

Stress Relaxation Behavior of the Annular Sealing Element - A Linear Modeling Approach

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Abstract. In the scope of this project, an annular sealing element was studied as a viscoelastic material. The main focus was to analyze the stress relaxation behavior of the annular sealer and investigate the performance of a linear viscoelastic mechanical model. Influence of viscoelastic behavior of the annular sealing element on its sealing ability is realized based on the hypothesis of elastic analogy. Based on a linear modeling approach a relationship allowing to determine the axial load ensuring seal tightness depending on its physic-mechanical properties and dimensions is established. The results of numerical calculations are represented in the form of graphs. It is shown that, viscous-elastic properties of sealer's material greatly influence on its sealing ability. Relaxation data suggested that for constant value of axial deformation in the section of application of external force with regard to heredity the stress greatly relaxes. This time stress relaxation for different velocities of deformation occurs differently.

Key Words and Phrases: contact pressure, sealing element, viscoelasticity, stress relaxation.

1. Introduction

Mechanism achieving seal tightness is strongly influenced by the application of mechanical loading, deformation, applied stress/stain rate, temperature and time. The main characteristic of their behavior is the viscoelastic response to the process achieving tightness. Achieving tightness by applying a minimum external load to sealing elements would improve their performance and determination of sealing parameters has an important scientific value [1-5], [7-10]. A major problem with these studies arises from the ignoring influence of edge effects and heredity and also, mechanism of achieving tightness was not studied enough. The current investigations for sealing elements based on viscoelastic materials are susceptible to heredity which may lead to the device failure. As the experience of using sealing elements shows, the edge effects and heredity have a significant influence on their sealing ability. In dynamic or static loading conditions, the sealer's materials will progressively accumulate permanent deformation, which part of it is internal damage, structure changes, due to qualitative changes [11-14].

In this paper, based on theoretical investigations, we will analyze the stress relaxation behavior of the annular sealer and investigate the performance of a linear viscoelastic

mechanical model. We will determine the influence of viscous-elastic behavior of the annular sealing element on the value of the axial load ensuring seal tightness when the subject to unidirectional compression. Numerical calculations will be conducted under different conditions, and the results of numerical calculations will be represented in the form of graphs of contact pressure and external force necessary achieving tightness and discussed.

Nomenclature

u, w	deformations of the protruding part of the sealing element in the radial and axial directions
$\varepsilon_r, \varepsilon_\theta, \varepsilon_z, \gamma_{rz}$	radial, tangential, axial and shear deformations
$\sigma_r, \sigma_z, \tau_{rz}$	radial, axial and tangential stress
s	hydrostatic pressure function
P	axial load
h	height of the protruding part of the annular sealing element
R_1, R_2	inner and outer radii of the sealer
Δ	axial displacement of the contact surface of the sealer
E_M	instantaneous modulus of elasticity
E_2	elasticity modulus
G	shear modulus of the sealing material
μ	friction coefficient between the smooth surface and the sealer
η	dynamical viscosity of the material of the sealing element
δ_{ij}	Kronecker's symbol

2. Statement of the problem. Elastic solution

Let us consider an annular sealing element in the form of a hollow cylinder, the lower part of which is inserted into the seat of the rigid valve (Fig.1). The protruding part of the annular sealing element creates contact pressure, leaning on a smooth rigid surface. When the width of the annular sealing element is much smaller than its other dimensions, let us assume that, the contact pressure, which is formed along the width, is systematically distributed, and its deformation condition is axially-symmetric. Then, by accepting the hypothesis of plane sections the axial deformation of the protruding part of the sealing element can be obtained depending only on the coordinate z in the axial direction.

We locate the origin of the coordinate system in the center of the cross-section of the sealing element, direct the coordinate axis z vertically-upwards, the axis r to the direction

of increasing the radius (see Fig.1).

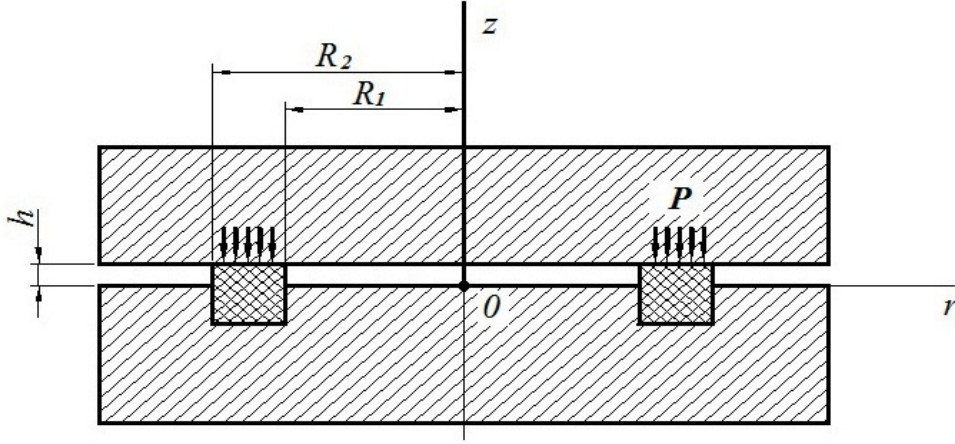


Figure 1: Calculation scheme

In the paper [1] problem was solved in the elastic statement using the methods of elasticity [4], [15]. For the potential energy of the annular sealing element, after its deformation with regard to axisymmetry, we have the equality [2], [4], [18], [19]

$$\Pi = 4\pi G \int_0^h \int_{R_1}^{R_2} \left(\varepsilon_r^2 + \varepsilon_\theta^2 + \varepsilon_z^2 + \frac{1}{2} \gamma_{rz}^2 \right) r dr dz - \int_0^h P f'(z) dz, \quad (1)$$

where h is height of the protruding part of the sealing element; R_1, R_2 are inner and outer radii of the annular sealer; G is a shear modulus of the sealing material; P is axial load; $\varepsilon_r, \varepsilon_\theta, \varepsilon_z$ and γ_{rz} are radial, tangential, axial and shear deformations, respectively [4], [15]:

$$\varepsilon_r = \frac{\partial u}{\partial r}; \varepsilon_\theta = \frac{u}{r}; \varepsilon_z = \frac{\partial w}{\partial z}; \gamma_{rz} = \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right). \quad (2)$$

where u, w are deformations of the protruding part of the sealing element in the radial and axial directions accordingly.

The boundary conditions are as the follows:

$$u|_{z=0} = 0, w|_{z=0} = 0, \sigma_r|_{r=R_2} = 0, \tau_{rz}|_{z=h} = \mu \sigma_z|_{z=h},$$

where $\sigma_r, \sigma_z, \tau_{rz}$ are radial, axial and tangential stress; μ is a friction coefficient between the smooth surface and the sealer.

Based on the principle of variation [2], [16], [17], the dependence between the magnitude of the axial load necessary for achieving tightness and geometrical sizes and character of contract pressure distribution were determined in the elastic statement [1]:

$$P = \frac{\pi G \Delta k R_2^2 (2\mu \sinh(kh) + k R_2 \cosh(kh)) \left[\frac{1}{1-\xi^2} - 3(1-\xi^2) \right]}{\mu (\cosh(kh) - 1) + \frac{kh}{2} (2\mu \sinh(kh) + k R_2 \cosh(kh))}, \quad (3)$$

$$\sigma_k = \frac{G k \Delta \left[\frac{1}{1-\xi^2} - 3(1-\xi^2) \right] (2\mu \sinh(kh) - kR_2 \cosh(kh))}{\left(1 - \frac{R_1^2}{R_2^2}\right) (\mu (\cosh(kh) - 1) + \frac{kh}{2} (2\mu \sinh(kh) + kR_2 \cosh(kh)))}, \quad (4)$$

where Δ is the displacement of the contact surface of the sealer; $\xi = \frac{R_1}{R_2}$;

$$k = \frac{1}{R_2} \sqrt{\frac{2 \left[\frac{1}{1-\xi^2} - 3(1-\xi^2) \right]}{\frac{1-\xi^4}{4} + 1 - \xi^2 - \ln \xi}}; B = \frac{\mu P_0 R_2^2 \left(\frac{1-\xi^4}{4} + 1 - \xi^2 - \ln \xi \right)}{(2\mu \sinh(kh) + kR_2 \cosh(kh)) \left[\frac{1}{1-\xi^2-3(1-\xi^2)} \right]}$$

$$P_0 = \frac{P}{\pi G R_2^4 \left(\frac{1-\xi^4}{4} + 1 - \xi^2 - \ln \xi \right)}.$$

3. Viscoelastic modeling. A linear modeling approach

Accounting of viscous-elastic properties of the material of the sealing element on sealing ability may be realized based on the hypothesis of elastic analogy [4], [14]. By this hypothesis when passing from elastic calculation to viscous-elastic one, only dependence between the stresses and strains change.

It should be noted that at elastic analogy, all stress components satisfy the dependence between stresses and strain obtained on the basis of the chosen model for a uniaxial stress-strain state.

The dependence between the stress-strain components for an arbitrary case of loading of a model that describes best the viscoelastic behavior of the material of a sealing element, is of the form [3], [4], [11], [12]

$$\dot{\tau}_{ij} + \lambda \tau_{ij} = G \left[2 \left(\dot{\varepsilon}_{ij} + \nu \varepsilon_{ij} \right) + \delta_{ij} \left(\dot{s} + \nu s \right) \right], \quad (5)$$

where $E_1 = E_M$, $\lambda = \frac{E_1+E_2}{\eta}$, $\nu = \frac{E_2}{\eta}$, η is dynamical viscosity of the material of the sealing element, E_M is instantaneous modulus of elasticity, E_2 is an elasticity modulus, τ_{ij} are stress components, ε_{ij} are relative strain components, δ_{ij} is Kronecker's symbol, $\dot{\tau}_{ij}$ and $\dot{\varepsilon}_{ij}$ is a time derivative from stress and strain components.

Based on elastic analogy we represent

$$\varepsilon(\bar{x}, t) = \varepsilon(\bar{x}) \varepsilon(t). \quad (6)$$

Substituting expression (6) in formula (5), we get

$$\dot{\tau}_{ij} + \lambda \tau_{ij} = G (2\varepsilon_{ij}(\bar{x}) + \delta_{ij}s(\bar{x})) \left(\dot{\varepsilon}_{ij}(t) + \nu \varepsilon_{ij}(t) \right). \quad (7)$$

Integrating expression (7) with the initial condition $\tau_{ij}(\bar{x}, 0) = G (2\varepsilon_{ij}(\bar{x}) + \delta_{ij}s(\bar{x}))$ we get

$$\tau_{ij} = (2\varepsilon_{ij}(\bar{x}) + \delta_{ij}s(\bar{x})) G \left[e^{-\lambda t} + \int_0^t \left(\dot{\varepsilon}_{ij}(\xi) + \nu \varepsilon_{ij}(\xi) \right) e^{-\lambda(t-\xi)} d\xi \right]. \quad (8)$$

Introducing the denotation

$$\bar{G} = G \left[e^{-\lambda t} + \int_0^t \left(\dot{\varepsilon}(\xi) + \nu \varepsilon(\xi) \right) e^{-\lambda(t-\xi)} d\xi \right], \quad (9)$$

we can represent the expression (8) in the form

$$\tau_{ij} = \bar{G} [2\varepsilon_{ij}(\bar{x}) + \delta_{ij}s(\bar{x})]. \quad (10)$$

For the considered case, when the sealing element at initial moment of deformation $w(z, t)|_{t=0} = w^*(z)$.

$$w(z, t) = w^*(\bar{z}) w(t), w(t) = 1. \quad (11)$$

Then allowing for (11), from expression (9) we get

$$\bar{G} = G \left[e^{-\lambda t} + \nu \int_0^t e^{-\lambda(t-\xi)} d\xi \right] = G \left[\left(1 - \frac{\nu}{\lambda}\right) e^{-\lambda t} + \frac{\nu}{\lambda} \right]. \quad (12)$$

Allowing for the expression (12), from the expression (3) we get formula for the axial load

$$P = \pi G \Delta k R_2^2 \left[\left(1 - \frac{\nu}{\lambda}\right) e^{-\lambda t} + \frac{\nu}{\lambda} \right] \frac{(2\mu \sinh(kh) + k R_2 \cosh(kh)) \left[\frac{1}{1-\xi^2} - 3(1-\xi^2) \right]}{\mu (\cosh(kh) - 1) + \frac{kh}{2} (2\mu \sinh(kh) + k R_2 \cosh(kh))}. \quad (13)$$

Allowing for the expression (12), from the expression (4) we get for the contact stress

$$\sigma_k = \frac{G k \Delta (2\mu \sinh(kh) - k R_2 \cosh(kh)) \left[\frac{1}{1-\xi^2} - 3(1-\xi^2) \right] \left[\left(1 - \frac{\nu}{\lambda}\right) e^{-\lambda t} + \frac{\nu}{\lambda} \right]}{\left(1 - \frac{R_1^2}{R_2^2}\right) (\mu (\cosh(kh) - 1) + \frac{kh}{2} (2\mu \sinh(kh) + k R_2 \cosh(kh)))}. \quad (14)$$

4. Stress relaxation depending on stain rate

We now consider the case when the sealing element deforms at a steady rate till the fixed time T and then it stays stable. Based on the elastic analogy [3], [4], [14] accepting the deformation of cross sections of the sealer in the form (Fig. 3)

$$\varepsilon(z, t) = \varepsilon(\bar{z}) \cdot \varepsilon(t), \quad (15)$$

$$\varepsilon_1(t) = w_1(t) = \frac{t}{T} [H(t) - H(t-T)] + H(t-T), \quad (16)$$

where $H(t)$ is a Heaviside function, T is time of deformation of the protruding part of the sealing element.

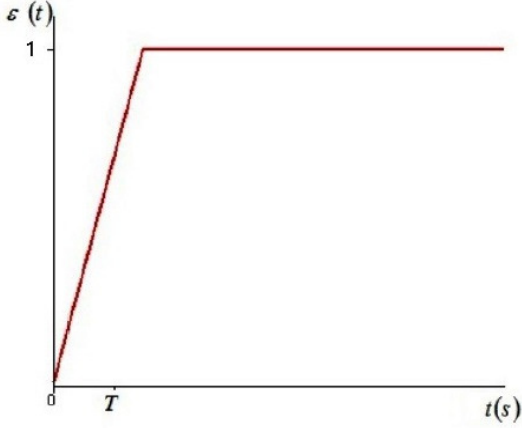


Figure 2: Graph of time dependence of relative axial strain of the protruding section
From formulas (9) and (16) we get

$$\bar{G} = G \left\{ e^{-\lambda t} + \int_0^t \left[\frac{1}{T} (H(\xi) - H(\xi - T)) + \frac{\xi}{T} (\delta(\xi) - \delta(\xi - T)) + \delta(\xi - T) + \right. \right. \\ \left. \left. + \nu \left(\frac{\xi}{T} (H(\xi) - H(\xi - T)) + H(\xi - T) \right) \right] e^{-\lambda(t-\xi)} d\xi \right\}, \quad (17)$$

where $\delta(t)$ is Dirac's function.

Integrating formula (17), we get

$$\bar{G} = \frac{G}{\lambda^2 T} \{ (\nu - \lambda) (H(-T) - H(t - T)) \exp(-\lambda(t - T)) + \\ + [-\lambda - \nu \lambda (t - T) + \nu] H(t - T) + [(-\nu - \nu \lambda T + \lambda) H(-T) + \\ + (\nu - \lambda) H(t) + \lambda^2 T] \exp(-\lambda t) + (-\nu + \nu \lambda t + \lambda) H(t) \}. \quad (18)$$

Then, allowing for formula (18) following from the expression (3) we get

$$P = \frac{\pi G \Delta k R_2^2}{\lambda^2 T} \{ (\nu - \lambda) (H(-T) - H(t - T)) \exp(-\lambda(t - T)) + \\ + [-\lambda - \nu \lambda (t - T) + \nu] H(t - T) + \\ + [(-\nu - \nu \lambda T + \lambda) H(-T) + (\nu - \lambda) H(t) + \lambda^2 T] \exp(-\lambda t) + \\ + (-\nu + \nu \lambda t + \lambda) H(t) \} \frac{(2\mu \sinh(kh) + k R_2 \cosh(kh)) \left[\frac{1}{1-\xi^2} - 3(1-\xi^2) \right]}{\mu (\cosh(kh) - 1) + \frac{kh}{2} (2\mu \sinh(kh) + k R_2 \cosh(kh))}. \quad (19)$$

From the expression (19) we define the axial load ensuring seal tightness depending on its physic-mechanical properties and dimensions.

Substituting formulas (18) in expression (4) for the contact stress with regard to heredity, we get

$$\begin{aligned} \sigma_k = & \frac{G k \Delta \left[\frac{1}{1-\xi^2} - 3(1-\xi^2) \right] (2\mu \sinh(kh) - kR_2 \cosh(kh))}{\lambda^2 T \left(1 - \frac{R_1^2}{R_2^2} \right) \left(\mu (\cosh(kh) - 1) + \frac{kh}{2} (2\mu \sinh(kh) + kR_2 \cosh(kh)) \right)} \times \\ & \times \{ (\nu - \lambda) (H(-T_2) - H(t - T_2)) \exp(-\lambda(t - T_2)) + \\ & + [-\lambda - \nu \lambda(t - T_2) + \nu] H(t - T_2) + [(-\nu - \nu \lambda T_2 + \lambda) H(-T_2) + \\ & + (\nu - \lambda) H(t) + \lambda^2 T_2] \exp(-\lambda t) + (-\nu + \nu \lambda t + \lambda) H(t) \}. \end{aligned} \quad (20)$$

5. Numerical calculation and discussion

The numerical calculation was made by formulas (13), (14) (19) and (20). Parameters and corresponding values are listed in Table 1. The results of numerical calculations are represented in the form of graphs of contact pressure and external force necessary achieving sightless (Fig.3 - Fig.10).

Table 1: The values of parameters

Variable	Unit	Value
R_2 , the outer radius of the sealer	m	$5 \cdot 10^{-2}$
$\xi, \xi = \frac{R_1}{R_2}$	-	$0.8 \div 0.95$
Δ , axial displacement of the contact surface of the sealer	m	$0.25 \cdot 10^{-3} \div 2 \cdot 10^{-3}$
h , the height of the protruding part of the annular sealing element	m	$1, 2, 5 \cdot 10^{-5}$
G , the shear modulus of the sealing material	Pa	$1.3 \cdot 10^8$
μ , the friction coefficient between the washer and the end of the sealer	-	0.5
$\nu, \nu = \frac{E_2}{\eta}$	-	0.01
$\lambda, \lambda = \frac{E_1 + E_2}{\eta}$	-	0.1
T , the time of deformation of the protruding part of the annular sealing element	s	10, 20, 30, 40, 50, 60

Fig.3 – Fig.5 shows the axial load/time dependence for the different values of the parameters Δ, h, ξ in the linear viscoelastic modeling under instantaneous loading.

Figure 3: Axial load/time dependence for the different value of the axial

displacement of the contact surface of the sealer ($h = 10^{-5}m, \xi = 0.8$)

Fig.3 shows the axial load/time dependence for the different value of the axial displacement of the contact surface of the sealer. It follows from Fig.3 that for every Δ , the value of the axial load decreases at a decreasing rate. With increasing the value of the Δ axial displacement both the value of the axial load and its relaxing rate increase.

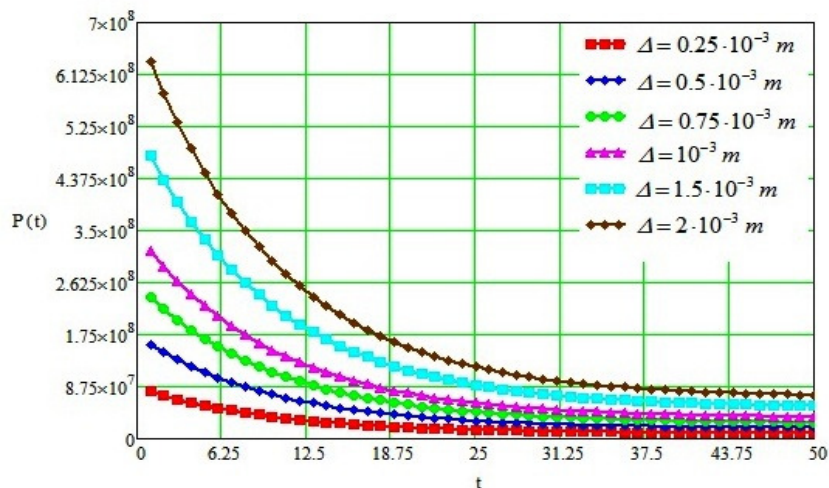


Figure 4: Axial load/time dependence for the different value of the height of the protruding part of the sealer ($\Delta = 10^{-3}m, \xi = 0.8$)

Fig.4 shows the relaxing of the axial load for the different value of the height of the protruding part of the annular sealing element. The axial load ensuring seal tightness relaxes by the time and for 30 second its value at the point of application of the external force decreases, and then stabilizes. While the element's height increases, the value of the axial load for tightness and its relaxing rate decrease, due to Fig.4.

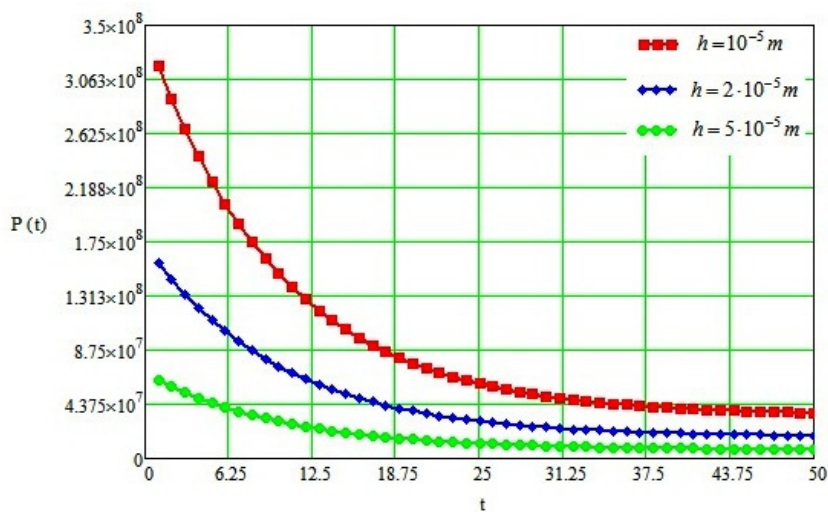


Figure 5: Axial load/time dependence for the different ratio of the inner and outer radii of the sealer ($h = 10^{-5}m, \Delta = 10^{-3}m$)

For Fig.5, the value at the point of application of the external load ensuring seal tightness relaxes at a decreasing rate by the time. With increasing the ratio of the sealer's inner and outer radii both the value of the axial load for tightness and its relaxing velocity increase. The model stress relaxation response under instantaneous loading are shown in Fig.6 – Fig.8 at different values of parameters Δ, h, ξ (Table 1). As is seen from the curves contact stress relaxes at a decreasing rate by time.

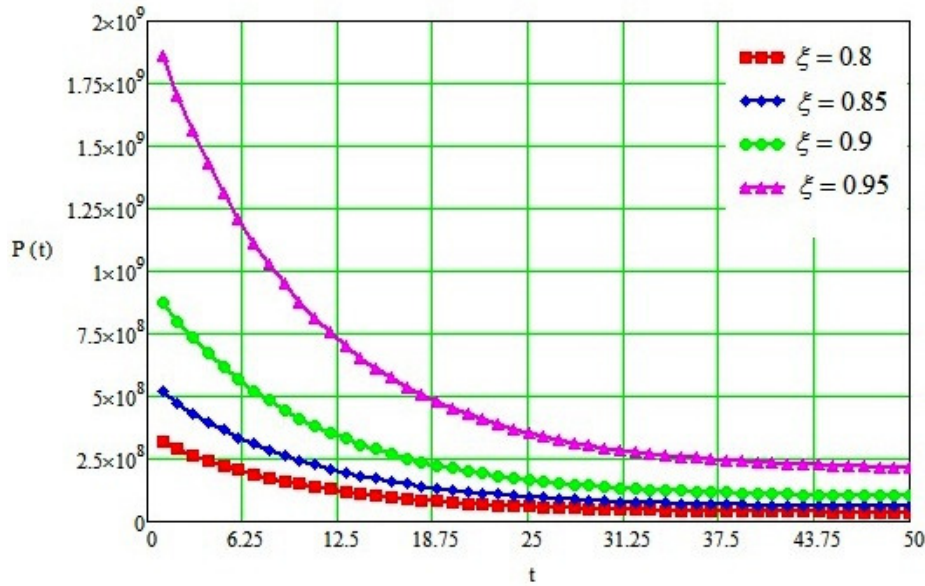


Figure 6: Stress relaxation curve for the different value of the axial displacement of the contact surface of the sealer ($h = 10^{-5}m, \xi = 0.8$)

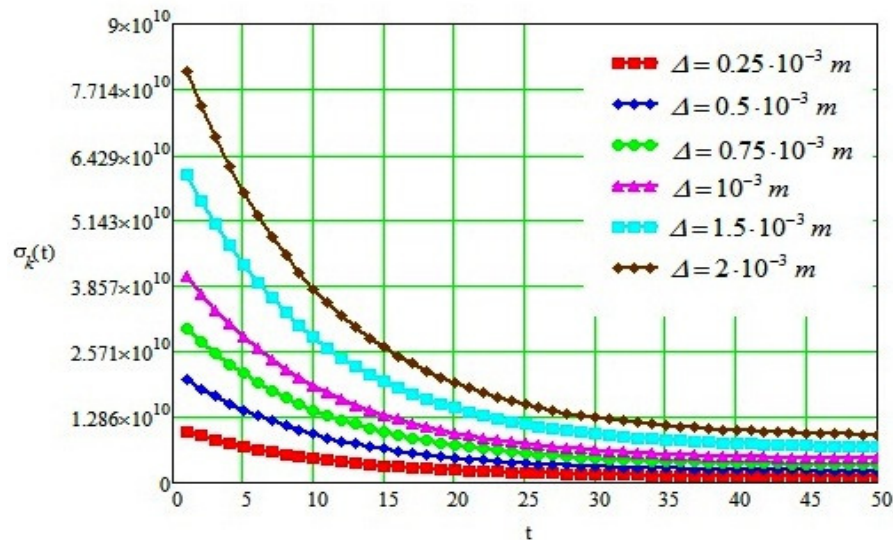


Figure 7: Stress relaxation curve for the different value of the

height of the protruding part of the sealer ($\Delta = 10^{-3}m, \xi = 0.8$)

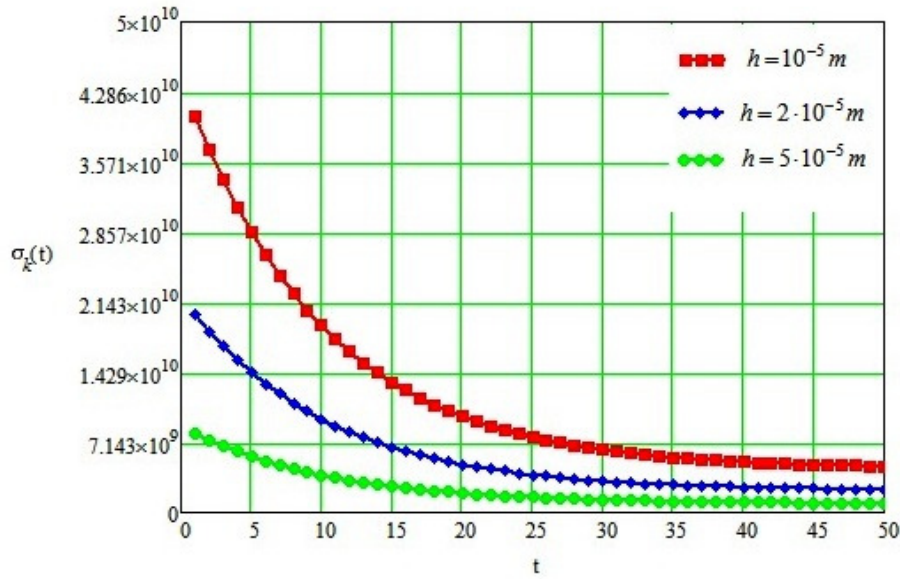


Figure 8: Stress relaxation curve for the different ratio of the

inner and outer radii of the sealer ($h = 10^{-5}m, \Delta = 10^{-3}m$)

The value of the contact stress and its relaxing rate increase by rise the value of the Δ axial displacement (Fig.6), but they decrease by increasing the values of the parameters h and ξ (Fig.7 and Fig.8).

Fig.9 and Fig.10 display relaxation process of the sealer due to gradual applied load. This time axial stress relaxation for different velocities of deformation occurs differently (Fig. 9 and Fig.10).

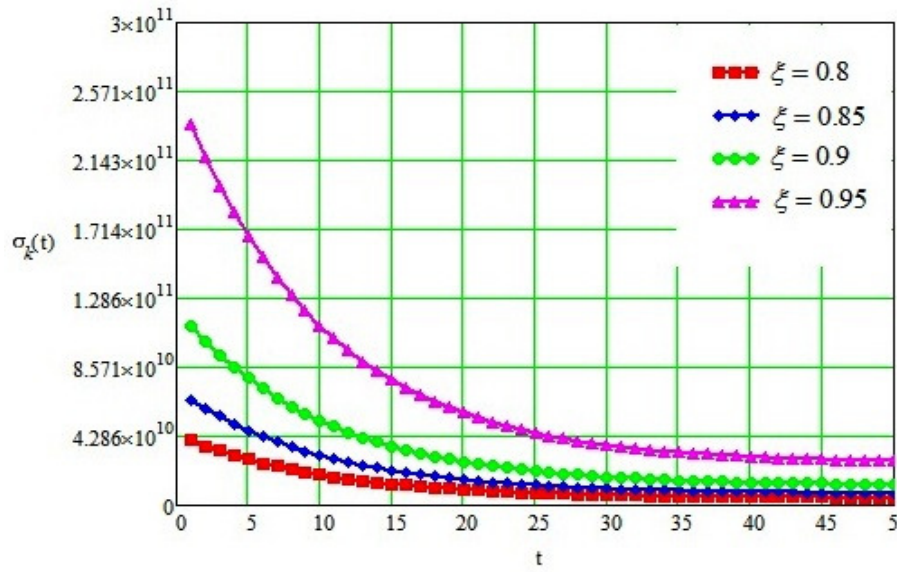


Figure 9 Axial load/time dependence due to gradual applied load with regard to heredity ($h = 10^{-5}m$, $\xi = 0.95$, $\Delta = 0.25 \cdot 10^{-3}m$).

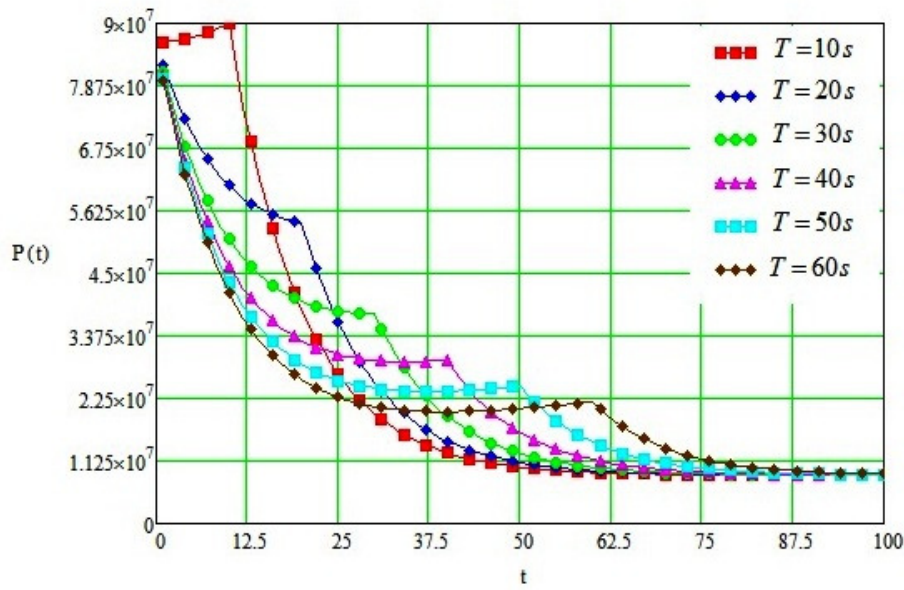


Figure 10: Stress relaxation curve due to gradual applied load

$$(h = 10^{-5}m, \xi = 0.8, \Delta = 0.25 \cdot 10^{-3}m)$$

6. Conclusion

In this article, the main focus is to analyze the stress relaxation behavior of the annular sealer and investigate the performance of a linear viscoelastic mechanical model. Influence of viscoelastic behavior of the annular sealing element on its sealing ability is realized based on the hypothesis of elastic analogy. We determine stress-strain state of the annular sealing element in the form of a hollow cylinder for two different loading type - instantaneous loading and gradual applied loading .

Under instantaneous loading the value of the axial load and contact stress decreases at a decreasing rate by the time in the linear viscoelastic modeling. The values of the axial load and contact stress and their relaxing rates increase by rise the value of the Δ axial displacement , but they decrease by increasing the values of the parameters h and ξ . Due to gradual applied loading axial stress relaxation for different velocities of deformation occurs differently. Contact stress relaxes by time and its value decreases, and then stabilizes.

The current work demonstrates that viscous elastic properties of the sealing material greatly influence on its sealing ability and their ignorance may lead to incorrect conclusions. It is shown that, viscous-elastic properties of sealer's material greatly influence on its sealing ability. Because of heredity of the sealer's material, the values of external forces in some cases drop about seven times.

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