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By the fractal geometry of the surface structures

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Abstract. Function and the van der Waerden used to assess the morphology of the surface structures of various systems. Them into account, the analysis of various elements of the scale nanostruktrurirovannyh semiconductor surfaces with the graph of the Van der Waerden.

Key Words and Phrases: fractals,non-differentiable,continuous,natural,iterative.

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The systems characterized by fractional dimension [1] are called fractals. At the study of fractal aggregates the computer modelling plays the special role so the possibility to consider the different modes of particle aggregation and their scale appears.

The numerical simulation leads to real aggregate structural organization on small scales.

The fractal models are usually constructed on the base of different mathematical algorithms with use of modern computer graphics [2].

In [1-4] the well known classic fractal functions: Weershtrass, Hankel, Bezikovich, Riman, Darbu, Kantor and Van-der-Varden have been considered.

The effective iterative process containing the several stages exists for the construction of the given function.

The aim of the work was to identify the signs of the fractal, the search for self-similar areas on the surfaces of the crystals on the basis of analysing the functions of Van der Waerden.

In 1875 Dubua-Reymon informed the World about continuous nowhere non-differentiable function constructed by K.Veyershtrass [1].

Let's give an example, its function is defined by$f(x) = \sum_{n=0}^{\infty} a^n \cdot \cos(b^n \pi\, x)$series where $0<a<1$, ab is uneven natural number (moreover$ab > 1 + \frac{3}{2}\pi$). This set is dominated by convergent progression $\sum_{n=0}^{\infty} a^n$consequently it converges equally and its sum is continuous function from x everywhere. Veyershtrass showed by detail investigation that the finite derivative doesn't exist in any its point.

Let's give the more simple example of Van-der-Varden[2] constructed on the same idea only oscillating curves$y = cos\omega x$ are exchanged by oscillating kinked curves. Let's designate the magnitude of difference between x number and nearest number to it through $u_0(x)$.This function is linear one in $\left[\frac{S}{2}, \frac{S+1}{2}\right]$interval where S is integral number; it is continuous one and has the period 1. The function plot presents itself the kinked curve, the separate elements of kinked curve have angular coefficient *±1*. For $k = 1, 2\ldots$ let's define the functions $u_k(x) = \frac{u_o(4^k \cdot x)}{4^k}$, these functions will be linear ones in $\left[\frac{S}{2\cdot 4^k}, \frac{S+1}{2\cdot 4^k}\right]$intervals, they are also continuous ones and have the period $\frac{1}{4^k}$. Its plot is also kinked curve but with small ripples. In all cases the angular coefficients of separate elements of kinked curve here are equal to *±1*.

Let's define the function *f(x)* for all real values of by$f(x) = \sum_{k=0}^{\infty} u_k(x)$equality. As it is obvious that$0 \leq u_k(x) \leq \frac{1}{2\cdot 4^k}(k = 0,\ 1,\ 2...)$then series is dominated by convergent progression$\sum_0^{\infty} \frac{1}{2\cdot 4^k}$(as in the case of Veyershtrass function), the series converges equally and *f(x)* function is continuous one everywhere.

Let's take any value of $x = x_0$. Calculating it with delicacy up to $\frac{1}{2\cdot 4^n}$ where n=0, 1, 2 ...)on defect and excess we put it between numbers of$\frac{S_n}{2\cdot 4^n} \leq x_0 < \frac{S_n+1}{2\cdot 4^n}$type where $Sn$is integral number. The closed intervals$\Delta_n = \left[\frac{S_n}{2\cdot 4^n}, \frac{S_n+1}{2\cdot 4^n}\right]$ *(n=0, 1, 2, ...)* are put the one into another. In each of them there is such point x_n that its distance from x_o point is equal to half of interval length:$|x_n - x_o| = \frac{1}{4^{n+1}}$, it is clear that $x_n > x_o$ with increase of n variant.

Let's $\frac{f(x_n)-f(x_o)}{x_n-x_o} = \sum_{k=o}^{\infty} \frac{u_k(x_n)-u_k(x_o)}{x_n-x_o}$. At $k > n$ number $\frac{1}{4^{n+1}}$ is whole multiple of $\frac{1}{4^k}$ period of $u_k(x)$ function so $u_k(x_n) = u_k(x_o)$, the corresponding members of series transform into zero and can be excluded. If $k < n$ then $u_k(x)$ function is linear one in Δk interval, it will be linear one in Δn interval including in it, moreover$\frac{u_k(x_n)-u_k(x_o)}{x_n-x_o} = \pm 1$ (k=0, 1, ..., n).

Then finally we have $\frac{f(x_n)-f(x_o)}{x_n-x_o} = \sum_{k=0}^{n}(\pm 1)$, by other words this relation is equal to even integral number at uneven n and to uneven integral number at even n.

It is clear that at the difference relation not to any finite bound can't strive for, this means that our function at $x = x_0$ doesn't have the finite derivative.

Thus Van-der-Varden function is simpler than known Veyershtrass function. On the base of [2-3] it is established that Van-der-Varden function is non-differentiable in all points.

The previous history of these questions is the following.In second half of previous century the representatives of mathschool criticizing the analysis basics and firstly Veyershtrass and Piano constructed the continuous functions not having derivatives everywhere and curves everywhere tightly filled the square.

The fractal conceptions in physical and natural structures have been earlier studied in [3].It is shown that fractal systems form the series of objects which can be described by functions introduced by Weershtrass and Van-der-Varden [6].

Let's *fo(x),* absolute difference between x value and nearest to it integral value

$$f\,(x) = \begin{cases} x \quad at\ \ 0 \leq \frac{1}{2} \\ 1 - x, \ \ at\ \ \frac{1}{2} \leq x \leq 1 \end{cases}$$

This $f_0(x)$ function is linear on each interval $\left[\frac{s-1}{2}; \frac{s}{2}\right]$ where s is integral number; continuous one on the whole number axis, periodic one with period 1. Function graph is polygonal line the separate sections of which have the angular coefficient ± 1. Let's define the function $f_n = \frac{f_0(4^n \cdot x)}{4^n}$ n>1, 2... These functions are linear in interval $\left[\frac{S_n}{2\cdot 4^n}, \frac{S_n+1}{2\cdot 4^n}\right]$. They are continuous also and have the period$\frac{1}{4^n}$. The developed iterative formula of Van-der-Varden function calculation has the form:

$$V(x) = \lim_{m\to\infty} \sum_{i=0}^{m} \sum_{j=0}^{i-1} F\left(4^i, x - \frac{j}{i}\right),\ F(a,x) = \begin{cases} x & at\ 0 \le ax < \frac{1}{2} \\ \frac{1}{a} - x, & at\ \frac{1}{2} \le ax \le 1 \end{cases}$$

Note than the further iterations without change of diagram scale made by us show the fit of function graph to o_x axis.

Thus, the considered structures have the determined fractal character. The self-similarity caused by the method peculiarities of their generation is their distinguishing characteristic. The self-similarity reveals on all levels. The determined fractals form in iteration process. They can be described by Van-der-Varden function.

Conclusion

The attention on functions which aren't smooth or regular providing the essentially better presentation of many natural phenomena is paid. The fractal perception is used at analysis of morphology of snake flake surface and morphology of interlayer crystal surface by telluride bismuth type. In both cases the structures look like to fractals of surrounding us nature.

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