Journal of Contemporary Applied Mathematics
V. 3, No 1, 2013, September
ISSN 2222-5498

Defining Optimum Regime in the Production Process Described Differential Equations with Variable Structure

B.M.Gasimov*, E.R.Shafizade, A.A.Murtuzayeva

Abstract. In this work considering an economical problem formulation as fuzzy optimal control problem. For investigation this problem at first define the space of the fuzzy numbers and derivative of the fuzzy function. Avter for this problem obtained optimality condition.

Key Words and Phrases: economical problem, fuzzy optimal control problem, fuzzy number, fuzzy function.
2000 Mathematics Subject Classifications: 49J15, 34A07

1. Introduction

Production dynamics of the enterprise is often characterized by a nonlinearity. Especially, there is a high rate of development in the early stages of production, but then it decreases. In this case the production function will be in the following form

$$P(t) = P_0 + \bar{p}(1 - e^{-A(t)})$$

Here, $P_0 = P(0)$ the early stage of production, $\bar{p}$ is saturation level. Then, the dynamics of the main producing assets are expressed in the following equation

$$\frac{dA(t)}{dt} = a_1(t) - a_2(t)e^{-A(t)} + I(t) + m\delta(t - t_0),$$

where

$$a_1(t) = a(t)(P_0 + \bar{p}), a_2(t) = a(t)\bar{p}.$$

But, after a certain stage the enterprise can earn new dynamics by the external factors, investment and credits. Therefore, the following question ensues. For optimality in the some sense the production function up to what time will be expressed by exponential function, later by linear function or power function. In other words, it's required to define the time moment $s \in [0, T]$ which provides optimum regime Here we'll investigate only one structural change - change from exponential case to linear one.

*Corresponding author.

2. The Space of the Pairs of Fuzzy Numbers

A fuzzy set A is characterized by a generalized characteristic function $\mu_A(.)$, called membership function, defined on a universe $X \subset R$, which assumes values in $[0,1]$. For any $\alpha \in [0,1]$ denote by

$$A^\alpha = \{x \in X : \mu_A(x) \geq \alpha\}$$

the α-cut of A. Let $\mu_A(.)$ is an upper semicontinuous function and

$$\sup p(A) = \{x \in X : \mu_A(x) > 0\}$$

is bounded set of X. A fuzzy set is a fuzzy number if $X = R$ and for any $\alpha \in [0,1]$, the α-cut A^α is convex and the height of A, that is, $\sup\limits_{x\in X} \mu_A(x)$ has to be equal to one. This fuzzy number usually is called convex normal fuzzy number.

Let's define by F the class of convex normal fuzzy numbers. Then for any $a \in F$the set of α-cut of fuzzy number a the interval $a^\alpha = [L_a(\alpha), R_a(\alpha)]$, $\alpha \in [0,1]$, is defined ([8]). Let $a \in F$, $b \in F$ and $a^\alpha = [L_a(\alpha), R_a(\alpha)]$, $b^\alpha = [L_b(\alpha), R_b(\alpha)]$. Then α-cut of fuzzy number $a+b$ and ka, $k \geq 0$, define as $a^\alpha + b^\alpha = [L_a(\alpha) + L_b(\alpha), R_a(\alpha) + R_b(\alpha)]$ and $ka^\alpha = [kL_a(\alpha), kR_a(\alpha)]$, respectively.
Note that F is not a linear space (the operation of subtraction is not defined in F).

We consider the set of pairs $(a,b) \in F \times F$and define the operation of addition, multiplication and equivalency as

$$\begin{array}{l} (a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2), \\ k \cdot (a,b) = (ka, \ \ kb), \quad k \geq 0 \\ (-1) \cdot (a,b) = (b,a) \\ (a_1, a_2) \approx (b_1, b_2) <=> a_1 + b_2 = a_2 + b_1 \end{array} \tag{1}$$

As zero element of this space is taken the pair (0, 0), i.e. the set of elements$(a, \ a)$, $a \in F$. The set of all pairs $(a,b) \in F \times F$forms a structure of a linear space. Let

$$x = (a_1, a_2) \in F \times F, \ y = (b_1, b_2) \in F \times F.$$

Then

$$a_i^\alpha = [L_{a_i}(\alpha), R_{a_i}(\alpha)], b_i^\alpha = [L_{b_i}(\alpha), R_{b_i}(\alpha)], \ \alpha \in [0,1].$$

For any $x, y \in F \times F$ define the scalar product as

$$x \circ y = \frac{1}{2}\int_0^1 [(L_{a_1}(\alpha) - L_{a_2}(\alpha))(L_{b_1}(\alpha) - L_{b_2}(\alpha)) +$$

$$+ (R_{a_1}(\alpha) - R_{a_2}(\alpha))(R_{b_1}(\alpha) - R_{b_2}(\alpha)]d\alpha \tag{2}$$

It may be shown that this definition satisfies all requirements of the scalar product. We denote this space by LF. Norm in this space is defined as

$$||x||^2 = \frac{1}{2}\int_0^1 [(L_{a_1}(\alpha) - L_{a_2}(\alpha))^2 + (R_{a_1}(\alpha) - R_{a_2}(\alpha))^2]d\alpha, \tag{3}$$

We define distance between two fuzzy numbers $a \in F$and $b \in F$ as

$$\rho(a,b) = \|x - y\|, \tag{4}$$

where$x = (a,\, 0), y = (b,\, 0)$.

3. Derivative of the Fuzzy Function

Now, let's consider fuzzy function $f(t) \in F$ for each $t \in [t_0, t_1]$ and define a derivative of the function $f(t)$.

For any $\alpha \in [0, 1]$,

$$f_\alpha(t) = [L_{f(t)}(\alpha), R_{f(t)}(\alpha)], \alpha \in [0, 1] \tag{5}$$

is called α-cut of the function $f(t)$.

Definition. Let there exists such $\varphi(t) \in F,\ \psi(t) \in F,\ t \in [t_0, t_1]$, that

$$\lim_{\Delta t \to 0} \frac{(f(t + \Delta t), 0) - (f(t), 0)}{\Delta t} = (\varphi(t), \psi(t)). \tag{6}$$

Then the pair $(\varphi(t), \psi(t)) \in F \times F$ is called a derivative of the function $f(t)$ at the point $t \in (t_0, t_1)$.This definition may be written in the following form

$$\lim_{\Delta t \to 0} \frac{(f_\alpha(t + \Delta t), 0) - (f_\alpha(t), 0)}{\Delta t} = (\varphi_\alpha(t), \psi_\alpha(t)), \tag{7}$$

where$\varphi_\alpha(t), \psi_\alpha(t)$ are α-cut for the functions $\varphi(t), \psi(t)$.

It is shown that, if $L_{f(t)}(\alpha)\,, R_{f(t)}(\alpha)$ is continuous differentiable relatively t , then $f(t)$ is differentiable. Each function $f(t)$may be considered as an element $(f(t), 0)$ from $F \times F$. Then

$$(f_1(t) \pm f_2(t))' = f_1'(t) \pm f_2'(t). \tag{8}$$

Now, let $f(t)$be a pair of fuzzy functions, i.e.

$$f(t) = (f_1(t), f_2(t),\ \ \forall t \in (t_0, t_1).$$

From relation

$$f(t) = (f_1(t), 0) + (0, f_2(t)) = f_1(t), 0) - (f_2(t), 0),$$

we see, that the derivative of the function $f(t)$also is a pair from $F \times F$.

For any $\eta = \eta(t) \in F \times F$, which $\eta'(t) \in F \times F,\ \ t \in [t_0, t_1]$, consider the scalar product $f'(t) \circ \eta(t)$ defined by the formulae (2). It can be shown that

$$\int_t^T f'(\tau)\circ\eta(\tau)d\tau = f(\tau)\circ\eta(\tau)/_t^T - \int_t^T f(\tau)\circ\eta'(\tau)d\tau, \forall t, T \in [t_0, t_1]. \qquad (9)$$

One may show that this derivative satisfies the "necessary natural" conditions.

4. Formulation of the Problem and Main Results

First, we assume that $m = 0$. It means that, there is not any affects to the production process in the enterprice. Let's the dynamics of the main producing assets are expressed by the following equation in $0 < t \le s$

$$\frac{dx_1(t)}{dt} = a_1(t) - a_2(t)e^{-x_1(t)} + I_1(t). \qquad (10)$$

start condition is

$$x_1(0) = x_1^0. \qquad (11)$$

Here the main producing assets are expressed by $x_1(t) = A(t)$.
In the interval $s < t \le T$ the main producing assets $x_2(t)$ function will be

$$\frac{dx_2(t)}{dt} = a(t)x_2(t) + I_2(t) \qquad s < t \le T. \qquad (12)$$

In the $s \in (0, T)$

$$x_2(s) = x_1(s), \qquad (13)$$

Let

$$V_1 = \{I_1 = I_1(t) \in C(0,T) : \; v_1(t) \in V_1, \forall t \in [0,T], \; \|I_i(t)\| \in C(0,T)\},$$

$$V_2 = \{I_2 = I_2(t) \in V_2, \forall t \in [0,T], \; \|I_2(t)\| \in C(0,T)\}.$$

Here $V_1 \subset R$, $V_2 \subset F$. it's required to find $s \in (0,T)$,$I_1 = I_1(t) \in V_1$, so that $x_2(T)$ will be close to given number z. In other words the control (s, v_1, v_2) given minimun the following functional

$$J(s, I_1, I_2) = \|x_2(T) - z\|^2 \qquad (14)$$

Here [14]

$$x_2(t) - z = (x_2(t),\, 0) - (z,\, 0) = (x_2(t), z)$$

and

$$\|x_2(T) - z\|^2 = \int_0^1 \left[(L_{x_2(T)}(\alpha) - L_z(\alpha))^2 + \ (R_{x_2(T)}(\alpha) - R_z(\alpha))^2 \right] d\alpha. \tag{15}$$

As wesee control $I_2(t)$ is fuzzy and so solution of the problem (12),(13) is fuzzy.

Let $\psi_1 = \psi_1(t)$, $t \in [0, s]$, $\psi_2 = \psi_2(t)$, $t \in [s, T]$ be solution of the problem

$$\frac{d\psi_1(t)}{dt} = -a_2(t)e^{-x_1(t)}\psi_1(t), t \in [0, s], \tag{16}$$

$$\dot{\psi}_2(t) = -a(t)\psi_2(t), \ t \in [s, T], \tag{17}$$

$$\psi_1(s) = \psi_2(s), \psi_2(T) = -2\left[x_2(T) - z\right]. \tag{18}$$

This problem is called adjoint problem to (10)-(14). As we see $\psi_1 = \psi_1(t)$, $t \in [0, s]$, $\psi_2 = \psi_2(t)$, $t \in [s, T]$ are fuzzy functions.
Denote

$$x(t) = \begin{cases} x_1(t), \ t \in [0, s], \\ \\ x_2(t), \ t \in [s, T], \end{cases} \quad \psi(t) = \begin{cases} \psi_1(t), \ t \in [0, s], \\ \\ \psi_2(t), \ t \in [s, T], \end{cases},$$

here $x_i(t)$, $\psi_i(t)$, $i = 1, 2$ are solutions of the problems (10)-(13) and (16)-(18), correspondly.

Theorem. Let $s \neq t_0$. Then the functional $J(s, I_1, I_2)$ is differentable on $(s, v_1, v_2) \in [0.T] \times V_1 \times V_2$ and for its first variation try the following formula

$$\begin{aligned} \delta J = {} & [a_1(s) - a_2(t)e^{-x_1(s)}x_1(s) - a(s)x_2(s) + I_1(s) - I_2(s)] \circ \psi_1(s)\delta s - \\ & - \textstyle\int_0^s \psi_1(t) \circ \delta I_1(t)dt - \int_{\bar{s}}^T \psi_2(t) \circ \delta I_2(t)dt. \end{aligned} \tag{19}$$

Proof. Take any $s, \bar{s} \in (0, T)$,$I_1, C \in V_1$, $I_2, \bar{I}_2 \in V_2$ and denote

$$I(t) = \begin{cases} I_1(t), \ t \in [0, s], \\ \\ I_2(t), \ t \in [s, T], \end{cases}, \bar{I}(t) = \begin{cases} I_2(t), \ t \in [0, \bar{s}], \\ \bar{I}_2(t), \ t \in [\bar{s}, T], \end{cases}$$

$x(t)$ and $\bar{x}(t)$, $t \in [0, T]$ are solutins of the problem (1)-(4) corresponding to controls $\{s, I(t)\}$, $\{\bar{s}, \bar{I}(t)\}$. Denote

$$\Delta s = \bar{s} - s, \ \Delta x_i(t) = \overline{x}_i(t) - x_i(t), \ \ \Delta x(t) = \overline{x}(t) - x(t),$$

$$\Delta I_i(t) = \bar{I}_i(t) - I_i(t), \ \ \Delta I(t) = \bar{I}(t) - I(t)$$

$$s_m = \min\{s, \bar{s}\}, \ \ s_M = \max\{s, \bar{s}\}$$

It is clear that

$$\frac{d\Delta x_1(t)}{dt} = a_2(t)e^{-x_1(t)}\Delta x_1(t) + \Delta I_1(t),\ \ i = 1,\ 2, \tag{20}$$

$$\frac{d\Delta x_2(t)}{dt} = a(t)\Delta x_2(t) + \Delta I_2(t),\ \ i = 1,\ 2, \tag{21}$$

$$\Delta x_1(0) = 0, \Delta x_1(s) = \Delta x_2(s), \tag{22}$$

Multyplying equation (20) to $\psi_1(t) \in F \times F$ and equation (21) to $\psi_2(t) \in F \times F$ we get

$$\int_0^{s_m} \left[\Delta\dot{x}_1(t) \circ \psi_1(t) -\ a_2(t)e^{-x_1(t)}\Delta x_1(t) \circ \psi_1(t) - \Delta I_1(t) \circ \psi_1(t)\right] dt = 0,$$

$$\int_{s_M}^{T} \left[\Delta\dot{x}_2(t) \circ \psi_2(t) -\ a(t)\Delta x_2(t) \circ \psi_2(t) - \Delta v_2(t) \circ \psi_2(t)\right] dt = 0$$

Take into account initial condition (22) we have

$$\Delta x_1(s_m) \circ \psi_1(s_m) - \int_0^{s_m} \left[\Delta x_1(t) \circ \dot{\psi}_1(t) + a_2(t)e^{-x_1(t)}\Delta x_1(t) \circ \psi_1(t)\right] dt-$$

$$-\int_0^{s_m} \Delta I_1(t) \circ \psi_1(t) dt = 0,$$

$$\Delta x_2(T) \circ \psi_2(T) - \Delta x_2(s_M) \circ \psi_2(s_M) - \int_{s_M}^{T} \left[\Delta x_2(t) \circ \dot{\psi}_2(t) + a(t)\Delta x_2(t) \circ \psi_2(t))\right] dt-$$

$$-\int_{s_M}^{T} \Delta I_2(t) \circ \psi_2(t) dt = 0 \tag{23}$$

Now calculate increment of the function

$$\begin{aligned} \Delta J &\equiv J(\bar{s}, \bar{I}_1, \bar{I}_2) - J(s, I_1, I_2) = \|\bar{x}_2(T) - z\|^2 - \|x_2(T) - z\|^2 = \\ &= 2(x_2(T) - z) \circ \Delta x_2(T) + \|\Delta x_2(T)\|^2 . \end{aligned} \tag{24}$$

here

$$\|\Delta x_2(T)\| = \|(\bar{x}_2(T), 0) - (x_2(T), 0)\| = \|(\bar{x}_2(T), x_2(T))\|$$

Is the norm of the pair fuzzy function.
Adding (**??**) and (23) to (24) we have

$$\Delta J = 2(x_2(T) - z) \circ \Delta x_2(T) + \Delta x_2(T) \circ \psi_2(T) + \Delta x_2(T) \circ \psi_2(T)-$$

$$-\Delta x_1(s_m)\circ\psi_1(s_m)-\int_0^{s_m}\left[\Delta x_1(t)\circ\dot{\psi}_1(t)+a_2(t)e^{-x_1(t)}\Delta x_1(t)\circ\psi_1(t)\right]dt-$$

$$-\int_0^{s_m}\Delta I_1(t)\circ\psi_1(t)dt-\int_{s_M}^{T}\Delta I_2(t)\circ\psi_2(t)dt+\|\Delta x_2(T)\|^2-$$

$$-\Delta x_2(s_M)\circ\psi_2(s_M)-\int_{s_M}^{T}\left[\Delta x_2(t)\circ\dot{\psi}_2(t)+a(t)\Delta x_2(t)\circ\psi_2(t))\right]dt$$

For the sake of symplicity let $s\leq\bar{s}$. In this case $s_m=s,\ s_M=\bar{s}$. Let $\bar{\psi}_1=\bar{\psi}_1(t),\ t\in[0,s],\ \bar{\psi}_2=\bar{\psi}_2(t),\ t\in[\bar{s},T]$ be solution of the problem

$$\dot{\bar{\psi}}_1(t)=-a_2(t)e^{-x_1(t)}\bar{\psi}_1(t),t\in[0,s], \tag{25}$$

$$\dot{\bar{\psi}}_2(t)=-a(t)\bar{\psi}_2(t),\ t\in[\bar{s},T], \tag{26}$$

$$\bar{\psi}_1(s)=\bar{\psi}_2(\bar{s}),\bar{\psi}_2(T)=-2\left[x(T)-z\right]. \tag{27}$$

Taking $\psi_i=\bar{\psi}_i$ we obtain

$$\begin{aligned}\Delta J=(\Delta x_1(s)-\Delta x_2(\bar{s}))\circ\bar{\psi}_1(s)-\int_0^{s}\Delta I_1(t)\circ\bar{\psi}_1(t)dt-\\ -\int_{\bar{s}}^{T}\Delta I_2(t)\circ\bar{\psi}_2(t)dt+\|\Delta x_2(T)\|^2.\end{aligned} \tag{28}$$

Take into account $x_1(s)=x_2(s)$ and $\bar{x}_1(\bar{s})=\bar{x}_2(\bar{s})$ let's calculate $\Delta x_1(s)-\Delta x_2(\bar{s})$

$$\Delta x_2(\bar{s})=\bar{x}_2(\bar{s})-x_2(\bar{s})=\bar{x}_1(\bar{s})-x_2(\bar{s})=$$

$$=[\bar{x}_1(s)-x_1(s)]+[\bar{x}_1(\bar{s})-\bar{x}_1(s)]+[x_2(s)-x_2(\bar{s})]=$$

$$=\Delta x_1(s)+\dot{\bar{x}}_1(s)\Delta s-\dot{\bar{x}}_2(s)\Delta s+o(\|\Delta s\|)=$$

$$=\Delta x_1(s)+[\dot{x}_1(s)-\dot{x}_2(s)]\Delta s+o(\|\Delta s\|)+\Delta\dot{x}_1(s)\Delta s+\Delta\dot{x}_2(\bar{s})\Delta s.$$

Then

$$\Delta x_2(\bar{s})-\Delta x_1(s)=[\dot{x}_1(s)-\dot{x}_2(s)]\Delta s+o(\|\Delta s\|)+\Delta\dot{x}_1(s)\Delta s+\Delta\dot{x}_2(\bar{s})\Delta s). \tag{29}$$

Putting this in (28) we get

$$\Delta J=\left[\frac{dx_1(s)}{dt}-\frac{dx_2(s)}{dt}\right]\circ\psi_1(s)\Delta s-\int_0^s\Delta I_1(t)\circ\psi_1(t)dt-$$

$$-\int_{\bar{s}}^{T}\Delta I_2(t)\circ\psi_2(t)dt+o(\|\Delta s\|)+\|\Delta\dot{x}_1(s)\|\,\Delta s+\|\Delta\dot{x}_2(s)\|\,\Delta s+$$

$$+O(\|\Delta\psi_1\|_{FC})\,\|\Delta I_1\|+O(\|\Delta\psi_2\|_{FC})\,\|\Delta I_2\|+o\left(\|\Delta x_2(T)\|\right)$$

where $\Delta\psi=\bar{\psi}-\psi$. It is no difficult to show

$$\|\Delta x(T)\|_{FC}\le C(|\Delta s|+\|\Delta I_1\|+\|\Delta I_2\|),$$

$$|\Delta s|+\|I_1\|+\|I_2\|\to 0,$$

when $\|\Delta\psi\|_{FC}+\|\Delta\dot{x}_1(s)\|+\|\Delta\dot{x}_2(s)\|\to 0$. Then from last elation we obtain

$$\Delta J=\left[\tfrac{dx_1(s)}{dt}-\tfrac{dx_2(s)}{dt}\right]\circ\psi_1(s)\Delta s-\int_0^s\Delta I_1(t)\circ\psi_1(t)dt-$$
$$-\int_{\bar{s}}^{T}\Delta I_2(t)\circ\psi_2(t)dt+o(\|\Delta s\|)+o(\|I_1\|)+o(\|I_2\|). \tag{30}$$

$x_i(t),\ i=1,2$ is continuously differentiable solution of the problem (10)-(13). In this case

$$\frac{dx_1(s)}{dt}=a_1(s)-a_2(s)e^{-x_1(s)}+I_1(s)),$$

$$\frac{dx_2(s)}{dt}=a(s)x_2(s)+I_2(s)$$

Putting this in (30) we get

$$\delta J=[a_1(s)-a_2(t)e^{-x_1(s)}x_1(s)-a(s)x_2(s)+I_1(s)-I_2(s)]\circ\psi_1(s)\delta s-$$

$$-\int_0^s\psi_1(t)\circ\delta I_1(t)dt-\int_{\bar{s}}^{T}\psi_2(t)\circ\delta I_2(t)dt+o(\|I_1\|)+o(\|I_2\|)+o(\|\Delta s\|).$$

From last relation we obtain formula (19).

Theorem is prooved.

Using formula (10) we can give the following optimal condition

Theorem. Let $(s^*,v_1^*,v_2^*)\in(0.T)\times V_1\times V_2$ be optimal for the problem (1)-(5) Then

$$[a_1(s)-a_2(t)e^{-x_1(s)}x_1(s)-a(s)x_2(s)+I_1(s)-I_2(s)]\circ\psi_1(s)=0 \tag{31}$$

and for any $\forall(I_1,I_2)\in V_1\times V_2$

$$\delta J=\int_0^s\psi_1(t)\circ(I_1(t)-I_1^*(t))dt+\int_{\bar{s}}^{T}\psi_2(t)\circ(I_2(t)-I_2^*(t))dt\le 0,\ \ . \tag{32}$$

Now, let the time s be fixed and $m\neq 0$, i.e. we consider the problem

$$\frac{dx_1(t)}{dt}=a_1(t)-a_2(t)e^{-x_1(t)}+I_1(t)+m\delta(t-t_0).$$

$$x_1(0) = x_1^0.$$

$$\frac{dx_2(t)}{dt} = a(t)x_2(t) + I_2(t) + m\delta(t - t_0) \qquad s < t \le T.$$

$$x_2(s) = x_1(s).$$

In this case we obtain following for the first variation of the functional (14)

$$\delta J = -\int_0^s \psi_1(t) \circ \delta I_1(t)dt - \int_s^T \psi_2(t) \circ \delta I_2(t)dt. \tag{33}$$

References

[1] Gerasimov B.I., Puchkov N.P., Protasov D.N. Differential dynamic models. Publishing house TGTU, 2010. – 80 .

[2] Vladimirov, V. S. Equations of mathematical physics, 1971, Marcel Dekker, ISBN 0-8247-1713-9.

[3] Barnabs Bede, Luciano Stefanini: Generalized differentiability of fuzzy-valued functions Fuzzy Sets and Systems (2013),230: 119-141.

[4] Angelov, P.: Fuzzy optimal control. Fuzzy Sets and Systems,1992; 47: 151-156.

[5] Niftiyev A.A., M.Poormanoochehri, Zeynalov C.I. Fuzzy optimal control problem with non-linear functional. International journal Fuzzy Information and Engineering, 2011, 3, p.311-320.

[6] Aliev F.A., Niftiyev A.A. , Zeynalov C.I. Optimal synthesis problem for the fuzzy systems in semi-infinite interval. Appl. and Comp. Mathematics, 2011, v.10, 1(Special Issue), pp. 97-105.

[7] Aliev R.A. Modelling and stability analysis in fuzzy economics. Applied and Computational Mathematics, 2008; 7(1): pp.31-53.

[8] T. Allahviranloo, Difference methods for fuzzy partial differential equations, Computational methods in appliead mathematics, 2(2002),No. 3,PP.233-242.169 (2005) 16-33.

[9] Dubois D., Prade H. Towards fuzzy differential calculas. II. Integration on fuzzy intervals,Fuzzy Sets and Systems, 1982; 8(2): 105-116.

[10] Dubois D., Prade H. Towards fuzzy differential calculas. III. Differentiation, Fuzzy Sets and Systems, 1982; 8(3): 225-233.

[11] Puri M.L., Ralescu D.A. Differentials for fuzzy function. J.Math.Anal. Appl.1978; 64:369-380.

[12] Seikkala S. On the fuzzy initial value problem, Fuzzy Sets and Systems, 1987; 24(3): 319-330.

[13] Demyanov V.F., Rubinov A.M. Basises of non-smooth analyses and quasidifferential calculas. M.: Nauka, 1990, 420 p.

[14] Aliev F.A., Niftiyev A.A., Zeynalov C.I. Optimal synthesis problem for the fuzzy systems. Optimal control, applications and methods. 2011. v. 32, 6, pp. 660–667.

B.M.Gasimov
Institute of Applied Mathematics Baku State University, Baku, country-Azerbaijan
E-mail: bayram.qasimov@mail.ru

E.R.Shafizade
Institute of Applied Mathematics Baku State University, Baku, country-Azerbaijan
E-mail: elnure_sh@mail.ru

A.A.Murtuzayeva
Institute of Applied Mathematics Baku State University, Baku, country-Azerbaijan
E-mail: beniz.aslanli@gmail.com

Received 23 May 2013
Accepted 29 August 2013