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One 3D contact-boundary value problem in the non-classical treatment and their application to the means of 3D technology in mathematical biology

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Abstract. In this paper substantiated for a higher-order equation with discontinuous coefficients a three dimensional contact-boundary value problem -3D contact-boundary value problem for the general case with non-classical boundary conditions is considered, which requires no matching conditions. Equivalence of these conditions three dimensional boundary condition is substantiated classical, in the case if the solution of the problem in the anisotropic S.L. Sobolev's space is found. The considered equation as a hyperbolic equation even in particular e.g. in the two-dimensional case generalizes not only classic equations of mathematical physics (heat-conductivity equations, string vibration equation) and also many models differential equations (telegraph equation, Aller's equation , moisture transfer generalized equation, Bianchi equation, Manjeron equation, Boussinesq-Love equation and etc.). It is grounded that the 3D contact-boundary conditions for the general case in the classic and non-classic treatment are equivalent to each other. Thus, namely in this paper, the non-classic problem with 3D contact-boundary conditions is grounded for a hyperbolic equation of higher- order. For simplicity, this was demonstrated for the general case in one of S.L. Sobolev anisotropic space $W_p^{(l_1,l_2,l_3)}(G)$.

Key Words and Phrases: 3D contact-boundary value problem, 3D mathematical modeling, mathematical biology, 3D technology, higher-order hyperbolic equations, equations with discontinuous coefficients, equations with dominating mixed derivative.

2000 Mathematics Subject Classifications: 35L25, 35L35

1. Introduction

Hyperbolic equations are attracted for sufficiently adequate description of a great deal of real processes occurring in the nature, engineering and etc. In particular, many processes arising in the theory of fluid filtration in cracked media are described by non-smooth coefficient hyperbolic equations.

Urgency of investigations conducted in this field is explained by appearance of local and non-local problems for non-smooth coefficients equations connected with different applied problems. Such type problems arise for example, while studying the problems of moisture, transfer in soils, heat transfer in heterogeneous media, diffusion of thermal neutrons in inhibitors, simulation of different biological processes, phenomena and etc.

It is evident that in this area 3D local and nonlocal boundary value problems for equations of mathematical biology and their application to the means of 3D technology in biology and medicine. Currently, three-dimensional technology-3D technology in the field of computer science finds wide application in medicine and biology. For example in medicine with great precision designs prostheses for patients.

In the present paper, for the general case here consider three dimensional contact-boundary problem- 3D contact boundary problem for higher- order hyperbolic equation with dominating mixed derivative. The coefficients in this hyperbolic equation are not necessarily differentiable; therefore, there does not exist a formally adjoint differential equation making a certain sense. For this reason, this question cannot be investigated by the well-known methods using classical integration by parts and Riemann functions or classical-type fundamental solutions. The theme of the present paper, devoted to the investigation 3D contact-boundary problem for higher-order differential equation with dominating mixed derivative of hyperbolic type, according to the above-stated is very actual for the solution of theoretical and practical problems. From this point of view, the paper even in particular, in the two-dimensional case is devoted to the actual problems of mathematical biology.

2. Problem statement

For the general case consider higher-order equation

$$(V_{l_1,l_2,l_3}u)(x) \equiv D_1^{l_1} D_2^{l_2} D_3^{l_3} u(x) + \sum_{i_1=0}^{l_1} \sum_{i_2=0}^{l_2} \sum_{i_3=0}^{l_3} a_{i_1,i_2,i_3}(x) D_1^{i_1} D_2^{i_2} D_3^{i_3} u(x) =$$

$$= \phi_{l_1,l_2,l_3}(x) \in L_p(G), \quad (a_{l_1,l_2,l_3}(x) \equiv 0). \tag{1}$$

Here $u(x) \equiv u(x_1, x_2, x_3)$ is a desired function determined on G; $a_{i_1,i_2,i_3}(x)$ are the given measurable functions on $G = G_1 \times G_2 \times G_3$, where $G_\zeta = \left(x_\zeta^0, h_\zeta\right)$, $\zeta = \overline{1,3}$; $\phi_{l_1,l_2,l_3}(x)$ is a given measurable function on G;$D_\zeta^\beta = \partial^\beta / \partial x_\zeta^\beta$ is a generalized differentiation operator in S.L.Sobolev sense, $D_\zeta^0$is an identity transformation operator.

Equation (1) is a hyperbolic equation with three multiple real characteristics$x_\zeta = const$, $\zeta = \overline{1,3}$.Therefore, in some sense we can consider equation (1) as a pseudoparabolic equation [1]. This equation is a Boussinesq-Love generalized equation from the vibrations theory [2] and Aller's equation under mathematical modeling [3, p.261] of the moisture absorption process in biology.

In the present paper equation (1) is considered in the general case when the coefficients $a_{i_1,i_2,i_3}(x)$ are non-smooth functions satisfying only the following conditions:

$$a_{i_1,i_2,i_3}(x) \in L_p(G), a_{l_1,i_2,i_3}(x) \in L_{\infty,p,p}^{x_1,x_2,x_3}(G),$$

$$a_{i_1,l_2,i_3}(x) \in L^{x_1,x_2,x_3}_{p,\infty,p}(G), a_{i_1,i_2,l_3}(x) \in L^{x_1,x_2,x_3}_{p,p,\infty}(G),$$

$$a_{l_1,l_2,i_3}(x) \in L^{x_1,x_2,x_3}_{\infty,\infty,p}(G), a_{i_1,l_2,l_3}(x) \in L^{x_1,x_2,x_3}_{p,\infty,\infty}(G),$$

$$a_{l_1,i_2,l_3}(x) \in L^{x_1,x_2,x_3}_{\infty,p,\infty}(G), i_\zeta = \overline{0, l_\zeta - 1},\ \zeta = \overline{1,3}.$$

Under these conditions, we'll look for the solution $u(x)$ of equation (1) in S.L.Sobolev anisotropic space

$$W_p^{(l_1,l_2,l_3)}(G) \equiv \left\{u(x) : D_1^{i_1} D_2^{i_2} D_3^{i_3} u(x) \in L_p(G),\quad i_\zeta = \overline{0, l_\zeta},\quad \zeta = \overline{1,3}\right\},$$

where $1 \le p \le \infty$. We'll define the norm in the space $W_p^{(l_1,l_2,l_3)}(G)$ by the equality

$$\|u\|_{W_p^{(l_1,l_2,l_3)}(G)} \equiv \sum_{i_1=0}^{l_1} \sum_{i_2=0}^{l_2} \sum_{i_3=0}^{l_3} \left\| D_1^{i_1} D_2^{i_2} D_3^{i_3} u \right\|_{L_p(G)}.$$

For hyperbolic equation (1) we can give the classic form 3D contact-boundary conditions in the form :

$$\left\{ \begin{array}{l} D_1^{i_1} u(x)\Big|_{x_1=h_1} = F_1^{i_1}(x_2,x_3), (x_2,x_3) \in G_2 \times G_3, i_1 = \overline{0, l_1 - 1}, \\ D_2^{i_2} u(x)\Big|_{x_2=x_2^0} = F_2^{i_2}(x_1,x_3), (x_1,x_3) \in G_1 \times G_3, i_2 = \overline{0, l_2 - 1}, \\ D_3^{i_3} u(x)\Big|_{x_3=h_3} = F_3^{i_3}(x_1,x_2), (x_1,x_2) \in G_1 \times G_2, i_3 = \overline{0, l_3 - 1}, \end{array} \right. \tag{2}$$

where $F_1^{i_1}(x_2,x_3)$, $F_2^{i_2}(x_1,x_3)$ and $F_3^{i_3}(x_1,x_2)$ are the given measurable functions on G. It is obvious that in the case of conditions (2), in addition to the conditions

$$F_1^{i_1}(x_2,x_3) \in W_p^{(l_2,l_3)}(G_2 \times G_3), i_1 = \overline{0, l_1 - 1},$$

$$F_2^{i_2}(x_1,x_3) \in W_p^{(l_1,l_3)}(G_1 \times G_3), i_2 = \overline{0, l_2 - 1},$$

and

$$F_3^{i_3}(x_1,x_2) \in W_p^{(l_1,l_2)}(G_1 \times G_2), i_3 = \overline{0, l_3 - 1},$$

the given functions should also satisfy the following agreement conditions:

$$\left\{ \begin{array}{l} D_2^{i_2} F_1^{i_1}(x_2,x_3)\Big|_{x_2=x_2^0} = D_1^{i_1} F_2^{i_2}(x_1,x_3)\Big|_{x_1=h_1}, \\ D_1^{i_1} F_3^{i_3}(x_1,x_2)\Big|_{x_1=h_1} = D_3^{i_3} F_1^{i_1}(x_2,x_3)\Big|_{x_3=h_3}, \\ D_3^{i_3} F_2^{i_2}(x_1,x_3)\Big|_{x_3=h_3} = D_2^{i_2} F_3^{i_3}(x_1,x_2)\Big|_{x_2=x_2^0}, \\ i_\zeta = \overline{0, l_\zeta - 1},\ \zeta = \overline{1,3}. \end{array} \right. \tag{3}$$

Consider the following non-classical boundary conditions :

$$\begin{cases} V_{i_1,i_2,i_3}u \equiv D_1^{i_1}D_2^{i_2}D_3^{i_3}u(x)/_{x_1=h_1,x_2=x_2^0,x_3=h_3} = \phi_{i_1,i_2,i_3} \in R,\ i_\xi = \overline{0,l_\xi-1}\,,\ \xi=\overline{1,3}; \\ (V_{l_1,i_2,i_3}u)(x_1) \equiv D_1^{l_1}D_2^{i_2}D_3^{i_3}u(x)/_{x_2=x_2^0,x_3=h_3} = \phi_{l_1,i_2,i_3}(x_1) \in L_p(G_1)\,,\ i_2=\overline{0,l_2-1}\,, \\ i_3=\overline{0,l_3-1}\,; \\ (V_{i_1,l_2,i_3}u)(x_2) \equiv D_1^{i_1}D_2^{l_2}D_3^{i_3}u(x)/_{x_1=h_1,x_3=h_3} = \phi_{i_1,l_2,i_3}(x_2) \in L_p(G_2)\,,\ i_1=\overline{0,l_1-1}\,, \\ i_3=\overline{0,l_3-1}\,; \\ (V_{i_1,i_2,l_3}u)(x_3) \equiv D_1^{i_1}D_2^{i_2}D_3^{l_3}u(x)/_{x_1=h_1,x_2=x_2^0} = \phi_{i_1,i_2,l_3}(x_3) \in L_p(G_1)\,,\ i_1=\overline{0,l_1-1}\,, \\ i_2=\overline{0,l_2-1}\,; \\ (V_{l_1,l_2,i_3}u)(x_1,x_2) \equiv D_1^{l_1}D_2^{l_2}D_3^{i_3}u(x)/_{x_3=h_3} = \phi_{l_1,l_2,i_3}(x_1,x_2) \in L_p(G_1\times G_2)\,, \\ i_3=\overline{0,l_3-1}\,; \\ (V_{i_1,l_2,l_3}u)(x_2,x_3) \equiv D_1^{i_1}D_2^{l_2}D_3^{l_3}u(x)/_{x_1=h_1} = \phi_{i_1,l_2,l_3}(x_2,x_3) \in L_p(G_2\times G_3)\,, \\ i_1=\overline{0,l_1-1}\,; \\ (V_{l_1,i_2,l_3}u)(x_1,x_3) \equiv D_1^{l_1}D_2^{i_2}D_3^{l_3}u(x)/_{x_2=x_2^0} = \phi_{l_1,i_2,l_3}(x_1,x_3) \in L_p(G_1\times G_3)\,, \\ i_2=\overline{0,l_2-1}. \end{cases} \tag{4}$$

3. Methodology

There with, the important principal moment is that the considered equation possesses nonsmooth coefficients satisfying only some p-integrability and boundedness conditions i.e. the considered hyperbolic operator V_{l_1,l_2,l_3} has no traditional conjugated operator. In other words, the Riemann function for this equation can't be investigated by the classical method of characteristics. In the papers [4]-[6] the Riemann function is determined as the solution of an integral equation. This is more natural than the classical way for deriving the Riemann function. The matter is that in the classic variant, for determining the Riemann function, the rigid smooth conditions on the coefficients of the equation are required.

The Riemann's method does not work for hyperbolic equations with discontinuous coefficients. Various classes of boundary value problems for differential and operator-differential equations with discontinuous coefficients investigated in the works [7]-[11] and etc. In the present paper, a method that essentially uses modern methods of the theory of functions and functional analysis is worked out for investigations of such problems. In the main, this method it requested in conformity to hyperbolic equations of higher-order with three multiple characteristics. Notice that, in this paper the considered equation even in particular e.g. in the two-dimensional case is a generation of many model equations of some processes (for example, heat-conductivity equations , telegraph equation, Aller's equation, moisture transfer generalized equation, three-dimensional Bianchi equation, Manjeron equation, string vibrations equations and etc).

If the function $u \in W_p^{(l_1,l_2,l_3)}(G)$ is a solution of the classical form 3D contact-boundary value problem (1), (2), then it is also a solution of problem (1), (4) for Z_{i_1,i_2,i_3}, defined by the following equalities:

$$\phi_{i_1,i_2,i_3} = D_2^{i_2} D_3^{i_3}\ F_1^{i_1}(x_2,x_3)\Big|_{\substack{x_2 = x_2^0 \\ x_3 = h_3}} = D_1^{i_1} D_3^{i_3} F_2^{i_2}(x_1,x_3)\Big|_{\substack{x_1 = h_1 \\ x_3 = h_3}} =$$
$$= D_1^{i_1} D_2^{i_2} F_3^{i_3}(x_1,x_2)\Big|_{\substack{x_1 = h_1 \\ x_2 = x_2^0}},\ i_\zeta = \overline{0, l_\zeta - 1},\ \zeta = \overline{1,3};$$

$$\phi_{l_1,i_2,i_3}(x_1) = D_1^{l_1} D_3^{i_3}\ F_2^{i_2}(x_1,x_3)\Big|_{x_3=h_3} = D_1^{l_1} D_2^{i_2} F_3^{i_3}(x_1,x_2)\Big|_{x_2=x_2^0},$$
$$i_2 = \overline{0, l_2 - 1},\ i_3 = \overline{0, l_3 - 1};$$
$$\phi_{i_1,l_2,i_3}(x_2) = D_2^{l_2} D_3^{i_3}\ F_1^{i_1}(x_2,x_3)\Big|_{x_3=h_3} = D_1^{i_1} D_2^{l_2} F_3^{i_3}(x_1,x_2)\Big|_{x_1=h_1},$$
$$i_1 = \overline{0, l_1 - 1},\ i_3 = \overline{0, l_3 - 1};$$

$$\phi_{i_1,i_2,l_3}(x_3) = D_2^{i_2} D_3^{l_3}\ F_1^{i_1}(x_2,x_3)\Big|_{x_2=x_2^0} = D_1^{i_1} D_3^{l_3} F_2^{i_2}(x_1,x_3)\Big|_{x_1=h_1},$$
$$i_1 = \overline{0, l_1 - 1},\ i_2 = \overline{0, l_2 - 1};$$

$$\phi_{l_1,l_2,i_3}(x_1,x_2) = D_1^{l_1} D_2^{l_2} F_3^{i_3}(x_1,x_2),\ i_3 = \overline{0, l_3 - 1};$$

$$\phi_{i_1,l_2,l_3}(x_2,x_3) = D_2^{l_2} D_3^{l_3} F_1^{i_1}(x_2,x_3),\ i_1 = \overline{0, l_1 - 1};$$

$$\phi_{l_1,i_2,l_3}(x_1,x_3) = D_1^{l_1} D_3^{l_3} F_2^{i_2}(x_1,x_3),\ i_2 = \overline{0, l_2 - 1}.$$

The inverse one is easily proved. In other words, if the function $u \in W_p^{(l_1,l_2,l_3)}(G)$ is a solution of problem (1), (4), then it is also a solution of problem (1), (2) for the following functions:

$$\begin{aligned} &F_1^{i_1}(x_2,x_3) = \textstyle\sum_{i_2=0}^{l_2-1}\sum_{i_3=0}^{l_3-1} \frac{(x_2-x_2^0)^{i_2}}{i_2!}\frac{(x_3-h_3)^{i_3}}{i_3!}\phi_{i_1,i_2,i_3} + \\ &+ \textstyle\sum_{i_3=0}^{l_3-1}\frac{(x_3-h_3)^{i_3}}{i_3!}\int_{x_2^0}^{x_2}\frac{(x_2-\mu_2)^{l_2-1}}{(l_2-1)!}\phi_{i_1,l_2,i_3}(\mu_2)d\mu_2 + \\ &+ \textstyle\sum_{i_2=0}^{l_2-1}\frac{(x_2-x_2^0)^{i_2}}{i_2!}\int_{h_3}^{x_3}\frac{(x_3-\mu_3)^{l_3-1}}{(l_3-1)!}\phi_{i_1,i_2,l_3}(\mu_3)d\mu_3 + \\ &+ \textstyle\int_{x_2^0}^{x_2}\int_{h_3}^{x_3}\frac{(x_2-\mu_2)^{l_2-1}}{(l_2-1)!}\frac{(x_3-\mu_3)^{l_3-1}}{(l_3-1)!}\phi_{i_1,l_2,l_3}(\mu_2,\mu_3)d\mu_2 d\mu_3,\ i_1 = \overline{0, l_1-1}; \end{aligned} \tag{5}$$

$$\begin{aligned} &F_2^{i_2}(x_1,x_3) = \textstyle\sum_{i_1=0}^{l_1-1}\sum_{i_3=0}^{l_3-1} \frac{(x_1-h_1)^{i_1}}{i_1!}\frac{(x_3-h_3)^{i_3}}{i_3!}\phi_{i_1,i_2,i_3} + \\ &+ \textstyle\sum_{i_3=0}^{l_3-1}\frac{(x_3-h_3)^{i_3}}{i_3!}\int_{h_1}^{x_1}\frac{(x_1-\gamma_1)^{l_1-1}}{(l_1-1)!}\phi_{l_1,i_2,i_3}(\gamma_1)d\lambda_1 + \\ &+ \textstyle\sum_{i_1=0}^{l_1-1}\frac{(x_1-h_1)^{i_1}}{i_1!}\int_{h_3}^{x_3}\frac{(x_3-\gamma_3)^{l_3-1}}{(l_3-1)!}\phi_{i_1,i_2,l_3}(\gamma_3)d\gamma_3 + \\ &+ \textstyle\int_{h_1}^{x_1}\int_{h_3}^{x_3}\frac{(x_1-\gamma_1)^{l_1-1}}{(l_1-1)!}\frac{(x_3-\gamma_3)^{l_3-1}}{(l_3-1)!}\phi_{l_1,i_2,l_3}(\gamma_1,\gamma_3)d\gamma_1 d\gamma_3,\ i_2 = \overline{0, l_2-1}; \end{aligned} \tag{6}$$

$$\begin{aligned}F_3^{i_3}(x_1,x_2) &= \textstyle\sum_{i_1=0}^{l_1-1}\sum_{i_2=0}^{l_2-1}\frac{(x_1-h_1)^{i_1}}{i_1!}\frac{(x_2-x_2^0)^{i_2}}{i_2!}\phi_{i_1,i_2,i_3}+\\ &+\textstyle\sum_{i_2=0}^{l_2-1}\frac{(x_2-x_2^0)^{i_2}}{i_2!}\int_{h_1}^{x_1}\frac{(x_1-\nu_1)^{l_1-1}}{(l_1-1)!}\phi_{l_1,i_2,i_3}(\nu_1)d\nu_1+\\ &+\textstyle\sum_{i_1=0}^{l_1-1}\frac{(x_1-h_1)^{i_1}}{i_1!}\int_{x_2^0}^{x_2}\frac{(x_2-\nu_2)^{l_2-1}}{(l_2-1)!}\phi_{i_1,l_2,i_3}(\nu_2)d\nu_2+\\ &+\textstyle\int_{h_1}^{x_1}\int_{x_2^0}^{x_2}\frac{(x_1-\nu_1)^{l_1-1}}{(l_1-1)!}\frac{(x_2-\nu_2)^{l_2-1}}{(l_2-1)!}\phi_{l_1,l_2,i_3}(\nu_1,\nu_2)d\nu_1d\nu_2,\; i_3=\overline{0,l_3-1}.\end{aligned} \tag{7}$$

Note that the functions (5)-(7) possess one important property, more exactly, for all ϕ_{i_1,i_2,i_3}, the agreement conditions (3) possessing the above-mentioned properties are fulfilled for them automatically. Therefore, equalities (5)-(7) may be considered as a general kind of all the functions $F_1^{i_1}(x_2,x_3)\,,F_2^{i_2}(x_1,x_3)$ and $F_3^{i_3}(x_1,x_2)$ satisfying the agreement conditions (3).

We have thereby proved the following assertion.

Theorem. *The 3D contact-boundary value problems of the form* (1), (2) *and the non-classical form* (1), (4) *are equivalent.*

Note that the 3D contact-boundary value problem in the non-classical treatment (1), (4) can be studied with the use of integral representations of special form for the functions $u\in W_p^{(l_1,l_2,l_3)}(G)$,

$$\begin{aligned}u(x) &= \textstyle\sum_{i_1=0}^{l_1-1}\sum_{i_2=0}^{l_2-1}\sum_{i_3=0}^{l_3-1}\frac{(x_1-h_1)^{i_1}}{i_1!}\frac{(x_2-x_2^0)^{i_2}}{i_2!}\frac{(x_3-h_3)^{i_3}}{i_3!}D_1^{i_1}D_2^{i_2}D_3^{i_3}u(h_1,x_2^0,h_3)+\\ &+\textstyle\sum_{i_2=0}^{l_2-1}\sum_{i_3=0}^{l_3-1}\frac{(x_2-x_2^0)^{i_2}}{i_2!}\frac{(x_3-h_3)^{i_3}}{i_3!}\int_{h_1}^{x_1}\frac{(x_1-\tau_1)^{l_1-1}}{(l_1-1)!}D_1^{l_1}D_2^{i_2}D_3^{i_3}u(\tau_1,x_2^0,h_3)d\tau_1+\\ &+\textstyle\sum_{i_1=0}^{l_1-1}\sum_{i_3=0}^{l_3-1}\frac{(x_1-h_1)^{i_1}}{i_1!}\frac{(x_3-h_3)^{i_3}}{i_3!}\int_{x_2^0}^{x_2}\frac{(x_2-\tau_2)^{l_2-1}}{(l_2-1)!}D_1^{i_1}D_2^{l_2}D_3^{i_3}u(h_1,\tau_2,h_3)d\tau_2+\\ &+\textstyle\sum_{i_1=0}^{l_1-1}\sum_{i_2=0}^{l_2-1}\frac{(x_1-h_1)^{i_1}}{i_1!}\frac{(x_2-x_2^0)^{i_2}}{i_2!}\int_{h_3}^{x_3}\frac{(x_3-\tau_3)^{l_3-1}}{(l_3-1)!}D_1^{i_1}D_2^{i_2}D_3^{l_3}u(h_1,x_2^0,\tau_3)d\tau_3+\end{aligned}$$

$$+\sum_{i_3=0}^{l_3-1}\frac{(x_3-h_3)^{i_3}}{i_3!}\int_{h_1}^{x_1}\int_{x_2^0}^{x_2}\frac{(x_1-\tau_1)^{l_1-1}}{(l_1-1)!}\frac{(x_2-\tau_2)^{l_2-1}}{(l_2-1)!}D_1^{l_1}D_2^{l_2}D_3^{i_3}u(\tau_1,\tau_2,h_3)d\tau_1d\tau_2+$$

$$+\sum_{i_2=0}^{l_2-1}\frac{(x_2-x_2^0)^{i_2}}{i_2!}\int_{h_1}^{x_1}\int_{h_3}^{x_3}\frac{(x_1-\tau_1)^{l_1-1}}{(l_1-1)!}\frac{(x_3-\tau_3)^{l_3-1}}{(l_3-1)!}D_1^{l_1}D_2^{i_2}D_3^{l_3}u(\tau_1,x_2^0,\tau_3)d\tau_1d\tau_3+$$

$$\begin{aligned}&+\textstyle\sum_{i_1=0}^{l_1-1}\frac{(x_1-h_1)^{i_1}}{i_1!}\int_{x_2^0}^{x_2}\int_{h_3}^{x_3}\frac{(x_2-\tau_2)^{l_2-1}}{(l_2-1)!}\frac{(x_3-\tau_3)^{l_3-1}}{(l_3-1)!}D_1^{i_1}D_2^{l_2}D_3^{l_3}u(h_1,\tau_2,\tau_3)d\tau_2d\tau_3+\\ &+\textstyle\int_{h_1}^{x_1}\int_{x_2^0}^{x_2}\int_{h_3}^{x_3}\frac{(x_1-\tau_1)^{l_1-1}}{(l_1-1)!}\frac{(x_2-\tau_2)^{l_2-1}}{(l_2-1)!}\frac{(x_3-\tau_3)^{l_3-1}}{(l_3-1)!}D_1^{l_1}D_2^{l_2}D_3^{l_3}u(\tau_1,\tau_2,\tau_3)d\tau_1d\tau_2d\tau_3.\end{aligned}$$

4. Result

So, the classical form 3D contact-boundary value problems (1), (2) and in non-classical treatment (1), (4) are equivalent in the general case. However, the 3D contact-boundary value problem in non-classical statement (1), (4) is more natural by statement than problem (1), (2). This is connected with the fact that in statement of problem (1), (4) the right sides of boundary conditions don't require additional conditions of agreement type. Note that some boundary-value problems in non-classical treatments for pseudo-parabolic and also hyperbolic equations were investigated in the author's papers [12]-[18].

5. Discussion and conclusions

In this paper a non-classical type 3D contact-boundary value problem for the general case is substantiated for a hyperbolic equation with non-smooth coefficients and with a higher-order dominating mixed derivative. Classic 3D contact-boundary conditions for the general case are reduced to non-classic 3D contact-boundary conditions by means of integral representations. Such statement of the problem has several advantages:

1) No additional agreement conditions are required in this statement;

2) One can consider this statement as a 3D contact-boundary problem for the general case formulated in terms of traces in the S.L. Sobolev anisotropic space $W_p^{(l_1,l_2,l_3)}(G)$;

3) In this statement the considered higher-order hyperbolic equation with dominating mixed derivative even in particular e.g. in the two-dimensional case is a generalization of many model equations of some processes (e.g. heat-conductivity equations , telegraph equation, Aller's equation, moisture transfer generalized equation, three-dimensional Bianchi equation, Boussinesq-Love equation, string vibrations equations and etc.).

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