigma_{11}^{(??),0}$	$\sigma_{\tau\tau}^+/\sigma_{11}^{(??),0}$
2	0.9	0.01	0.069	-0.067	0.001	1.007	1.016
		0.04	0.285	-0.261	0.011	1.030	1.069
		0.05	0.361	-0.322	0.017	1.037	1.0.87
	0.01	0.01	0.062	-0.061	0.000	1.006	1.012
		0.04	0.252	-0.242	0.003	1.024	1.050
		0.05	0.316	-0.300	0.013	1.029	1.062
100	0.9	0.01	0.036	-0.032	-0.038	0.974	0.928
		0.04	0.166	-0.102	-0.154	0.910	0.752
		0.05	0.217	-0.116	-0.191	0.891	0.709
	0.01	0.01	0.033	-0.031	-0.036	0.975	0.932
		0.04	0.140	-0.112	-0.147	0.910	0.756
		0.05	0.178	-0.135	-0.183	0.891	0.708

Table 3

Analysis of the results shows that, with decreasing $\frac{E^{(2)}}{E^{(1)}}$and increasing $\frac{E^{(2)}}{G_{12}^2}$the value of $\sigma_{nn}^+/\sigma_{11}^{(1),0}$is decreasing. Impact of $\frac{E^{(2)}}{G_{12}^2}$to the $\sigma_{nn}^+/\sigma_{11}^{(1),0}$is more significant than impact of

$\frac{E^{(2)}}{E^{(1)}}$. This conclusion is valid for all$\eta^{(2)}$. For all $\eta^{(2)}$increasing $\frac{E^{(2)}}{G_{12}^2}$reduces the absolute value of $\sigma_{nn}^{-}/\sigma_{11}^{(1),0}$. Increasing the value of $\frac{E^{(2)}}{G_{12}^2}$increases, decreasing of $\frac{E^{(2)}}{E^{(1)}}$decreases the absolute value of $\sigma_{n\tau}^{+}/\sigma_{11}^{(1),0}$. Increasing $\frac{E^{(2)}}{G_{12}^2}$and decreasing $\frac{E^{(2)}}{E^{(1)}}$increases the value of $\sigma_{\tau\tau}^{+}/\sigma_{11}^{(1),0}$and $\sigma_{\tau\tau}^{-}/\sigma_{11}^{(1),0}$. Impact of $\frac{E^{(2)}}{G_{12}^2}$to $\sigma_{\tau\tau}^{+}/\sigma_{11}^{(1),0}$and $\sigma_{\tau\tau}^{-}/\sigma_{11}^{(1),0}$is more significant than $\frac{E^{(2)}}{E^{(1)}}$.

In this paper problem of stress distribution in the composite materials with curved layers was investigated. Piecewise homogenous body model was considered, and by the use of three-dimensional linearized theory, stress distributions in the composite materials with antiphased locally curved layers, when external forces acted at infinity considered. Analyzing results show that obtained results agree with the results found in the previous investigations.

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