Journal of Contemporary Applied Mathematics
V. 2, No 1, 2012, July
ISSN 2222-5498

Model of optimal resources using related with uncertainties

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Abstract. In this paper, first a space of fuzzy numbers is constructed and a scalar product is introduced. Using this it is created model of the piece-wise homogeneous area problem. This model is reduced to classical linear programming problem by discretization.

Key Words and Phrases: Fuzzy numbers, linear space, linear programming, discretization

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1. Introduction

As we know, we use many simplifies and assumptions for creating mathematical models of economic problems. n opposite case the created model will be very difficult and will be many problems in its investigation. But we've to do simplifies with such method which reduced the creating model to real process. Such as the results of mathematical model have to correspond to the results of real process.

Many observations, estimatings and etc. in the nature associate with uncertainties in different levels. It will be better if these uncertainties will be considered in these mathematical models. Founded by professor of Bercly University, Lotfi A. Zadeh, Fuzzy theory playes important role in the controlling systems associated with uncertainties [2].

2. The Space of the Pairs of Fuzzy Numbers

Let's define by F the class of convex normal fuzzy numbers. Then for any $a \in F$the set of α-cut of fuzzy number a the interval $a^\alpha = [L_a(\alpha), R_a(\alpha)]$, $\alpha \in [0,1]$, is defined ([3;5]). Let $a \in F$, $b \in F$ and $a^\alpha = [L_a(\alpha), R_a(\alpha)]$, $b^\alpha = [L_b(\alpha), R_b(\alpha)]$. Then α-cut of fuzzy number $a+b$ and ka, $k \geq 0$, define as $a^\alpha + b^\alpha = [L_a(\alpha) + L_b(\alpha), R_a(\alpha) + R_b(\alpha)]$ and $ka^\alpha = [kL_a(\alpha), kR_a(\alpha)]$, respectively.
Note that F is not a linear space (the operation of subtraction is not defined in F).

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We consider the set of pairs $(a, b) \in F \times F$and define the operation of addition, multiplication and equivalency as

$$\begin{array}{l} (a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2), \\ k \cdot (a, b) = (ka, \;\; kb), \quad k \geq 0 \\ (-1) \cdot (a, b) = (b, a) \\ (a_1, a_2) \approx (b_1, b_2) <=> a_1 + b_2 = a_2 + b_1 \end{array} \tag{1}$$

As zero element of this space is taken the pair (0, 0), i.e. the set of elements$(a, \; a), \; a \in F$. The set of all pairs $(a, b) \in F \times F$forms a structure of a linear space. Let

$$x = (a_1, a_2) \in F \times F, \; y = (b_1, b_2) \in F \times F.$$

Then

$$a_i^\alpha = [L_{a_i}(\alpha), R_{a_i}(\alpha)], b_i^\alpha = [L_{b_i}(\alpha), R_{b_i}(\alpha)], \; \alpha \in [0, 1].$$

For any $x, y \in F \times F$ define the scalar product as

$$x \circ y = \frac{1}{2} \int_0^1 [(L_{a_1}(\alpha) - L_{a_2}(\alpha))(L_{b_1}(\alpha) - L_{b_2}(\alpha)) +$$

$$+ (R_{a_1}(\alpha) - R_{a_2}(\alpha))(R_{b_1}(\alpha) - R_{b_2}(\alpha)]d\alpha \tag{2}$$

It may be shown that this definition satisfies all requirements of the scalar product. We denote this space by LF. Norm in this space is defined as

$$||x||^2 = \frac{1}{2} \int_0^1 [(L_{a_1}(\alpha) - L_{a_2}(\alpha))^2 + (R_{a_1}(\alpha) - R_{a_2}(\alpha))^2] d\alpha, \tag{3}$$

We define distance between two fuzzy numbers $a \in F$and $b \in F$ as

$$\rho(a, b) = \|x - y\|, \tag{4}$$

where$x = (a, \, 0), y = (b, \, 0)$.

If $x, \; y$ are vectors, i.e. $x = [x_1, x_2, ..., x_n], \quad y = [y_1, y_2, ..., y_n]$, where $x_i \, , \, y_i \in F \times F$, then we define the scalar product and norm as follows

$$x \circ y = x_1 \circ y_1 + x_2 \circ y_2 + ... + x_n \circ y_n,$$

$$\|x\|^2 = \|x_1\|^2 + \|x_2\|^2 + ... + \|x_2\|^2 .$$

For the sake of simplicity, instead $x \in F^{(n)} \times F^{(n)}$, we will write $x \in F \times F$.

3. Statement of the Problem and Main Result

In many references it was noted that modeling of undefined processes and its' investigation is very difficult . But the above linear space allows to solve these problems. Let's investigate following model and apply fuzzy approach to this one.

Here we'll investigate model, which is piece-wise homogeneous problem. For the simplicity we'll consider the agricultural area.

Let's the sowing area consists of m parts for its' character and productivity parameters and areas of each part are $b_1, b_2, ..., b_m$. We've to cultivate p type of products (goods, seeds) here. a_ij is the productivity of i-th type of product (seed) which is cultivated in the j-th area. c_i- is income from realization i-th type of product. Then problem will be as: how many hectare we‘ve to cultivate of each product in the each type of area for maksimization incomes? And it's required that i-th type of product isn't less than d_i.

The mathematical model of this problem is given in the diferent works [1,13]. Let's x_ij will be the i-th product from the j-th area. Then the problem will be as:

$$F(x) = \sum_{i=1}^{p} \sum_{j=1}^{m} c_i a_{ij} x_{ij} \to \max, \tag{5}$$

$$\sum_{j=1}^{m} a_{ij} x_{ij} \geq d_i,\ \ i = \overline{1,p}. \tag{6}$$

$$\sum_{i=1}^{p} x_{ij} = b_j,\ \ \ j = \overline{1,m.},. \tag{7}$$

$$x_{ij} \geq 0, \quad i = \overline{1,p}, \quad j = \overline{1,m}. \tag{8}$$

There is simplified some parameters in this model. For example: here is assumed that a_ij -is the middle productivity of i-th product from j-th area. But productivity parameter carried different uncertainties. In other words we have to take productivity as fuzzy parameter. x_ij depends on a_ij. So x_ij will be fuzzy parameter too. Then we've to define product a_ijx_ij with new approach. Let's write datas α-cut for fuzzy modeling of previous problem:

$$a_{ij}^{\alpha} = [L_{a_{ij}}(\alpha), R_{a_{ij}}(\alpha)],$$

$$x_{ij}^{\alpha} = [L_{x_{ji}}(\alpha), R_{x_{ij}}(\alpha)], \alpha \in [0,1].$$

In this situation product a_ijx_ij. The product $a_ij \circ x_ij$ can be taken as middle value. From (2) we've get

$$a_{ij} \circ x_{ji} = \frac{1}{2} \int_0^1 [L_{a_{ij}}(\alpha) L_{x_{ij}}(\alpha) + R_{a_{ij}}(\alpha) R_{x_{ij}}(\alpha)] d\alpha.$$

Let's calculate this product for one simple problem.

Sample1. Assume, a, x are correspondingly fuzzy numbers 2 and 3 of triangle type

$$a^{\alpha} = [1+\alpha, 3-\alpha],\ \ x^{\alpha} = [2+\alpha, 4-\alpha], \alpha \in [0,1],$$

Let's calculate this product for simple problem.
In crisp version this product equals to 6. Now let's calculate product $a^o x$

$$a \circ x = \tfrac{1}{2}\int_0^1 [L_a(\alpha)L_x(\alpha) + R_a(\alpha)R_x(\alpha)]d\alpha =$$

$$= \tfrac{1}{2}\int_0^1 [(1+\alpha)(2+\alpha) + (3-\alpha)(4-\alpha)]d\alpha = 6\tfrac{1}{3}.$$

So, difference beetwen defined product and crisp one is $\dfrac{1}{3}$.
So, replacing in previous model $a_{ij}x_{ij}$ with $a_{ij} \circ x_{ij}$ we get following fuzzy model (9)-(10).

$$\tilde{F}(x) = \frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{m}\int_0^1 [c_i L_{a_{ij}}(\alpha)L_{x_{ij}}(\alpha) + c_i R_{a_{ij}}(\alpha)R_{x_{ij}}(\alpha)]d\alpha \to \max. \qquad (9)$$

$$\frac{1}{2}\sum_{j=1}^{m}\int_0^1 [L_{a_{ij}}(\alpha)L_{x_{ij}}(\alpha) + R_{a_{ij}}(\alpha)R_{x_{ij}}(\alpha)]d\alpha \geq d_i, \quad i = \overline{1,p}, \qquad (10)$$

$$\sum_{i=1}^{p} L_{x_{ij}}(1) = b_j, \ \sum_{i=1}^{p} R_{x_{ij}}(1) = b_j \ j = \overline{1,m}. \qquad (11)$$

Here conditions

$$x_{ij} \geq 0, \quad i = \overline{1,p}, \quad j = \overline{1,m}$$

can be written as (12).

$$0 \leq L_{x_{ij}}(\alpha) \leq R_{x_{ij}}(\alpha), \ \ i = \overline{1,p}, \quad j = \overline{1,m}, \quad \alpha \in [0,1]. \qquad (12)$$

Note that condition (7) can be written not only (11) form. But it's can be written in other form too. For example,

$$\sum_{i=1}^{p} \tilde{1} \circ x_{ij} = b_j, \ \ j = \overline{1,m}.$$

Here $\tilde{1}$ is fuzzy one. If we'll take 1 in place of $\tilde{1}$ and will write the condition in open form, we'll get

$$\frac{1}{2}\sum_{i=1}^{p}\int_0^1 [L_{x_{ij}}(\alpha) + R_{x_{ij}}(\alpha)]d\alpha = b_j, \quad j = \overline{1,m}. \qquad (13)$$

So, considering some fuzzy factors in the model of area resourses optimal using we've got (9)- (12) model. In the solving (9)- (12) problem we have to cconsider that required $L_{x_{ij}}(\alpha)$, $R_{x_{ij}}(\alpha)$, $i = \overline{1,p}$, $j = \overline{1,m}$, functions satisfy monotony condition.$L_x(\alpha)$ is non-decreasing, $R_x(\alpha)$ is nonincreasing.
Conditions (9) and (10) can be written in the form:(14)- (16).

$$F(x) = \sum_{i=1}^{p}\sum_{j=1}^{m} c_i a_{ij} \circ x_{ij} \to \max, \qquad (14)$$

$$\sum_{j=1}^{m} a_{ij} \circ x_{ij} \geq d_i, \;\; i = \overline{1,p}. \tag{15}$$

$$\sum_{i=1}^{p} \tilde{1} \circ x_{ij} = b_j, \;\; j = \overline{1,m}. \tag{16}$$

Let's note that of this linear form (14)-(16), these relations are nonlinear in the form (9) and (10).

We've got (14)-(16) model considering $\alpha_i j$ as fuzzy parameter. Now let's suppose productivities $\alpha_i j$ are crisp parameters, but prices are fuzzy. Let's search solution of this problem in pair form

$$x_{ij}^{\alpha} = \left(\left[L_x^{(1)}(\alpha), R_x^{(1)}(\alpha) \right[, \left[L_x^{(2)}(\alpha), R_x^{(2)}(\alpha) \right[\right)$$

In this case functional (5) can be written as follow:

$$\tilde{F}(x) = \frac{1}{1} \sum_{i=1}^{p} \sum_{j=1}^{m} \int_0^1 [a_{ij} L_c(\alpha) \left(L_(x)^{(1)}(\alpha) - L_(x)^{(2)}(\alpha) \right)$$

$$a_{ij} R_c(\alpha) \left(R_(x)^{(1)}(\alpha) - R_(x)^{(2)}(\alpha) \right)] d\alpha \to max \tag{17}$$

As we seach solution of the problem in pair form, conditions (6), (7) won't be changed. We'll suppose that, conditions (6) are gotten as equals with entering new variables. Then relations (6) and (7) will be as following form:

$$\sum_{j=1}^{m} \left(\left[a_{ij} L_x^{(1)}(\alpha), a_{ij} R_x^{(1)}(\alpha) \right], \left[a_{ij} L_x^{(2)}(\alpha), a_{ij} R_x^{(2)}(\alpha) \right] \right) = \left(\left[\overline{d_i}, \bar{d}_{ij} \right], 0 \right), i = \overline{1,p}, \tag{18}$$

$$\sum_{j=1}^{m} \left(\left[L_x^{(1)}(\alpha), R_x^{(1)}(\alpha) \right], \left[L_x^{(2)}(\alpha), R_x^{(2)}(\alpha) \right] \right) = \left([b_j, b_j], 0 \right), j = \overline{1,m} \tag{19}$$

Acknowledgement

This work was supported by the Science Development Foundation under the President of the Republic of Azerbaijan - Grant EIF-2011-1(3)-82/26/1 and Baku State University Internal Grant Project "50+50", 2011

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Received 4 June 2012
Accepted 25 June 2012