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The Third Compilation is a Mixed Discrete Additive and Derivative Equation for Discrete Multiplicative Investigation of the Solution of Issues

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Abstract. Cauchy and boundary problems for a two-dimension discrete derivative equation of the third order will be reviewed here.
Thus, the solution of the problems for the second-order discrete additive derivative from one variable, the third-order equation with the first-order discrete multiplicative derivative from the other variable is investigated.
As in previous works, analytical expressions will be justified here as well by setting up the problems.

multivariate discrete derivative equation, discrete additive derivative, discrete multiplicative derivative, Cauchy problem, boundary problem, analytical expression of problem solving.

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1. Introduction

The derivative taught in "Algebra and the beginning of analysis" in secondary school and "Mathematical Analysis" course in Higher School is mainly additive derivative [1] – [2]. Although the multiplicative derivative has been created for around nearly a century [3], problems for the multiplicative derivative equations have been considered recently [4] - [5]. Here we will talk about the discrete cases of these additive and multiplicative derivatives [6] - [8]. We began to look at the problems for ordinary discrete additivo-multiplicative and multiplicativo - additive derivative equations, [9] - [11]. It should be noted that the markings for discrete derivatives and integrals also belong to us [12].

2. Solution of the problem

Here we look at Cauchy and boundary value problems for a two-dimensional third-order equation (the second-order discrete additive relative to one variable, which holds a discrete multiplicative derivative relative to the other variable).

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Let's look at the equation as follows

$$D_2^{(\prime)} D_1^{(\prime\prime)} u_{ij} = f_{ij}, \ \ i \geq 0, \ \ j \geq 0, \tag{1}$$

the real-valued sequence given here $f_{ij}, i \geq 0, j \geq 0$ is the sequence sought $u_{ij}, i \geq 0, j \geq 0$ if there is one.

Using the definition of discrete additive and discrete multiplicative derivatives (1), let's bring the equation to the equation clearly with the following differences:

$$U_{i+2,j+1} = 2U_{i+1,j+1} - U_{i,j+1} + f_{ij}(u_{2+ij} - 2u_{i+1j}), \ \ i \geq 0, \ \ j \geq 0 \tag{2}$$

By taking in the linear equation $j = 0$ we get (2) we get it by giving values starting from zero,

$$u_{i1} = iu_{11} - (i-1)u_{01} + \sum_{k=0}^{i-2}(i-1-k)\times$$
$$\times f_{k0}(u_{i+k,0} - 2u_{i-1+k,0} + u_{i-2+k,0}), i \geq 2, \tag{3}$$

we'll get his expression. By the same rule u_{i2} and u_{i3} after receiving the expressions for

$$u_{ij} = iu_{1j} - (i-1)\, u_{0j} + \sum_{k=0}^{i-2} (i-1-k)\cdot$$

$$\cdot f_{kj-1} \left(u_{i-3+k,j-1} - 2u_{i-4+k,j-1} + u_{i-5+k,j-1} \right), \ \ i \geq 0, j \geq 0 \tag{4}$$

the result concluded, which is given through u_{1j}, u_{0j} and u_{sj-1}.

3. The solution is solved by integration

Since it is not possible to get the expression u_{ij} from the expression we get (4) through u_{s0} we return to the equation (1) and get it by applying the definition of a discrete multiplier derivative:

$$\frac{D_1^{(\prime\prime)} u_{ij+1}}{D_1^{(\prime\prime)} u_{ij}} = f_{ij}, \ \ i \geq 0, j \geq 0.$$

By setting the value here to j:

$$\frac{D_1^{(\prime\prime)} u_{i1}}{D_1^{(\prime\prime)} u_{i0}} = f_{i0},$$
$$\frac{D_1^{(\prime\prime)} u_{i2}}{D_1^{(\prime\prime)} u_{i1}} = f_{i1},$$
$$\vdots$$
$$\frac{D_1^{(\prime\prime)} u_{ij}}{D_1^{(\prime\prime)} u_{ij-1}} = f_{ij-1},$$

If we make cross multiplication of the expressions, we will get:

$$\frac{D_1^{('')}u_{ij}}{D_1^{('')}u_{i0}} = \prod_{k=0}^{j-1} f_{ik}, i \geq 0, j \geq 1,$$

or

$$D_1^{('')}u_{ij} = D_1^{('')}u_{i0} \cdot \prod_{k=0}^{j-1} f_{ik}, \quad i \geq 0, j \geq 1. \tag{5}$$

Let's accept the marking as follows:

$$g_{ij} = g_{ij}\left(D_1^{('')}u_{i0}, f_{ik}\right) = D_1^{('')}u_{i0} \cdot \prod_{k=0}^{j-1} f_{ik}, \quad i \geq 0, j \geq 1, \tag{6}$$

then (5) the equation will be as follows:

$$D_1^{('')}u_{ij} = g_{ij}, \quad i \geq 0, j \geq 1, \tag{7}$$

By this method, we have turned a third-order discrete derivative (1) to the second-order discrete additive derivative (linear) (7) equation.

We now take from (7) using the definition of discrete additive derivative:

$$D_1^{(')}u_{i+1j} - D_1^{(')}u_{ij} = g_{ij},$$

or

$$D_1^{(')}u_{i+1j} = D_1^{')}u_{ij} + g_{ij}, \quad i \geq 0, j \geq 1, \tag{8}$$

we get this expression. If here is $i = 0$ then

$$D_1^{(')}u_{1j} = D_1^{(1)}u_{0j} + g_{0j},$$

if here is $i = 1$, then

$$D_1^{(')}u_{2j} = D_1^{(')}u_{1j} + g_{1j} = D_1^{(')}u_{0j} + g_{0j} + g_{1j},$$

we get the expression. If we continue this process:

$$D_1^{(')}u_{ij} = D_1^{(')}u_{0j} + \prod_{k=0}^{i-1} g_{kj}, \quad i \geq 1, j \geq 1, \tag{9}$$

we would have got the relation. Finally, if we use the definition of a discrete additive derivative once again (for the left side) by giving values to i in (9), we get:

$$u_{2j} - u_{1j} = D_1^{(\prime)} u_{0j} + \sum_{k=0}^{0} g_{kj},$$

$$u_{3j} - u_{2j} = D_1^{(\prime)} u_{0j} + \sum_{k=0}^{1} g_{kj},$$

$$u_{4j} - u_{3j} = D_1^{(\prime)} u_{0j} + \sum_{k=0}^{2} g_{kj},$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$u_{ij} - u_{i-1j} = D_1^{(\prime)} u_{0j} + \sum_{k=0}^{i-2} g_{kj}.$$

If we add them together:

$$u_{ij} = u_{1j} + (i-1)\, D_1^{(1)} u_{0j} + \sum_{s=0}^{i-2} \sum_{k=0}^{s} g_{kj}, \;\; i \geq 2, j \geq 1. \tag{10}$$

Thus received the following verdict:

Theorem 3.1. *If f_{ij}, $i \geq 0$, $j \geq 0$ is sequence that satisfies the true value $f_{ij} \neq 0$ given by g_{ij}, then there is general solutions that can be expressed as equation* (10)*, where u_{i0}, u_{0j} and u_{ij} are arbitrary constants.*

4. Cauchy problem:

Let's give the starting conditions for the given (1) equation as follows:

$$u_{ij} = \alpha_{ij}, \;\; i \geq 0, \; j = 0; \;\; i = 0, \; j \geq 0; \;\; i = 1, j \geq 0, \tag{11}$$

then

$$D_1^{(\prime\prime)} u_{i0} = (u_{i+1,0} - u_{i0})^{(\prime)} = u_{i+2,0} - 2u_{i+1,0} + u_{i0} = \alpha_{i+2,0} - 2\alpha_{i+1,0} + \alpha_{i,0}, i \geq 0,$$

$$D_1^{(\prime)} u_{0j} = u_{1j} - u_{0j} = \alpha_{1j} - \alpha_{0j}, j \geq 0, \;\; u_{1j} = \alpha_{1j}, j \geq 0, \tag{12}$$

(6) we get from (9) and (10)

$$g_{ij} = (\alpha_{i+2,0} - 2\alpha_{i+1,0} + \alpha_{i,0}) \prod_{k=0}^{j-1} f_{ik}, \; j \geq 1, i \geq 0, \tag{13}$$

$$D_1^{(\prime)} u_{ij} = (\alpha_{1j} - \alpha_{0j}) + \sum_{s=0}^{i-1} g_{sj}, \;\; i \geq 1, j \geq 1, \tag{14}$$

so we get

$$u_{ij} = \alpha_{1j} + (i-1)(\alpha_{1j} - \alpha_{0j}) + \sum_{s=0}^{i-2}\sum_{k=0}^{s} g_{kj}, \ \ i \geq 2, \ \ j \geq 1. \tag{15}$$

Theorem 2. Under the terms of the Theorem 1, if- α_{ij} s are the real-valued sequences given, then (1), (11) has the solution of the Cauchy problem, and this solution is in the form of (15), so as g_{ij} are defined by means of (13).

5. Boundary problem

Now let's look at the equation (1) at the values of i, C, for this equation $0 \geq i \geq N-1$ we $0 \geq j \geq M-1$ set the boundary conditions as follows:

$$\begin{cases} D_1^{(\prime\prime)} u_{i0} + a_i u_{NM} = A_i, i \geq 0, \\ D_1^{(I)} u_{0j} + b_j u_{NM} = B_j, j \geq 0, \\ u_{1j} + c_j u_{Nj} = C_j, j \geq 0. \end{cases} \tag{16}$$

Here a_j, A_j, b_j, B_j, c_j and C_j - the real-valued sequences given. Then from (6) and (10) we will get:

$$g_{ij} = (A_i - a_i u_{NM}) \prod_{k=0}^{j-1} f_{ik}, \ \ i \geq 0, j \geq 1, \tag{17}$$

$$u_{ij} = (C_j - c_j u_{Nj}) + (i-1)(B_j - b_j u_{NM}) + \sum_{s=0}^{i-2}\sum_{k=0}^{s} g_{kj}, \ \ i = 2, j \geq 1, \tag{18}$$

if we take from these expressions (18), $i = N$, $j = M$ we will get:

$$u_{Nj} = (C_j - c_j u_{Nj}) + (N-1)(B_j - b_j M u_{Nj}) + \sum_{s=0}^{N-2}\sum_{k=0}^{s} g_{kj}, j \geq 1, \tag{5.3$_1$}$$

$$u_{NM} = (C_M - c_M u_{NM}) + (N-1)(B_M - b_M u_{NM}) + \sum_{s=0}^{N-2}\sum_{k=0}^{s} g_{kM}, \tag{5.3$_2$}$$

now, by writing in (5.3$_2$) the expression we get from (17) if $i = k$, $j = M$

$$u_{NM} = [C_M + (N-1) B_M] - [c_M + (N-1) b_M] u_{NM} +$$

$$+ \sum_{s=0}^{N-2}\sum_{k=0}^{s} (A_k - a_k u_{NM}) \prod_{p=0}^{M-1} f_{kp} =$$

$$= [C_M + (N-1) B_M] - [c_M + (N-1) b_M] u_{NM} +$$

$$+\sum_{s=0}^{N-2}\sum_{k=0}^{s}A_k\prod_{p=0}^{M-1}f_{kp}-u_{NM}\sum_{s=0}^{N-2}\sum_{k=0}^{s}a_k\prod_{p=0}^{M-1}f_{kp}$$

And from here,

$$\left\{1+c_M+(N-1)b_M+\sum_{s=0}^{N-2}\sum_{k=0}^{s}a_k\prod_{p=0}^{M-1}f_{kp}\right\}u_{NM}=$$

$$=C_M+(N-1)B_M+\sum_{s=0}^{N-2}\sum_{k=0}^{s}A_k\prod_{p=0}^{M-1}f_{kp}, \tag{19}$$

finally, if

$$1+c_M+(N-1)b_M+\sum_{s=0}^{N-2}\sum_{k=0}^{s}a_k\prod_{p=0}^{M-1}f_{kp}\neq 0, \tag{20}$$

condition is true, then from (19) we get:

$$u_{NM}=\frac{C_M+(N-1)\,B_M+\sum\limits_{s=0}^{N-2}\sum\limits_{k=0}^{s}A_k\prod\limits_{p=0}^{M-1}f_{kp}}{1+c_M+(N-1)\,b_M+\sum\limits_{s=0}^{N-2}\sum\limits_{k=0}^{s}a_k\prod\limits_{p=0}^{M-1}f_{kp}}, \tag{21}$$

The same rule take from (5.3_1) we obtain

$$u_{ij}=(C_j-c_ju_{Nj})+(N-1)\,(B_j-b_ju_{NM})+\sum_{s=0}^{i-2}\sum_{k=0}^{s}(A_k-a_ku_{NM})\prod_{p=0}^{M-1}f_{kp},$$

Here considering (21)

$$(1+c_j)u_{Nj}=C_j-\left[(N-1)b_j+\sum_{k=0}^{s}(A_k)\prod_{p=0}^{M-1}f_{kp}\right]\times$$

$$\times\frac{C_M+(N-1)\,B_M+\sum\limits_{s=0}^{N-2}\sum\limits_{k=0}^{s}A_k\prod\limits_{p=0}^{M-1}f_{kp}}{1+c_M+(N-1)\,b_M+\sum\limits_{s=0}^{N-2}\sum\limits_{k=0}^{s}a_k\prod\limits_{p=0}^{M-1}f_{kp}}+$$

$$+(N-1)b_j+\sum_{k=0}^{s}(A_k)\prod_{p=0}^{M-1}f_{kp}, \tag{$*$}$$

$1 + c_j \neq 0,$

$$u_{Nj} = \frac{1}{1+c_j}\left\{C_j - \left[(N-1)b_j + \sum_{k=0}^{s}(A_k)\prod_{p=0}^{M-1} f_{kp}\right]\times\right.$$

$$\times\frac{C_M + (N-1)\,B_M + \sum\limits_{s=0}^{N-2}\sum\limits_{k=0}^{s} A_k \prod\limits_{p=0}^{M-1} f_{kp}}{1 + c_M + (N-1)\,b_M + \sum\limits_{s=0}^{N-2}\sum\limits_{k=0}^{s} a_k \prod\limits_{p=0}^{M-1} f_{kp}} +$$

$$\left. + (N-1)b_j + \sum_{k=0}^{s}(A_k)\prod_{p=0}^{M-1} f_{kp}\right\}, j \geq 1, \qquad (**)$$

If we write (21) this expression in (17), it does not hold any discretion for by writing

$$g_{ij} = \left[A_i - a_i\frac{C_M + (N-1)\,B_M + \sum\limits_{s=0}^{N-2}\sum\limits_{k=0}^{s} A_k \prod\limits_{p=0}^{M-1} f_{kp}}{1 + c_M + (N-1)\,b_M + \sum\limits_{s=0}^{N-2}\sum\limits_{k=0}^{s} a_k \prod\limits_{p=0}^{M-1} f_{kp}}\right]\prod_{q=0}^{j-1} f_{iq}, \quad i \geq 0, j \geq 1, \qquad (22)$$

the analytical expression, finally (21), (*), (**) and (22) in (18), we get the solution of the boundary problem (1), (16).

So, we get the following solution.

Theorem 3. Under the terms of the Theorem 1, if the and conditions are true (20) with the given real-valued sequences, then there is a solution of the boundary problem (1), (16), and this solution is taken from (21),(*), (**) and (22) out of (18).

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