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Development and Implementation of a Computational Algorithm for Solving Ordinary Differential Equations

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Abstract. In this paper, we developed a numerical algorithm which aimed to solve some first order initial value problems of ordinary differential equations. We explicitly present the breakdown derivation of the new numerical algorithm. The implementation of this new numerical algorithm is on some real life problems leading to first order initial value problem of ordinary differential equations. Results comparison is also made with some existing methods.
Numerical Scheme, Ordinary Differential Equation, Scheme Development

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1. Introduction

Different authors have developed lot of methods which are suitable for solving some sets of Initial Value Problems (IVPS) in Ordinary Differential Equations (ODEs). But the efficiency of any method in numerical analysis depends solely on the characterization of the scheme such as: stability, accuracy, convergence and consistency properties of the method. In numerical analysis, the accuracy properties of different methods are usually compared by considering the order of convergence as well as the truncation error coefficients of the various methods . Numerical analysts like, Fatunla (1987a), Ibijola, (1997 and 1998) and Ogunrinde, (2010, 2012, 2013, 2015, and 2016) have developed numerical schemes for solving initial value problems. In this paper, we improved on Ogunrinde, (2010) work which was based on the local representation of the theoretical solution to initial value problem of the form:

$$y' = f(x, y); y(a) = \eta$$

in the interval $(x_n, x_{(n+1)})$ by interpolating function

$$F(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + bR_e(e^{(kx+\mu)})$$

where a_0, a_1a_2, a_3 and b are real undetermined coefficients and $k, \mu$are complex parameters. But in this paper, we shall be using the same assumptions but different interpolating function such as:

$$F(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + bxR_e(e^{(kx+\mu)})$$

where a_0, a_1a_2, a_3 and b are real undetermined coefficients and k, μ are complex parameters.

2. Derivation of the Scheme

Considering an interpolating function:

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + bxR_e(e^{kx+\mu}) \tag{1}$$

Where$a_{0,}a_{1,}a_{2,}a_{3,}$and$b$are real undetermined coefficients and k, μ are complex parameters. Since $kand\mu$ are complex parameters, then we have:

$$k = k_1 + ik_2 \tag{2}$$

Also, $\mu = i\theta$, where $i^2 = -1$,therefore putting this together with (2)in (1), we have the Interpolating function to be:

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + bxe^{k_1x}\cos(k_2x + \sigma) \tag{3}$$

Let us define $R(x)$ and $\theta(x)$ as follows:

$$R(x) = xe^{k_1x}, \theta(x) = k_2x + \sigma \tag{4}$$

Putting (4) in (3), we have:

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + bR(x)cos\theta(x) \tag{5}$$

By assumption,$y_n$is a numerical estimate to the theoretical solution$y(x_n)$and also $f_n = f(x_n, y_n)$. Let our mesh points (self length) be define as follows:

$$x_n = a + nh; n = 0, 1, 2\ldots, a = 0, x_n = nh, x_{n+1} = (n+1)h \tag{6}$$

Imposing the following constraints on the interpolating function (5), we have:

1. The interpolating function must coincide with the theoretical solution at $x = x_n$ and $x = x_{n+1}$. This required that:

$$f(x_{n+1}) = a_0 + a_1x_{n+1} + a_2x_{n+1}^2 + a_3x_{n+1}^3 + bR(x_{n+1})\cos(\theta(x_n)) \tag{7}$$

That is, $f(x_n) = y(x_n)$ and

$$f(x_{n+1}) = a_0 + a_1x_{n+1} + a_2x_{n+1}^2 + a_3x_{n+1}^3 + bR(x_{n+1})\cos(\theta(x_{n+1})) \tag{8}$$

It implies that $f(x_{n+1}) = y(x_{n+1})$.

2. The first, second, third and fourth derivatives with respect to x of the interpolating function respectively coincide with the differential equation as well as its first, second, third and fourth derivatives with respect to $xatx_n$, i.e.

$$F^1(x_n) = f_n, F^2(x_n) = f_n^1, F^3(x_n) = f_n^2, F^4(x_n) = f_n^3 \tag{9}$$

From equation (9) implies:

$$f^{'}(x_n) = f_n + 2a_2x_n + 3a_3x_n^2 + [\cos(\theta(x_n))\frac{d}{dx}(bR(x_n)) + bR(x_n)\frac{d}{dx}(\cos\theta(x))] \tag{10}$$

Where

$$\frac{d}{dx}(bR(x_n)) = \frac{d}{dx}(bxe^{k_1x}) = e^{k_1x}\cdot b + bx\cdot k_1e^{k_1x} = be^{k_1x} + bk_1xe^{k_1x}$$

$$= be^{k_1x} + bk_1R(x_n) \tag{11}$$

$$\frac{d}{dx}\cos(\theta(x_n))\frac{d}{dx}[cos(k_2x+\sigma)] = -k_2\sin(\theta(x_n)) \tag{12}$$

Putting (11) & (12) in (10) we have:

$$f^{'}(x_n) = f_n = a_1 + 2a_2x_n + 3a_3x_n^2[cos(\theta(x_n)\{be^{k_1x} + bk_1R(x_n)\} + bR(x_n)(-k_2\sin\theta(x_n))]$$

$$= a_1 + 2a_2x_n + 3a_3x_n^2 + [be^{k_1x}\cos(\theta(x_n)) + bk_1R(x_n)\cos(\theta(x_n)) - bR(x_n)(-k_2\sin\theta(x_n))]$$

$$f_n = a_1 + 2a_2x_n + 3a_3x_n^2 + b[e^{k_1x_n}cos(\theta(x_n)$$

$$+k_1R(x_n)\cos(\theta(x_n)) - k_2R(x_n)\sin(\theta(x_n))] \tag{13}$$

That is,

$$F^{'}(x_n) = f_n^1$$

$$F''(x_n) = f_n^1 = 2a_2x_n + 6a_3x_n + b[(e^{k_1x_n}\frac{d}{dx}\cos\theta(x_n) + \cos\theta(x_n)\frac{d}{dx}e^{k_1x_n})$$

$$+(k_1R(x_n)\frac{d}{dx}\cos\theta(x_n) + \cos\theta(x_n)\frac{d}{dx}k_1R(x_n)$$

$$-(R(x_n)\frac{d}{dx}\sin\theta(x_n) + sin(\theta(x_n)\frac{d}{dx}R(x_n)))] \tag{14}$$

Where

$$k_1R(x_n) = k_1[xe^{k_1x_n}] = k_1(x\frac{d}{dx}e^{k_1x_n} + e^{k_1x_n}\frac{d}{dx}x) = k_1^2xe^{k_1x_n} + k_1e^{k_1x_n}$$

$$= k_1^2R(x_n) + k_1e^{k_1x_n} \tag{15}$$

Since

$$R(x_n) = x_ne^{k_1}x_n = e^{k_1x_n} + k_1R(x_n)$$

Putting (15) in (14)

$$F''(x_n) = f_n^1 = 2a_2x_n + 6a_3x_n + b[\{e^{k_1x_n}(-\sin\theta(x_n) + \cos\theta(x_n)(k_1e^{k_1x_n})\}\}$$

$$+b[\{k_1R(x_n) - sin(\theta(x_n) + \cos\theta(x_n)(k_1^2R(x_n) + ke^{k_1}x_n) - R(x_n)cos\theta(x_n) + \sin\theta(x_n)(e^{k_1x_n} + k_1R(x_n))]\}$$

$$F''(x_n) = 2a_2 + 6a_3x_n + b[-e^{k_1x_n}\sin\theta(x_n) + k_1e^{k_1x_n}\cos\theta(x_n) - k_1R(x_n)sin?(\theta(x_n) + k_1^2R(x_n)cos\theta(x_n)]$$

$$+b[k_1e^{k_1x_n}cos\theta(x_n) - R(x_n)\cos\theta(x_n) - e^{k_1x_n}\sin\theta(x_n) - k_1R(x_n)sin(\theta(x_n))]$$

$$f_n' = 2a_2x_n + 6a_3x_n + b\{-2e^{k_1x_n}\sin(\theta(x_n)) + 2k_1e^{k_1x_n}\cos(\theta(x_n))$$

$$-2k_1R(x_n)\sin\theta(x_n) + k_1^2R(x_n)\cos\theta(x_n) - R(x_n)cos\theta(x_n)\} \qquad (16)$$

$$F'''(x_n) = f_n^2 = 6a_3$$

$$+b[-\{+2e^{k_1x_n}\frac{d}{dx}\sin\theta(x_n) + \sin\theta(x_n)\frac{d}{dx} + 2e^{k_1x_n}\}$$

$$+\{2k_1e^{k_1x_n}\frac{d}{dx}cos\theta(x_n) + cos\theta(x_n)\frac{d}{dx}2k_1e^{k_1x_n}\}$$

$$-2k_1R(x_n)\frac{d}{dx}\sin\theta(x_n) + \sin\theta(x_n)\frac{d}{dx}2k_1R(x_n)$$

$$+\{k_1^2R(x_n)\frac{d}{dx}cos\theta(x_n) + cos\theta(x_n)\frac{d}{dx}k_1^2R(x_n)\}$$

$$-\{R(x_n)\frac{d}{dx}cos\theta(x_n) + cos\theta(x_n)\frac{d}{dx}R(x_n)\}]$$

$$F'''(x_n) = 6a_3 + b[-(2e^{k_1x_n}cos\theta(x_n) + 2k_1e^{k_1x_n}\sin\theta(x_n)) + (2k_1e^{k_1x_n}(-\sin\theta(x_n))$$

$$+2k_1^2e^{k_1x_n}cos\theta(x_n) - 2k_1R(x_n)cos\theta(x_n)$$

$$+[2k_1e^{k_1x_n} + 2k_1^2R(x_n)]\sin\theta(x_n) + (k_1^2R(x_n)(-\sin\theta(x_n)$$

$$+k_1^2e^{k_1x_n} + k_1^3R(x_n)cos\theta(x_n) - R(x_n)(-\sin\theta(x_n) + (e^{k_1x_n} + k_1R(x_n))cos\theta(x_n)]$$

$$F'''(x_n) = f_n^2 = 6a_3 + b[-3e^{k_1x_n}cos\theta(x_n) - 6k_1e^{k_1x_n}\sin\theta(x_n) + 3k_1^2R(x_n)\sin\theta(x_n)$$

$$-2k_1R(x_n)cos\theta(x_n) + k_1^3R(x_n)cos\theta(x_n)$$

$$+R(x_n)\sin\theta(x_n) - k_1e^{k_1x_n}cos\theta(x_n)] \qquad (17)$$

$$F^{'v}(x_n) = f^3(x_n)$$

$$= b[-3\{e^{k_1x_n}\frac{d}{dx}\cos\theta(x_n) + \cos\theta(x_n)\frac{d}{dx}e^{k_1x_n}\} - 6\{e^{k_1x_n}\frac{d}{dx}\sin\theta(x_n) + \sin\theta(x_n)\frac{d}{dx}e^{k_1x_n}\}$$

$$+3k_1^2\{e^{k_1x_n}\frac{d}{dx}\cos\theta(x_n) + \cos\theta(x_n)\frac{d}{dx}e^{k_1x_n}\}$$

$$-3k_1^2[R(x_n)\frac{d}{dx}\sin\theta(x_n) + \sin\theta(x_n)\frac{d}{dx}R(x_n)]$$

$$-2k_1\{R(x_n)\frac{d}{dx}\cos\theta(x_n) + \cos\theta(x_n)\frac{d}{dx}R(x_n)\}$$

$$+k_1^3R(x_n)\frac{d}{dx}cos\theta(x_n) + cos\theta\frac{d}{dx}R(x_n)$$

$$+\{R(x_n)\frac{d}{dx}\sin\theta(x_n)+\sin\theta(x_n)\frac{d}{dx}R(x_n)\}$$
$$-k_1\{e^{k_1x_n}\frac{d}{dx}\cos\theta(x_n)+\cos\theta(x_n)\frac{d}{dx}e^{k_1x_n}\}]$$

$$\begin{aligned}F^{iv}(x_n) = b[&4e^{k_1x_n}\sin\theta(x_n)-11k_1e^{k_1x_n}cos\theta(x_n)-12k_1^2e^{k_1x_n}\sin\theta(x_n)+4k_1^3e^{k_1x_n}cos\theta(x_n)\\ &-5k_1^2R(x_n)\cos\theta(x_n)-4k_1^3R(x_n)\sin\theta(x_n)+3k_1R(x_n)\sin\theta(x_n)\\ &+R(x_n)\cos\theta(x_n)+k_1e^{k_1x_n}\sin\theta(x_n)\\ &-k_1^2e^{k_1x_n}cos\theta(x_n)+k_1^4R(x_n)\cos\theta(x_n)]\end{aligned} \quad (18)$$

$$b=\frac{f_n^3}{\Upsilon} \quad (19)$$

where

$$\begin{aligned}\Upsilon = &e^{k_1x_n}(4\sin(\theta(x_n))-11k_1\cos(\theta(x_n))-12k_1^2\sin(\theta(x_n))\\ &+4k_1^3\cos(\theta(x_n))+k_1\sin(\theta(x_n))-k_1^2cos(\theta(x_n)))+R(x_n)(-5k_1^2cos(\theta(x_n))\\ &-4k_1^3sin(\theta(x_n))+3k_1sin(\theta(x_n))+\cos(\theta(x_n))+k_1^4\cos(\theta(x_n)))\end{aligned}$$

From (17), we have:

$$\begin{aligned}a_3=\frac{1}{6}[&f_n^2-b\{-3e^{k_1x_n}\cos\theta(x_n)-6k_1e^{k_1x_n}sin\theta(x_n)+3k_1^2e^{k_1x_n}\cos\theta(x_n)\\ &-3k_1^2R(x_n)\sin\theta(x_n)-2k_1R(x_n)\cos\theta(x_n)\\ &+k_1^3R(x_n)\cos\theta(x_n)+R(x_n)\sin\theta(x_n)-k_1e^{k_1x_n}\cos\theta(x_n)\}]\end{aligned} \quad (20)$$

Putting (19) in (20), we have:

$$a_3=\frac{1}{6}\left[f_n^2-\frac{f_n^3}{\Upsilon}v\right] \quad (21)$$

where

$$\begin{aligned}\Upsilon = &e^{k_1x_n}(4\sin(\theta(x_n))-11k_1\cos(\theta(x_n))-12k_1^2\sin(\theta(x_n))+4k_1^3\cos(\theta(x_n))\\ &+k_1sin(\theta(x_n)-k_1^2cos(\theta(x_n)+R(x_n)(-5k_1^2\cos(\theta(x_n))\\ &-4k_1^3\sin(\theta(x_n)))+3k_1\sin(\theta(x_n))+\cos(\theta(x_n)+k_1^4\cos(\theta(x_n)\end{aligned} \quad (22a)$$

$$\begin{aligned}v = &-3e^{k_1x_n}\cos\theta(x_n)-6k_1e^{k_1x_n}sin\theta(x_n)\\ &+3k_1^2e^{k_1x_n}\cos\theta(x_n)\\ &-3k_1^2R(x_n)\sin\theta(x_n)-2k_1R(x_n)\cos\theta(x_n)\\ &+k_1^3R(x_n)\cos\theta(x_n)+R(x_n)\sin\theta(x_n)-k_1e^{k_1x_n}\cos\theta(x_n)\end{aligned} \quad (22b)$$

Therefore, (21) becomes

$$a_3=\frac{1}{6}\left[f_n^2-\left(\frac{f_n^3}{\Upsilon}\right)v\right] \quad (23)$$

From (16), we have:

$$a_2 = \frac{1}{2}\left[f_n^1 - \left(f_n^2 - \left(\frac{f_n^3}{\Upsilon}\right)v\right)x_n - b\{u\}\right] \tag{24}$$

where

$$u = -2e^{k_1x_n}\sin\theta(x_n) - 2k_1e^{k_1x_n}\cos\theta(x_n)$$
$$-2k_1R(x_n)\sin\theta(x_n) + k_1^2R(x_n)\cos\theta(x_n) - R(x_n)\cos\theta(x_n)$$

Putting (19) and (23) in (24) we have:

$$a_2 = \frac{1}{2}\left[f_n^1 - \left(f_n^2 - \left(\frac{f_n^3}{\Upsilon}\right)v\right)x_n - \left(\frac{f_n^3}{\Upsilon}\right)u\right] \tag{25}$$

From (13) we have:

$$a_1 = f_n - 2a_2x_n - 3a_3x_n^2 - b[e^{k_1x_n}cos?(\theta(x_n)$$
$$+k_1R(x_n)\cos(\theta(x_n)) - R(x_n)\sin(\theta(x_n))] \tag{26}$$

Putting (19), (23) and (25) in (26), we have:

$$a_1 = f_n - \left(\left(\frac{A}{\Upsilon}\right)x_n - \frac{D}{\Upsilon}\right)x_n + \frac{1}{2}\left[\frac{A}{\Upsilon}\right]x_n^2$$
$$-\left(\frac{f_n^3}{\Upsilon}\right)[e^{k_1x_n}cos(\theta(x_n) + k_1R(x_n)\cos(\theta(x_n)) - R(x_n)\sin(\theta(x_n))] \tag{27}$$

where

$$A = f_n^2 - (-3e^{k_1x_n}\cos\theta(x_n) - 6k_1e^{k_1x_n}sin\theta(x_n)$$
$$+3k_1^2e^{k_1x_n}\cos\theta(x_n) - 3k_1^2R(x_n)\sin\theta(x_n)$$
$$-2k_1R(x_n)\cos\theta(x_n))f_n^3 \tag{28}$$

$$D = -2e^{k_1x_n}\sin\theta(x_n) - 2k_1e^{k_1x_n}\cos\theta(x_n) - 2k_1R(x_n)\sin\theta(x_n)$$
$$+k_1^2R(x_n)\cos\theta(x_n) - R(x_n)\cos\theta(x_n) \tag{29}$$

For preservative of the scheme, then we can write the new scheme in a compact form as:

$$y_{n+1} = y_n + a_1h + a^2a_2(2n+1) + h^3a_3(3n^2+3n+1)$$
$$+bR_n[he^{k_1}h(\cos\theta_n\cos k_2h - \sin\theta_n\sin k_2h) - \cos\theta_n] \tag{30}$$

Putting a_1, a_2 and a_3 as derived above, we arrived at a new scheme. But to test the scheme, we shall proceed to write a programme which will command the scheme to solve some first order differential equations.

3. Implementation of the Scheme

Problem 1

Let us consider the initial value problem of the form (Lambert 1973a, Fatunla 1988)

$$y' = y, y(0) = 1$$

with the exact solution

$$y(x) = \exp(x)$$

in the interval $0 \leq x \leq 1$. The parameter ρ_1, ρ_2 and θ_t were obtained with $\rho_1 = 1.01009762, \rho_2 = 0.72269428, \theta_t = 1.04708743$. Taking $h = 0.01$.The numerical experiments are shown below

Note; The new method is named Olubunmi all through.

Table 1: At H = 0.01.

T	IBIJOLA 1997	OGUNRINDE 2010	THEORETICAL SOLUTION	OLUBUNMI
0	1000000D+01	1	1	1
1	.1010050D+01	1.0100502	1.0100502	1.00999963
2	.1020201D+01	1.0202013	1.0202013	1.02009833
3	.1030454D+01	1.0304545	1.0304545	1.03029692
4	.1040811D+01	1.0408108	1.0408108	1.04059637
5	.1051271D+01	1.0512712	1.0512711	1.05099761
6	.1061836D+01	1.0618367	1.0618366	1.06150162
7	.1072508D+01	1.0725083	1.0725082	1.07210946
8	.1083287D+01	1.0832872	1.0832871	1.08282208
9	.1094174D+01	1.0941745	1.0941743	1.09364045
10	.1105170D+01	1.1051712	1.1051710	1.10456562

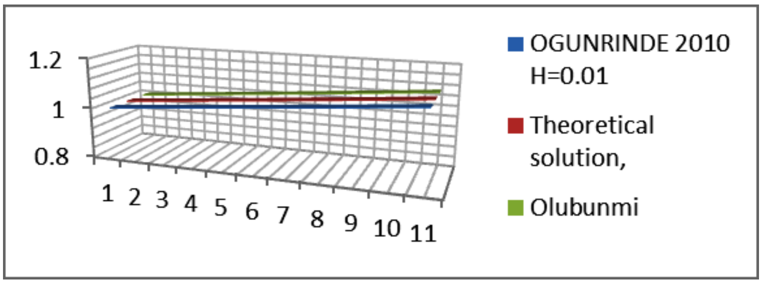

Figure 1: The Graphical representation of Table 1

Problem 2

The new cereal product was introduced into a company through an advertising campaign to a population of 1 million potential customers. The rate at which the population hears about the product is assume to be proportional to the number of people who are yet aware of the product. By the end of 1 year, half of the population has heard of the product.

Table 2: At H = 0.001.

T	OGUNRINDE 2010	THEORETICAL SOLUTION	OLUBUNMI
0	1	1	1
1	1.0010005	1.0010005	1.00100005
2	1.002002	1.002002	1.00200105
3	1.0030046	1.0030046	1.00300312
4	1.0040081	1.0040081	1.00400615
5	1.0050126	1.0050125	1.00501013
6	1.0060182	1.006018	1.00601518
7	1.0070248	1.0070245	1.00702119
8	1.0080323	1.0080321	1.00802815
9	1.0090408	1.0090406	1.00903618
10	1.0100504	1.0100502	1.01004517

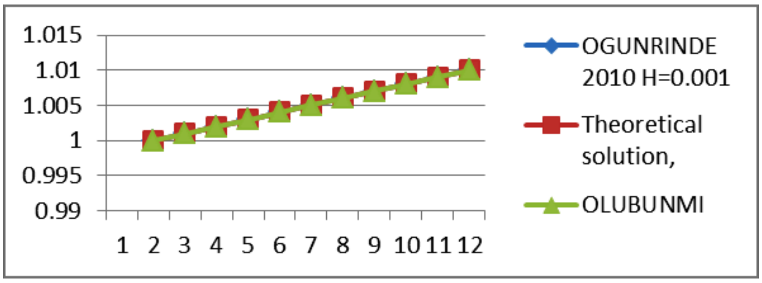

Figure 2: The Graphical representation of Table 2

Table 3: At H = 0.0001.

T	OGUNRINDE 2010	THEORETICAL SOLUTION	OLUBUNMI
0	1	1	1
1	1.0001	1.0001	1.00010002
2	1.0002	1.0002	1.00020003
3	1.0003	1.0003	1.00029993
4	1.0004001	1.0004001	1.00039995
5	1.0005001	1.0005001	1.00049996
6	1.0006001	1.0000602	1.00059998
7	1.0007002	1.0007002	1.00070012
8	1.0008004	1.0008004	1.00080013
9	1.0009005	1.0009004	1.00090015
10	1.0010006	1.0010005	1.00100029

How many will have heard of it by the end of 2 years? Mathematical Interpretation of the Problem Let y be the number (in millions) of people at time t who have heard of the product. This means that $1 - y$ is the number of people who have not heard, and $\frac{dy}{dt}$ is the rate at which the population hears about the product. From the given assumption, you can write the differential equation as follows.

$$\frac{dy}{dt} = k(1 - y)$$

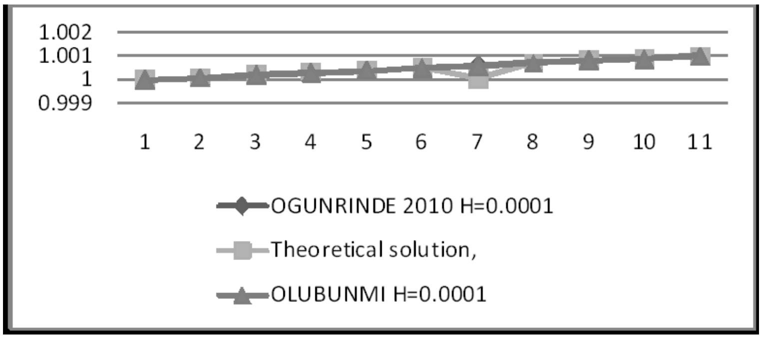

Figure 3: The Graphical representation of Table 3

Using separation of variables or a symbolic integration utility, you can find the general solution to be

$$y = 1 - Ce^{-kt}$$

To solve for the constant C and k use the initial conditions. That is, because $y = 0$ when $t = 0$, you can determine that $C = 1$, similarly, because $y = 0.5$ when $t = 1$, it follows that $0.5 = 1 - e^{-k}$, which implies that

$$k = \ln 2 \approx 0.693$$

So that the particular solution is obtained as

$$y = 1 - e^{-0.693t}$$

You can determine that the number of people who have heard of the product after 2 years is

$$y = 1 - e^{-0.693t}$$

Table 4: At H = 0.1.

T	OGUNRINDE 2010	THEORETICAL SOLUTION	OLUBUNMI
0	3	3	3
1	3	2.9900498	3
2	2.9789658	2.9607894	2.98030829
3	2.9377823	2.9139311	2.94229841
4	2.8786058	2.8521438	2.88866377
5	2.8046827	2.7788007	2.82338285
6	2.7200534	2.6976762	2.75192475
7	2.629179	2.6126263	2.68288875
8	2.5365391	2.5272923	2.64216757
9	2.4462538	2.4448581	2.35845251
10	2.3617744	2.3678794	2.31527181

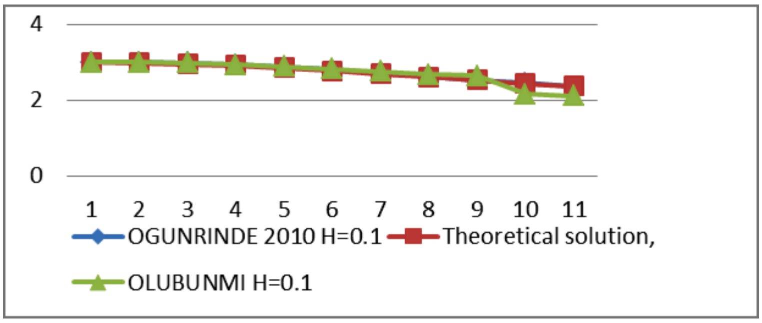

Figure 4: The Graphical representation of Table 4

4. Conclusion

We have developed a new numerical scheme which favourably compares with the existing methods for solving some initial value problems of ordinary differential equations. Clearly, this paper has been able to show the development of the new numerical scheme, the implementation of new scheme to solve some first order initial value problems of ordinary differential equations and also compare the results with the existing methods. In our subsequent research, we shall pay more attention on the implementation of this new scheme to solve some second order initial value problems of ordinary differential equation and also compare the results with the existing methods and thereafter examine the characteristics properties such as the stability, convergence, accuracy and consistency of the scheme.

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