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Residual Stresses and Deformations of an Annular Plate

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Abstract. An annular plate loaded with distributed moment along the inner contour, is considered. It is supposed that when deformed, a part of the plate passes to a plastic state. The plate material properties are described by the equations of plastic flow theory; hardening of the plate material is excluded. The solution of the appropriate elastico-plastic problem when loading from the natural No deformedstate obtained by Nordgren and Nahdy, is used.
In the elastic domain $r_s \leq r \leq b$ we have :

$$u_\theta = -\frac{(1+\nu)\ M\,(t)}{2\pi\,E}\left(\frac{1}{r}-\frac{r}{b^2}\right).$$

In the plastic domain $a \leq r \leq r_s$ we have :

$$u_\theta = -\frac{(1-\nu)\ M\,(t)}{2\pi E}\left(\frac{1}{r}-\frac{r}{b^2}\right)-\frac{2}{\pi}\left(1-\frac{r}{b}\right)\int_0^t \dot{\lambda} M\,(\tau)\ d\tau\,.$$

The process of plate unloading after preliminary elastico-plastic deformation is studied. The moment distributed along the contour attains such value that at unloading in addition to appearance of elastic unloading domain there alsoappears a domain of secondary plastic deformations. Residual stresses and deformations in the domain of secondary plastic deformations are found by using the theorem on V.V. Moskvitin'ssecondary plastic deformations, in the domain of elastic unloading these quantities are determined by using A.A. Ilyushin's theorem on elastic unloading. At the same time, the boundary between the domains of elastic unloading with secondary deformations was found.
Key Words and Phrases: annular plate, distributed moment, elastico-plastic deformation, elastic unloading, secondary plastic deformation

1. Introduction

An annular plate as a structural element has a wide application in various branches of mechanical engineering and construction industry. In the framework of the theory of plastic flow, the statement of the mathematical problem on the deformation of this plate from the natural state was presented. In this connection, the studies in the field of strength analysis of annular plates can always be considered relevant. Here we used a cylindrical system of coordinates (r, θ, z) whose origin coincides with the center of the annular plate. This time $a \leq r \leq b$; $0 \leq \theta \leq 2\pi$.

Furthermore, the conjugation conditions u_r and σ_r hold for $r = r_s$ where r_s is the radius of the circle separating the elastic domain from plastic one. The materials of the plate have the property of elastic-plastic deformation. Early researches of elastic-plastic deformation of an annular plate belong to Nadai and Mizes [1]. In the framework of theory of small elastic-plastic deformations, problems on deformation of annular plate with various force factors were solved in [2-4]. In [1], the Tresc-Saint Venant condition, while in [5-6] the Mizes condition was used as a plasticity condition. Note that in the indicated works only the process of loading of an annular plate from a natural undeformed state was considered. Using the above solution, we determine residual stresses, strains and displacements that which will remain in the annular plate after removing the distributed moment $M(t)$. We used V.V.Moskvitin's theorem on secondary plastic deformations [9, 10]. Unlike the plate under consideration, the dummy plate material shear yield point is $2\sigma_s$. We determined residual stresses, strains and displacements in the domain $r_0 \leq r \leq r_s$. In this domain the unloading process is elastic. Consequently, residual stresses, strains and displacements in the domain $r_0 \leq r \leq r_s$ are determined as differences of appropriate formulas (3.8)-(3.14) when allowing for (3.15) and replacing t by $t_1/2$ and the elastic solution formulas. At the same time, the study of the process of unloading from the loaded state of a plate of considerable independent interest. This process is considered here. With this, along with elastic unloading we study the loading with the appearance of secondary plastic deformations. The results obtained here are some generalization of [12]

2. Statement of an elastic-plastic problem

An annular plate with inner radius a, outer radius b is loaded along the inner contour by the distributed moment $M(t)$ where t is time. In the framework of theory of plastic yield [7, 49] we give the statement of a mathematical problem on deformation of this plate from the natural state.

According to [7, 49] we accept equations of theory of plastic yield in the form:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda} \frac{\partial F}{\partial \sigma_{ij}} \,. \tag{2.1}$$

Here $\dot{\varepsilon}_{ij}^{p}$ is the rate of plastic deformations ε_{ij}^{p} ; $\dot{\lambda}$ is a loading parameter; σ_{ij} are current stresses, , *(i, j=1,2,3); F* is a loading function. We think the material of the plate does not have hardening. Therefore, in the plastic domain we have $F = 0$, $\dot{\lambda} \geq 0$ [7, p.50]

To relations (2.1) we should add relations for elastic components of deformations [8, 62].

$$\varepsilon_{ij}^{e} = \frac{1+\nu}{E} \left(\sigma_{ij} - \frac{3\nu}{1+\nu} \sigma \, \delta_{ij} \right) , \tag{2.2}$$

where ν is the Poisson ratio; E is a longitudinal elasticity modulus; δ_{ij} are the Kronecker symbols, $\sigma = \sigma_{ij}\delta_{ij}/3$ is average stress.

We will use a cylindrical system of coordinates $(r,\, \theta,\, z)$,whose center coincides with the center of the annular plate. This time $a \leq r \leq b\,; \quad 0 \leq \theta \leq 2\pi$. The stress components σ_r, σ_θ , $\sigma_{r\,\theta}$ should satisfy the following equilibrium equations [8, 72]:

$$\frac{\partial \sigma_r}{\partial r} = \frac{\sigma_\theta - \sigma_r}{r}\,, \frac{\partial \sigma_{r\theta}}{\partial r} = \frac{2}{r}\,\sigma_{r\theta} = 0 \tag{2.3}$$

Between the components of displacement u_r, u_θ and deformations ε_r, ε_θ, $\varepsilon_{r\theta}$ there is Cauchy's kinematic relation [8, 72]:

$$\varepsilon_r = \frac{\partial u_r}{\partial r}; \quad \varepsilon_\theta = \frac{u_r}{r}\,, \quad \varepsilon_{r\theta} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)\,. \tag{2.4}$$

The following boundary conditions should be satisfied:

$$\sigma_r|_{r=a} = 0, \quad \sigma_r|_{r=b} = 0, \quad u_\vartheta|_{r=b} = 0; \tag{2.5}$$

$$\int_0^{2\pi} \sigma_{r\theta} r^2\, d\theta = M\ (t)\ . \tag{2.6}$$

Furthermore, we have the conjugation conditions u_r and σ_r for $r = r_s$, where r_s is a radius of the circle separating the elastic domain from the plastic one.

3. The Nordgren and Nahdi solution

The elastic-plastic problem (2.1)-(2.6) was solved in the paper of Nordgren and Nahdi [2]. Further we will give this solution.

The loading function F is chosen in the form

$$F = \frac{1}{4}\,(\sigma_r - \sigma_\theta)^2 + \sigma_{r\theta}^2 - \tau_s^2\,, \tag{3.1}$$

where τ_s is pure shear yield point.

$$\sigma_{r\theta} = \frac{M\,(t)}{2\pi r^2} fora \leq r \leq b\,. \tag{3.2}$$

In the elastic domain $r_s \leq r \leq b$ we have

$$\left.\begin{matrix}\sigma_\theta \\ \sigma_r\end{matrix}\right\} = \frac{\left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{b^2}\left(1 \pm \frac{b^2}{r^2}\right)\tau_s\,, \tag{3.3}$$

$$\left.\begin{matrix}\varepsilon_\theta \\ \varepsilon_r\end{matrix}\right\} = \frac{\tau_s}{E}\left[r_s^4 - \left(\frac{M\,(t)}{2\pi\tau_s}\right)^2\right]^{1/2}\left(\frac{1-\nu}{b^2} \pm \frac{1+\nu}{r^2}\right)\,, \tag{3.4}$$

$$\varepsilon_{r\theta} = \frac{1+\nu}{E}\,\frac{M\,(t)}{2\pi r^2}\,, \tag{3.5}$$

$$u_r = \frac{\tau_s}{E}\left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}\left[\frac{(1-\nu)\, r}{b^2} + \frac{1+\nu}{2}\right], \tag{3.6}$$

$$u_\theta = -\frac{(1+\nu)\, M(t)}{2\pi\, E}\left(\frac{1}{r} - \frac{r}{b^2}\right). \tag{3.7}$$

In the plastic domain $a \le r \le r_s$ we have

$$\sigma_r = \tau_s\left\{\ln\frac{r^2 + \left[r^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{a^2 + \left[a^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}} + \frac{\left[a^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{a^2} - \frac{\left[r^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{r^2}\right\}, \tag{3.8}$$

$$\sigma_\vartheta = \sigma_r + \frac{2\tau_s}{r^2}\left[r^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}, \tag{3.9}$$

$$\varepsilon_r = \frac{1-\nu}{E}\sigma_r + \frac{2\tau_s}{Er^2}\left\{\left[r^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}(1-\nu) - \left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}\right\} \tag{3.10}$$

$$\varepsilon_\theta = \frac{1-0}{E}\sigma_r + \frac{2\tau_s}{Er^2}\left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}. \tag{3.11}$$

$$\varepsilon_{r\theta} = \frac{1+\nu}{2\pi\, r^2 E}M(t) + \frac{1}{\pi r^2}\int_0^t \dot{\lambda}\, M(\tau)\, d\tau, \tag{3.12}$$

$$u_r = \frac{1-\nu}{E}r\sigma_r + \frac{2\tau_s}{rE}\left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}, \tag{3.13}$$

$$u_\theta = -\frac{(1-\nu)\, M(t)}{2\pi E}\left(\frac{1}{r} - \frac{r}{b^2}\right) - \frac{2}{\pi}\left(1 - \frac{r}{b}\right)\int_0^t \dot{\lambda} M(\tau)\, d\tau. \tag{3.14}$$

The quantity included in (3.9)-(3.11), (3.13) is represented by formula (3.8). The loading parameter $\dot{\lambda}$ is determined by the formula

$$\dot{\lambda} = \frac{2}{E}\left\{\frac{M\dot{M}/\left(4\pi^2\tau_s^2\right)}{\left[r^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}} + \frac{2r_s^3\dot{r}_s - M\dot{M}/\left(4\pi^2\tau_s^2\right)}{\left[\left(r^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right)\left(r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right)\right]^{1/2}}\right\}. \tag{3.15}$$

The unknown radius r_s is the solution of the following equation:

$$\ln\frac{r_s^2 + \left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{a^2 + \left[a_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}} + \frac{\left[a^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{a^2} - \frac{\left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{r^2{}_s}. \tag{3.16}$$

The represented solution holds for $\frac{M(t)}{2\pi\tau_s} \leq a^2$.

4. Residual stresses and deformations

Assume that the moment $M\,(t)$ monotonically increases in time interval $0 \leq t \leq \frac{t_1}{2}$. Let starting from time $t = \frac{t_1}{2}$ it begins to decrease and at time $t = t_1$ it becomes $M\;(t_1) = 0$. Unloading occurs in the plate in the time interval $\frac{t_1}{2} \leq t \leq t_1$. We consider that the unloading process is accompanied by the appearance of secondary plastic deformations.

Using the above solution we determine residual stresses, deformations and displacements that will remain in the annular plate after deleting the distributed moment $M\,(t)$

In this case we will consider that the moment $M\,(t)$ under loading from the natural state attains such a value that in the unloading process the secondary plastic deformations will definitely appear.

We will use M.M.Moskvitin's theorem on secondary plastic deformations [9,10]. According to this theorem, the residual stresses, σ_r^0, σ_θ^0 , $\sigma_{r\theta}^0$, residual deformations ε_r^0, ε_θ^0 , $\varepsilon_{r\theta}^0$, residual displacements u_r^0 , u_θ^0 can be determined by the following formulas:

$$\begin{array}{lll} \sigma_r^0 = \sigma_r - \sigma_r^* \,; & \sigma_\theta^0 = \sigma_\theta - \sigma_\theta^* \,; & \sigma_{r\theta}^0 = \sigma_{r\theta} - \sigma_{r\theta}^* \,; \\ \varepsilon_r^0 = \varepsilon_r - \varepsilon_r^* \,; & \varepsilon_\theta^0 = \varepsilon_\theta - \varepsilon_\theta^* \,; & \varepsilon_{r\theta}^0 = \varepsilon_{r\theta} - \varepsilon_{r\theta}^* \,; \\ u_r^0 = u_r - u_r^* \,; & u_\theta^0 = u_\theta - u_\theta^* \,. & \end{array} \tag{4.1}$$

Here σ_r, σ_θ, $\sigma_{r\theta}$, ε_r, ε_θ, $\varepsilon_{r\theta}$, u_r , u_θ are stresses, deformations and displacements before unloading and are determined by formulas (3.3)-(3.14) allowing for (3.15).

In the indicated formulas $M\,(t)$ should be replaced by $M\,(t_1/2)$.

The quantities σ_r^*, $\sigma_{r\theta}^*$, ε_r^*, ε_θ^* , $\varepsilon_{r\theta}^*$, u_r^*, u_θ^* are stresses, deformations and displacements that hold in some dummy annular plate at its elastic-plastic deformation when loaded along the inner contour by the moment $M\,(t_1/2)$. Unlike the plate under consideration the yield point at the dummy plate material shear is $2\sigma_s$.

The second plastic deformations will appear in the zone adjacent to the inner contour of the plate. Let r_0 be a radius of a circle that separates the domain of secondary plastic deformations: $a \leq r \leq r_0$. To compose an equation determining r_0 we should use equation (3.16) replacing in it τ_s by $2\tau_s$

$$\begin{array}{l} \ln \dfrac{r_0^2 + \left[a_0^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}.}{a^2 + \left[a^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}.} + \\ + \dfrac{\left[a^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}}{a^2} - \dfrac{\left[r_0^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}}{b^2} = 0. \end{array} \tag{4.2}$$

According to (4.1) the condition of appearance of secondary plastic deformations

$$\frac{M\;(t_1/2)}{4\pi\tau_s} \leq a^2 \,; \tag{4.3}$$

$$M > 4\pi a^2 \tau_s \,.$$

In the field of secondary plastic deformations $a \le r \le r_0$ the residual stresses, deformations and displacements will be

$$\sigma_r^0 = \tau_s \left\{ \ln \frac{\left[r^2 + \left(r^4 - (M(t_1/2)/(2\pi\tau_s))^2\right)^{1/2}\right]\left[a^2 + \left(a^4 - (M(t_1/2)/(4\pi\tau_s))^2\right)^{1/2}\right]^2}{\left[r^2 + \left(r^4 - (M(t_1/2)/(4\pi\tau_s))^2\right)^{1/2}\right]^2\left[a^2 + \left(a^4 - (M(t_1/2)/(2\pi\tau_s))^2\right)^{1/2}\right]} + \right.$$
$$+ \frac{1}{a^2}\left[\left(a^4 - \left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right)^{1/2} - 2\left(a^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right)^{1/2}\right] -$$
$$\left. - \frac{1}{r^2}\left[\left(r^4 - \left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right)^{1/2} - 2\left(r^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right)^{1/2}\right]\right\},$$

$$\sigma_\theta^0 = \sigma_r^0 + \frac{2\tau_s}{r^2}\left\{\left[r^4 - \left(\frac{M(t_1/2)}{2\pi\,\tau_s}\right)^2\right]^{1/2} - \left[r^4 - \left(\frac{M(t_1/2)}{4\pi\,\tau_s}\right)^2\right]^{1/2}\right\} \tag{4.4}$$

$$\sigma_{r\theta}^0 = 0. \tag{4.5}$$

$$\varepsilon_r^0 = \frac{1-\nu}{E}\sigma_r^0 + \frac{2\tau_s}{Er^2}\left\{(1-\nu)\left[\left(r^4 - \left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right)^{1/2} - 2\left(r^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right)^{1/2}\right] - \right.$$
$$\left. - \left(r_s^4 - \left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right)^{1/2} + 2\left(r_0^4 - \left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right)^{1/2}\right\},$$

$$\varepsilon_\theta^0 = \frac{1-\nu}{E}\sigma_r^0 + \frac{2\tau_s}{Er^2}\left\{\left[r_s^4 - \left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right]^{1/2} - 2\left[r_0^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}\right\}, \tag{4.6}$$

$$\varepsilon_{r\theta}^0 = \frac{1}{\pi\, r^2}\int_0^{t_1/2}\left(\dot\lambda - \dot\lambda_*\right) M(\tau)\, d\tau\,, \tag{4.7}$$

$$u_r^0 = \frac{1-\nu}{E}\, r\sigma_r^0 + \frac{2\tau_s}{rE}\left\{\left[r_s^4 - \left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right]^{1/2} - 2\left[r_0^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}\right\}. \tag{4.8}$$

$$u_\theta^0 = -\frac{2}{\pi}\left(1 - \frac{r}{b}\right)\int_0^{t_1/2}\left(\dot\lambda - \dot\lambda_*\right) M(\tau)\; d\tau\,. \tag{4.9}$$

The quantity σ_r^0 included in formulas (4.4), (4.6), (4.8) is determined by the first formula of (4.4). The formula for the quantity $\dot\lambda_*$ is written similar to formula (3.15) replacing in the later $\dot\lambda$ by $\dot\lambda_*$, τ_s by $2\tau_s$, r_s by r_0, $t$by $t_1/2$.

We now determine residual stresses, deformations and displacements in the domain $r_0 \le r \le r_s$. In this domain the unloading process happens elastically. The residual stresses may be found by A.A.Ilyushins theorem on elastic unloading [11] by formulas (4.1). The quantities σ_r, σ_θ, $\sigma_{r\theta}$, ε_r, ε_θ, $\varepsilon_{r\theta}$, u_r, u_θ are determined as in the previous case of unloading. The quantities σ_r^*, σ_θ^*, $\sigma_{r\theta}^*$, ε_r^*, ε_θ^*, $\varepsilon_{r\theta}^*$, u_r^*, u_θ^* are stresses, deformations, displacements that arise in a dummy annular plate under its elastic deformation by the moment $M(t_1/2)$ To find him we will use independent solution of an elastic problem on the action of the distributed moment $M\,(t_1/2)$ along the inner contour of this plate. On the bases of this, the quantities σ_r^*, σ_θ^* , $\sigma_{r\theta}^*$, ε_r^*, ε_θ^* , $\varepsilon_{r\theta}^*$, u_r^*, u_θ^* are determined by the following formulas

$$\left.\begin{matrix}\sigma_\theta^* \\ \sigma_r^*\end{matrix}\right\} = 0\sigma_{r\theta}^* = \frac{M\,(t_1/2)}{2\pi\,r^2}\,, \tag{4.10}$$

$$\left.\begin{matrix}\varepsilon_\theta^* \\ \varepsilon_r^*\end{matrix}\right\} = 0\varepsilon_{r\theta}^* = \frac{1+\nu}{E}\frac{M\,(t_1/2)}{2\pi\,r^2}\,, \tag{4.11}$$

$$u_r^* = 0u_\theta^* = -\frac{(1+\nu)\;M\,(t_1/2)}{2\pi\,E}\left(\frac{1}{r}-\frac{r}{b^2}\right). \tag{4.12}$$

Consequently, residual stresses, deformations and displacements in the domain $r_0 \leq r \leq r_s$ are determined as difference of appropriate formulas (3.8)-(3.14) allowing for (3.15) and replacing t by $t_1/2$ and elastic solution formulas.

It remains to determine the residual desired quantities in the domain $r_s \leq r \leq b$. These formulas are also found on the basis of the formula (4.1). The quantities σ_θ , σ_r , ε_θ , ε_r , $\varepsilon_{r\theta}$, u_r , u_θ and $\sigma_{r\theta}$ are expressed by formulas (3.8)-(3.14), where r_s is determined by formula (3.16). The quantities σ_θ^*, σ_r^*, $\sigma_{r\theta}^*$, ε_θ^*, ε_r^*, $\varepsilon_{r\theta}^*$, u_r^* , u_θ^* are represented by the formulas (4.10)-(4.12).

The results obtained in this paper are some generalizations of the paper [12].

5. Conclusions

1. The conditions of appearance of secondary plastic deformations under full unloading after preelasticoplastic deformation of an annular plate under the action of distributed moment along the inner contour were determined.
2. Residual stresses and deformations with elastic loading and unloading in domain with appearance of secondary residual deformations were determined.
3. The boundary between the domains of elastic loading and unloading with secondary plastic deformations was found .
4. The number of cycles of loading to fatigue damage and total failure of an annular plate under its cyclic elastico-plastic deformation under pulsating pressure and moment acting along the inner contour were determined.

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