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A comparative study of MHD flow Analysis in a Porous Medium by Using Differential Transformation Method and Variational Iteration Method

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Abstract. The purpose of the current study is to examine Magnetohydrodynamic (MHD) boundary layer flow having magnetic characteristics and behavior of electrically transmitted viscous incompressible fluids over a stretching sheet fixed in a porous medium.This porosity on the flow field is typically analyzed under the effects of magnetic field(MF) and permeability. The results for various prandtl numbers on temperature profile has also been observed. By using Darcy Model in order to a variable MF, we have considered the flow of a viscous fluid through porous media. The series solution of the non-linear boundary layer problem of flow field is obtained by using DTM as well as by VIM along with Pade approximant(PA).For solving BVPs the combination of DTM-Pade and VIM-Pade approximation is shown to be a powerful techniques.The methods are effective and convenient subject to the comparison of the obtained results and with already available results shown in the literature.It is observed that both magnetic field and the permeability parameter of porous medium impart to thinning of the boundary layer.Same in the case of thermal effects,temperature profile decreases with the increment of Prandtl number.

Key Words and Phrases: Magnetohydrodynamics, DTM, VIM,Pade-approximation, Porous Medium, Magnetic field.

1. Introduction

The aim of the current work is to investigate porous medium for steady-state incompressible viscous flow of an electrically conducting fluids under the influence of MF. For this purpose, to overcome the difficulties concerning the closed form solution in non linear boundary value problem we have used semi-analytical methods name as Differential transformation method and Variational iteration method with the combination of PÀ[1]. Although there exists many other research studies with great interest have largely been used such as Perturbation techniques , Homotopy P̀erturbation method (H̀PM),Homotopy Ànalysis methods(H̀AM),Àdomian D̀ecomposition method(ÀDM) and Successive approximation method. But this work mainly focuses on the behavior of DTM ,VIM and PÀ [2, 3]. For the solution of linear and non linear Differential equations (NLDE) these methods are strongly effective and reliable which can be directly applied to NLDE in physics, engineering and mathematics.They do not require any linearization and perturbation.Although these methods has some drawbacks , they are applicable in the small region but invalid in an unbound domain.To overcome this issue we have consider PÀ . By using the above mentioned methods we obtain the solution in the form of series, the series may diverge when the variable involved in the problem approaches to infinity. For the settlement of this situation a combination of PÀ [2]has been used to get approximate solutions.

In the manifestation of a MF the flow of an electrically conducting fluids having much momentousness in multiple disciplines of technology and engineering such as M̀HD pumps, M̀HD power generation and M̀HD flow meter etc.In many extents of engineering and industrial areas of interest the porous media flow plays a significant role [3]. Moreover,in polymer industries wide applications are to be found in particular flow on a stretching sheet. A little while back,Mohammadreja et.al. and Peker et. al.have studied viscous fluid flows on a stretching sheet with a constant rate of stretching and the fluid flow in order to a variable MF. In their work, they did not consider the porous media.Later,a stretching sheet with uniform matrix subjected to a MF strength fixed in a porous medium propotional to $x^{(n-1)/2}$ and non linear stretching sheet x^n had been discussed[4]. Many authors used to consider the strength of MF as constant. In the current work we have focused on the following two main objectives; the first one is the study of wedge flow through porous media subjected to a MF leads towards a modified model of Falkner-skan equation while the second purpose is to investigavte the stability of the DTM and VIM empowered with PÀ[10, 12]. The following are the Prandtl Boundary layer(B.l) D̀arcian flow equations,[1]

$$u_x + v_y = 0 \tag{1}$$

$$uu_x + vu_y = \nu u_{xx} - \sigma\frac{D^2(x)u}{\rho} - \frac{u\nu}{kp(x)} \tag{2}$$

$$uT_x + vT_y = \alpha T_{yy} \tag{3}$$

variable magnetic field $D(x)$ and $kp(x)$ variable Porosity are as follows,
$D(x) = D_0(x)x^{\frac{n-1}{2}}$, $kp(x) = kp'x^{1-n}$
subject to the boundary conditions(BCs),

$$\begin{cases} y = 0, \\ u = cx^n, v = 0, T = T_w, \\ y \to \infty, \\ u \to 0, T \to T(\infty), \end{cases}$$

Where c is stretching rate. The continuity equation is satisfied by choosing $\psi(x, y)$ as a stream f̀unction which is,

$u = \psi_y$

and $v = -\psi_x$

Introducing similarity transformation,

$$\begin{cases} \eta(x,y) = x^{\frac{n-1}{2}} y \sqrt{\frac{c(n+1)}{2\nu}}, \\ \psi(x,y) = x^{\frac{n+1}{2}} f(\eta) \sqrt{\frac{2c\nu}{(n+1)}} \end{cases}$$

$$u = cx^n f' \tag{4}$$

$$v = -\sqrt{\frac{c\nu(n+1)}{2}} x^{\frac{n-1}{2}} [f + (\frac{n-1}{n+1})\eta f'] \tag{5}$$

so the transformed ODEs along their boundary conditions are

$$f''' - (M + \frac{1}{kp})f' + ff'' - \beta f'^2 = 0 \tag{6}$$

$$\frac{1}{Pr}\theta'' + \theta' f = 0, \tag{7}$$

$$\begin{cases} f(0) = 0, f'(0) = 1, f''(0) = 2\alpha(say), f'(\infty) = 0, \theta(0) = 1, \theta(\infty) = 0, \theta'(0) = A(say), \end{cases}$$

$M = \frac{2\sigma D_0^2(x)}{\rho c(n+1)}$, and $\frac{1}{kp} = \frac{2\nu}{c(n+1)kp'}$,

where M stands for the magnetic parameter, kp stands for permeability parameter , β is power index,
α and A are unknowns to be determined.

1.1. Differential Transformation Method

Differential transformation method (DTM) was purposed by $Zhou$.The method was developed to tackle initial value problems (IVP) in electric circuit theory. DTM is based on Taylor series expansion.In this method we have applied some transformation rules. The set of fundamental equations are reduced to Ordinary Differential equations(ODEs), then these equations along with boundary conditions are transformed under the rules described by DTM to yields the desired solution of the problem. Basic definitions and operations of DTM are introduced for the function $f(\eta)$ as given below,[13]

$$F(r) = \frac{1}{r!}\frac{d^r}{d\eta^r} f(\eta), \tag{8}$$

Where basic original function is $f(\eta)$ while transformed function is $F(r)$ known as Spectrum of $f(\eta)$ at $\eta = \eta_0$ in the r^{th} domain. Also the inverse of $F(r)$ is given as,

$$f(\eta)=\sum_{r=0}^{\infty}F(r)(\eta-\eta_0)^r, \quad (9)$$

On formulation of equation (10) and (11) $f(\eta)$ is,

$$f(\eta)=\sum_{r=0}^{\infty}[\frac{d^r}{d\eta^r}f(\eta)]_{(\eta=\eta_0)}\frac{(\eta-\eta_0)^r}{r!}, \quad (10)$$

Although the method is based on Taylor series èxpansion but it does not s̀ymbolically calculate the derivatives .So, the desired derivatives are obtained by an iterative method,

In fact, $f(\eta)$ in equation (9) is expressed by a finite series which c̀an be formulated as,

$$f(\eta)=\sum_{r=0}^{N}F(r)(\eta-\eta_0)^r, \quad (11)$$

where N is a series size. here we have considered the differential t̀ransformed f̀unction(DTF) about the point $\eta = 0$ in table-1 and it is assumed that $\eta_0 = 0$ in this section.

1.2. V̀ariational Ìteration M̀ethod (VIM)

The VIM was proposed by Ji-Huan-He which is a modified general L̀agrange multiplier used to handle a variety of homogeneous,inhomogeneous ,linear and non-linear,problems with approximations which rapidly converges to exact solution.This method tackle both linear and nonlinear problems with same manners and this method is free from any specification, linearization and purturbation such as ÀDM, H̀PM and H̀AM. The V̀IM gives the series solution that converges to closed form solution if an analytical solution exists.If the analytical solution does not exist then the computed series is used for numerical purposes.[3, 4] The main steps of the methods are as follows: consider the non-linear equation

$$L_1g+M_1g=y(t) \quad (12)$$

Where L_1 is linear and M_1 is non-linear operators and y(t) represents the inhomogeneous term. The correction functional of above equation is

$$g_{n+1}(x)=g_n(x)+\int_0^x\lambda(t)(L_1g_n(t)+M_1g_n(t)-y(t))dt \quad (13)$$

Where λ is a Lagrange-multiplier and it may be constant or a function and g_n is a restricted to behave as a constant. The initial guess can be taken as follows

$$\begin{cases} g_0(x) = g(0), forg'_n \\ g_0(x) = g(0) + xg'(0), forg''_n \\ g_0(x) = g(0) + xg'(0) + \frac{1}{2!}x^2 g''(0), forg'''_n \\ \dots \\ \dots \end{cases}$$

the compact form of the solution is

$$g(x) = \lim_{n \to \infty} g_n(x) \tag{14}$$

1.3. P̀ade-Àpproximation

Suppose a p̀ower s̀eries $\sum_{r=0}^{\infty} \hat{a}_r t^r$ r̀epresenting a f̀unction $f(\eta)$, this implies,

$$f(\eta) = \sum_{r=0}^{\infty} \hat{a}_r t^r, \tag{15}$$

Basically,PÀ is a r̀ational f̀unction and the notation for such PÀ is,

$$[I, J] = \frac{S_I(t)}{Q_J(t)}, \tag{16}$$

where $S_I(t)$ is a polynomial of degree at most I and $Q_J(t)$ is a polynomial of degree at most J,we have,

$$f(\eta) = \hat{a}_0 + \hat{a}_1 t + \hat{a}_2 t^2 + \hat{a}_3 t^3 + \hat{a}_4 t^4 + ..., \tag{17}$$

$$S_I(t) = \hat{s}_0 + \hat{s}_1 t + \hat{s}_2 t^2 + \hat{s}_3 t^3 + \hat{s}_4 t^4 + ... + \hat{s}_I(t)^I, \tag{18}$$

$$Q_J(t) = \hat{q}_0 + \hat{q}_1 t + \hat{q}_2 t^2 + \hat{q}_3 t^3 + \hat{q}_4 t^4 + ... + \hat{q}_J(t)^J, \tag{19}$$

equation (16) reveals that $I+1$ numerator coefficients and $J+1$ denomenator coefficients are there. since we can clearly multiply numerator and denomenator by a constant and $[I, J]$ remains unvaried Now imposing the normalized conditions,[8, 9]

$$Q_J(0) = 1 = \hat{q}_0 \tag{20}$$

so, there are $I+1$ independent numerator coefficients and J independent denomenator coefficients making $I+J+1$ unknown coefficients.This number gives $[I, J]$ ought to fit the power series with the orders,

$1, t, t^2, t^3, t^4, ..., t^{I+J},$

The formal power series is,

$$\sum_{r=0}^{\infty} \hat{a}_r t^r = \frac{\hat{s}_0 + \hat{s}_1 t + \hat{s}_2 t^2 + \hat{s}_3 t^3 + \hat{s}_4 t^4 + ... + \hat{s}_I (t)^I}{\hat{q}_0 + \hat{q}_1 t + \hat{q}_2 t^2 + \hat{q}_3 t^3 + \hat{q}_4 t^4 + ... + \hat{q}_J (t)^J} + O(t^{I+J+1}), \tag{21}$$

by cross multiplication and comparing we will get the following set of equations,

$$\begin{cases} \hat{a}_0 = \hat{s}_0, \\ \hat{a}_1 + \hat{a}_0 \hat{q}_1 = \hat{s}_1, \\ \hat{a}_2 + \hat{a}_1 \hat{q}_1 + \hat{a}_0 \hat{q}_2 = \hat{s}_2, \\ . \\ . \\ \hat{a}_I + \hat{a}_{I-1} \hat{q}_1 + ... + \hat{a}_0 \hat{q}_I = \hat{s}_I \end{cases} \tag{22}$$

and

$$\begin{cases} \hat{a}_{I+1} + \hat{a}_I \hat{q}_1 + ... + \hat{a}_{I-J+1} \hat{q}_J = 0, \\ \hat{a}_{I+2} + \hat{a}_{I+1} \hat{q}_1 + ... + \hat{a}_{I-J+2} \hat{q}_J = 0, \\ . \\ . \\ \hat{a}_{I+J} + \hat{a}_{I+J-1} \hat{q}_1 + ... + \hat{a}_I \hat{q}_J = 0, \end{cases} \tag{23}$$

$\hat{a}_n = 0$ for $n < 0$ and $\hat{q}_{M_1} = 0$ for $M_1 > J$. for non-singular solution of equation (22) and (23) then the direct solution is,

$$[I,J]=\frac{\begin{vmatrix}\hat{a}_{I-j+1} & \hat{a}_{I-j+2} & \cdots & \hat{a}_{I+1}\\ \vdots & \vdots & \ddots & \vdots\\ \hat{a}_{I} & \hat{a}_{I+1} & \cdots & \hat{a}_{I+J}\\ \sum_{M_1=J}^{I}\hat{a}_{M_1-J}t^{I} & \sum_{M_1=J-1}^{I}\hat{a}_{M_1-J+1}t^{M_1}\cdots & \sum_{M_1=0}^{I}\hat{a}_{M_1}t^{M_1} & \end{vmatrix}}{\begin{vmatrix}\hat{a}_{I-j+1} & \hat{a}_{I-j+2} & \cdots & \hat{a}_{I+1}\\ \vdots & \vdots & \ddots & \vdots\\ \hat{a}_{L_1} & \hat{a}_{L_1+1} & \hat{a}_{I+J} & \\ t^{J} & t^{J-1} & \cdots & 1\end{vmatrix}} \tag{24}$$

In PÀ , every choice of I and J (the degree of numerator and denominator respectively) leads to an approximant. In order to find a best approximant we have considered $I = J$ to avoid any complexity regarding the shape of the solution. We directly apply PÀ about the point $x = 0$, by using the s̀ymbolic c̀omputational s̀oftware *MAPLE*. It is essential to get the best convergence of the truncated series to employ the PÀ,without using this technique the semi-analytical solution obtained by VIM and DTM can not satisfy the boundary conditions in an infinite domain. Although to get desired accuracy ,the higher order approximation procedure is required,[5, 6]

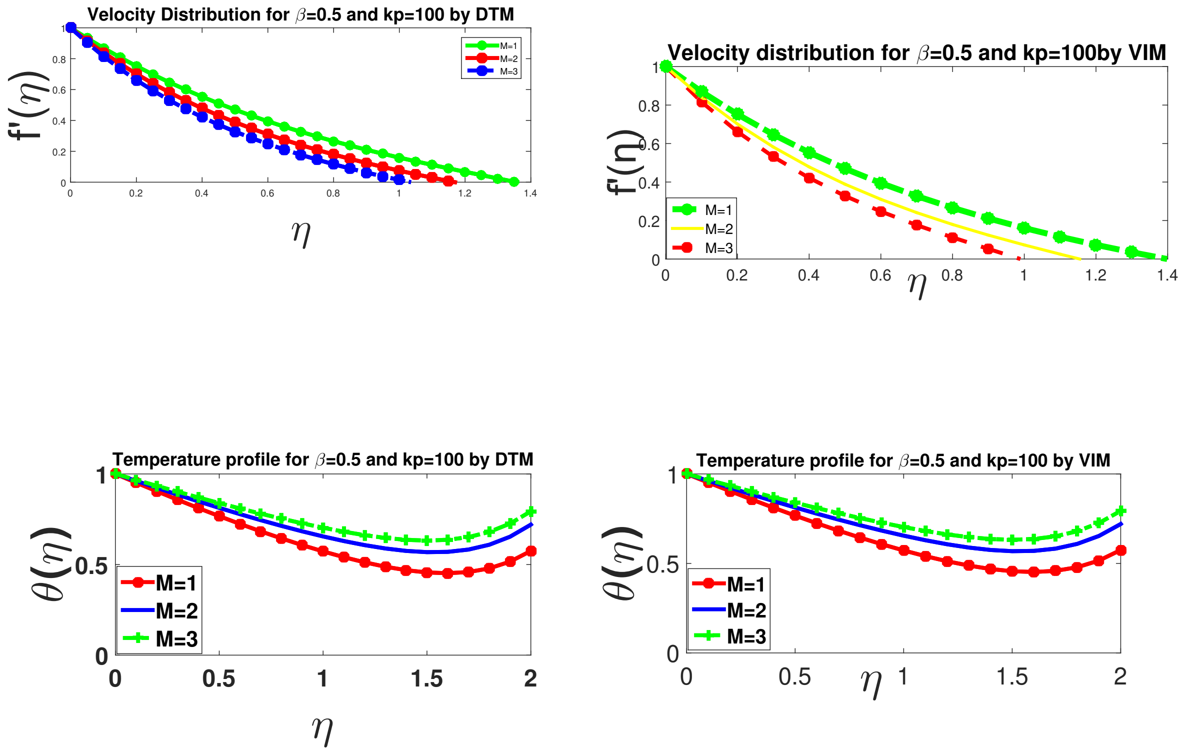

Figure 1: Velocity and Temperature profile for various values of Magnetic parameter M by DTM and VIM

Table-1: Some fundamental operations of DTM

$Sr.no$	$Orignal\ Function$	$Transformed\ Function$
1	$s(\eta) = \lambda t(\eta)$	$S(k) = \lambda T(k)$
2	$s(\eta) = \alpha t(\eta) \pm \beta u(\eta)$	$S(k) = \alpha T(k) \pm \beta U(k)$
3	$s(\eta) = \frac{d^r}{d\eta^r} t(\eta)$	$S(k) = \frac{(k+r)!}{k!} T(k+r)$
4	$s(\eta) = t(\eta)u(\eta)$	$S(k) = \sum_{r=0}^{k} T(r)U(k-r)$
5	$s(\eta) = t(\eta)\frac{d^2}{d\eta^2}u(\eta)$	$\sum_{r=0}^{k}(k-r+1)(k-r+2)T(r)U(k-r+2)$

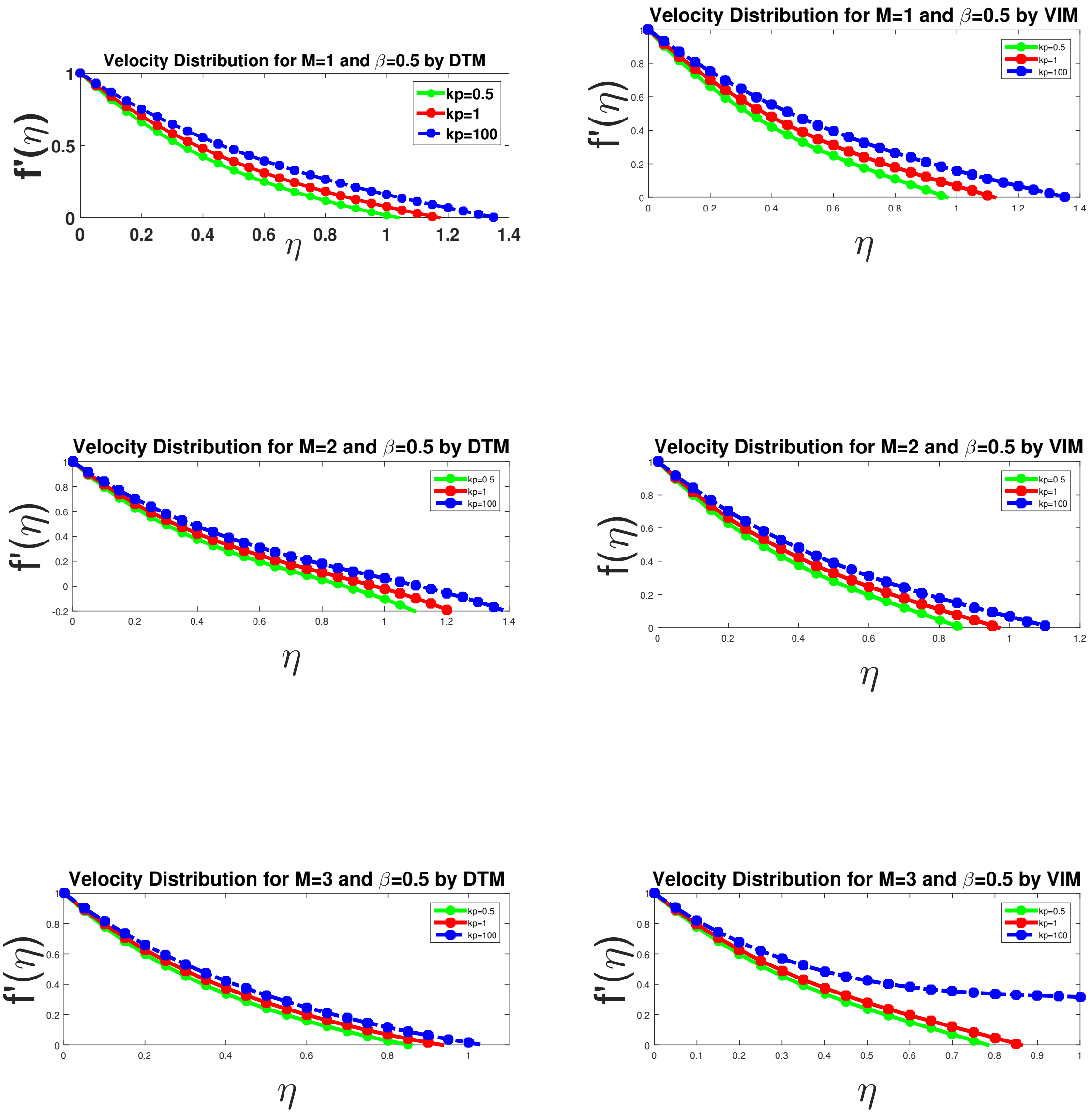

Figure 2: Velocity profile kp=0.5,kp=1,kp=100 by DTM and VIM

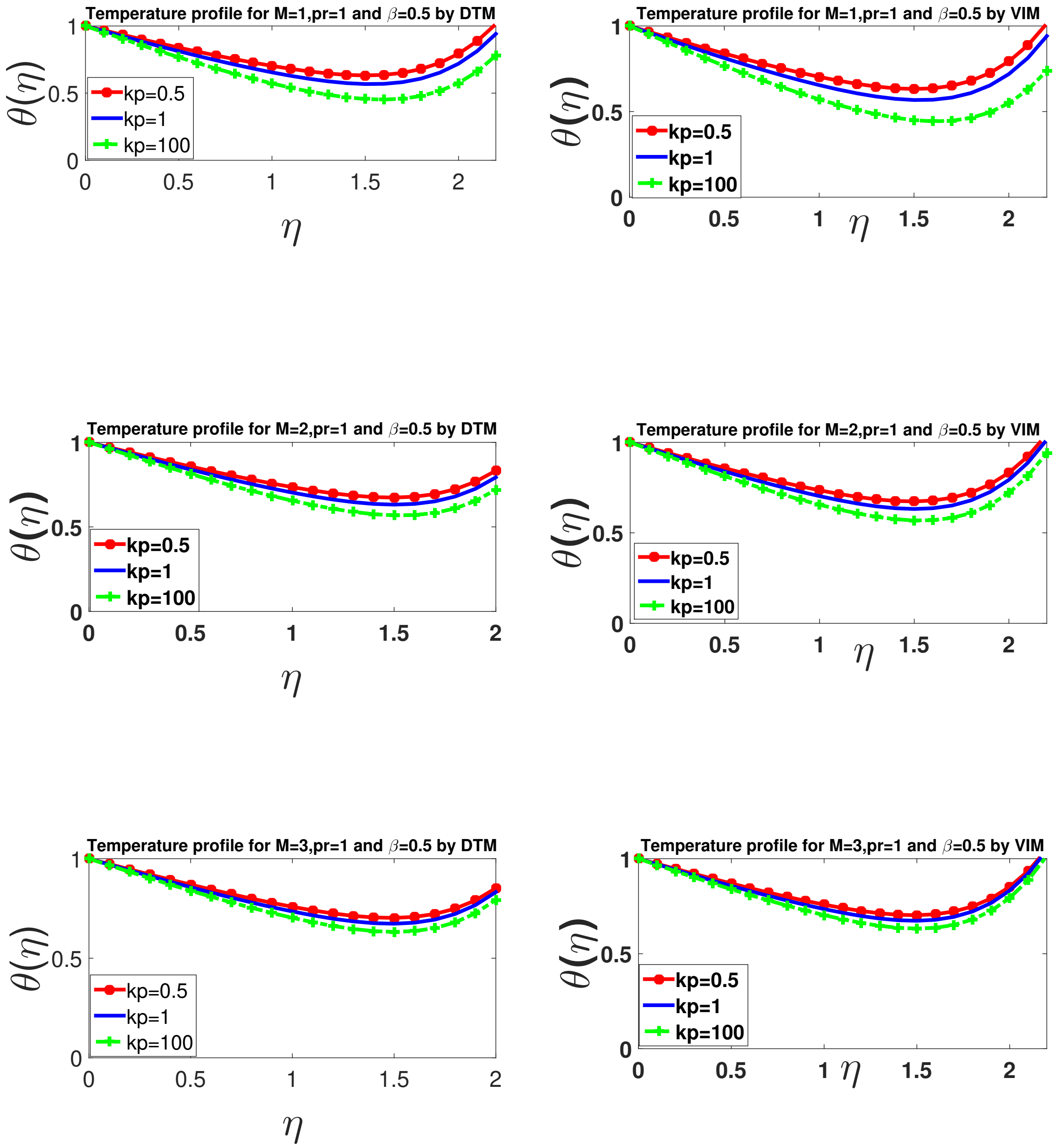

Figure 3: Temperature profile kp=0.5,kp=1,kp=100 by DTM and VIM

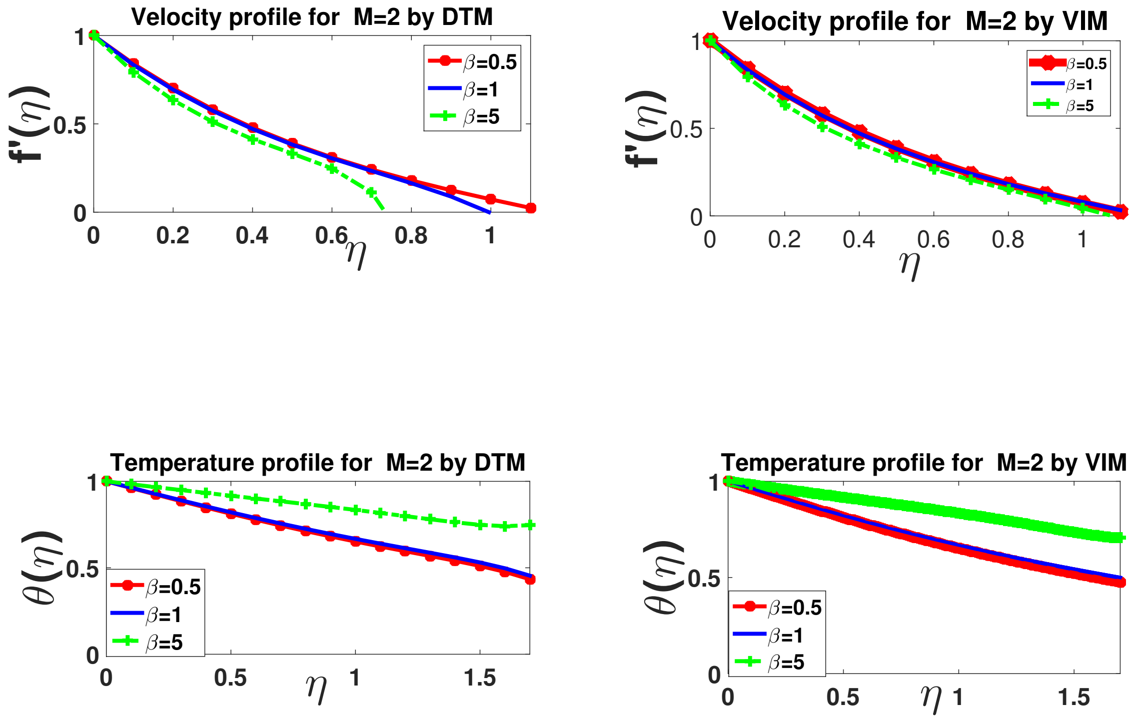

Figure 4: Velocity and Temperature profile for various values of β when M=2 by DTM and VIM

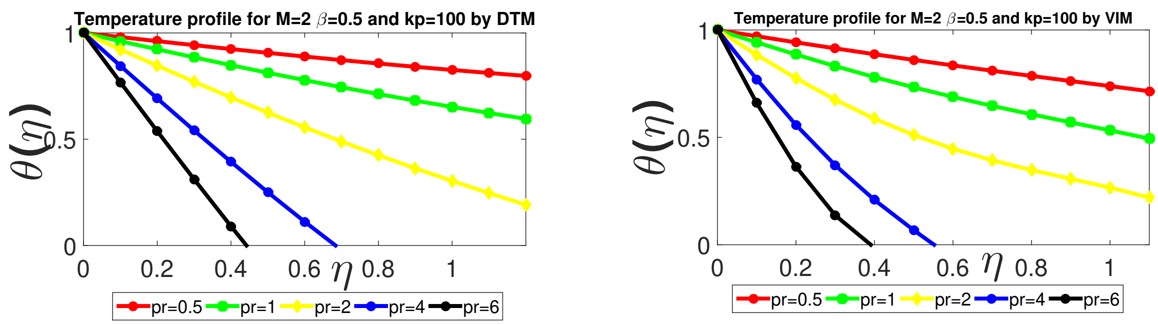

Figure 5: Temperature profile for different prandtl numbers $M = 2$, $\beta = 0.5$ and $kp = 100$ by DTM and VIM

Table-2: Comparison between the numerical results of velocity distribution obtained by DTM and VIM when $M = kp = \beta = 0.5$

η	$f(\eta)$,DTM	$f(\eta)$,VIM
0.0	0.0000000000	0.0000000000
0.1	0.0911529382	0.0911529382
0.2	0.1663892400	0.1663892427
0.3	0.2280668237	0.2280668491
0.4	0.2781691442	0.2781691674
0.5	0.3183491031	0.3183484809
0.6	0.3499694111	0.3499648276
0.7	0.3741380536	0.3741186098
0.8	0.3917372430	0.3916754343
0.9	0.4034433232	0.4032818451
1.0	0.4097333975	0.4093706527

Table-3: Numerical analysis of results obtained by DTM and VIM when

$M = kp = \beta = 0.5$

η	$f'(\eta)$,DTM	$f'(\eta)$,VIM	$\theta(\eta)$,DTM	$\theta(\eta)$,VIM
0.0	1.00000	1.0000000000	1.0000000000	1.0000000000
0.1	0.8276814941	0.8276814963	0.9643221086	0.9643221087
0.2	0.6809667103	0.6809667961	0.9289630514	0.9289630519
0.3	0.5558833678	0.5558837292	0.8941832201	0.8941831843
0.4	0.4489111212	0.4489099215	0.8601839810	0.8601837706
0.5	0.3569532942	0.3569376075	0.8271142983	0.8271135457
0.6	0.2772946240	0.2772191571	0.7950776250	0.7950758469
0.7	0.2075451639	0.2072965134	0.7641377641	0.7641355735
0.8	0.1455646023	0.1449166342	0.734217077	0.7342545255
0.9	0.08935417176	0.08793396542	0.7056161362	0.7056511922
1.0	0.03689496702	0.03419985184	0.6779519531	0.6780950993

Table-4: Analysis of s̀kin fricţion c̀oefficient $f''(0)$ and and the rate of heat transfer $\theta'(0)$ by DTM and VIM

β	M	kp	$f''(0)$,DTM	$f''(0)$,VIM	$\theta'(0)$,DTM	$\theta'(0)$,VIM
0.5	1	100	-1.381589474	-1.381589474	-0.7238040090	-0.7238040090
0.5	2	100	-1.720476298	-1.720476298	-0.5812343950	-0.5812343950
0.5	3	100	-2.005888928	-2.005888928	-0.4985320902	-0.4985320902
1	2	100	-1.801186973	-1.801186973	-0.5551894470	-0.5551894470
5	2	100	-2.389820330	-2.389820330	-0.4184414985	-0.4184414985
0.5	3	0.5	-2.479707362	-2.479707362	-0.4032733924	-0.4032733924

1.4. Results and Discussion

Under the usual Boundary layer approximations, the boundary layer equations such as continuity equation,momentum equation and energy equation are transformed in to ODEs. This set of nonlinear ODEs along with the boundary conditions are calculated by using semi-analytical techniques DTM and VIM by systematic guessing of $f''(0)$ and $\theta'(0)$ by PÀ until the boundary conditions are satisfied at infinity with the help of computer softwares. Figure (1) by DTM and VIM depicts that the velocity and temperature profiles for different values of Magnetic parameter. From graphical representation it is observed that in the boundary layer region, the velocity profiles decreases when the magnetic parameter M increases. This result validates the physical behavior of Magnetic field. But transverse effects are observed on the Temperature profiles for varoius values of Magnetic parameter M.Clearly using the two methods it can be seen in figures,as the M increases through 1 to 3 the temperature boundary layer thickness also increases. It is also noticed that graphically both the methods are in well agreement. In figures 2 and 3 when the permeability parameter kp increases from 0.5 to 100 the velocity profile thickness increases due to more injection or suction of the fluid. But increase in the permeability parameter (kp) leads to decrease in fluid temperature profile,which is due the fact the *Darcian* body force transfers heat from solid surface to the fluid layers.The DTM and VIM shows g̀ood agreement that can be seen in velocity profile as well as in temperature profiles.The velocity profile is also calculated for various values of β as shown in figure 4 by applying the two methods under consideration and it is observed that the velocity profile reduces at all points when β varies from 0.5 to 5.These values of β leads to ǹon-linear v̀ariation of p̀late velocity as well as MF strength. While $\beta = 1$ corresponds the linear v̀ariation of velocity as well as constant MF.From this we conclude that, to increase the fluid velocity, v̀ariation of plate velocity contributes more than the MF strength.Also it is useful to mention that the temperature profile increases with the increment of the values of β. Effects of Prandtl number on thermal boundary layer can be seen via figure.5 which reveals that fluid with larger prandtl number have a thinner thermal boundary layer,due to more heat transfer between the stretching sheet and viscous fluid.In Table (2) the skin friction coefficient is also computed by DTM and VIM.It is observed that in the existence of porous media and MF the magnitude of skin friction increases but adverse effects are to be seen due to the power index of MF.

1.5. Conclusion

In this paper, DTM and VIM along with PÀ has been employed to investigate the heat and mass transfer of a steady MHD flow. We have seen that the obtained results are in good agreement with the previous work. The velocity and temperature profiles are obtained by DTM and VIM along with PÀ.In this work to avoid the complexity,we have considered the diagonal pade [2/2] that is the about η^5.It is quite obvious as the order increases it will yeild better approximation and produces higher accuracy.It is observed that due to resistive force of electromagnetic which is lorentz force the velocity decreases.Also the velocity profile at all the points reduces under the influence of power index and increases with the increase in the permeability parameter. Due to magnetic field and permeability the shear stress increases but the power index of magnetic field shows reverse effects.Same as in the case of thermal profile,an increase in magnetic parameter M will increase the thermal boundary layer but reverse effects are to be seen in the case of permeability parameter (kp) Also as expected,by increasing the prandtl number the temperature profile gets thinner.

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