Journal of Contemporary Applied Mathematics
V. 11, No 2, 2021, December
ISSN 2222-5498

Asymptotics of the Solution of a Boundary Value Problem Stated in an Infinite Half Strip for a Non-Classical Type Singularly Perturbed Differential Equation

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Abstract. In the paper complete asymptotics of the solution of a boundary value problem stated in an infinite strip for a third order singularly perturbed non-classical type differential equation was constructed with respect to a small parameter and the obtained residual term was estimated.

Key Words and Phrases: Asymptotic expansion, boundary layer type function, remainder
2010 Mathematics Subject Classifications: 35B25, 35B40

1. Introduction

When passing from one physical characteristics to other ones, mathematical modules of non-smooth processes are described by differential equations with a small parameter in front of some higher order derivatives. Such differential equations are called singularly perturbed differential equations. The papers devoted to the study of singularly perturbed differential equations began to be published in the fourties of the past century. A.N. Tikhonov's works [1], [2] should be mentioned among these studies. As present there exist various asymptotic methods for studying the dependence of the solution of singularly perturbed differential equations on a small parameter. From the point of view of application area and accuracy of mathematical justification the most effective one is the method developed by M.I. Vishik and L.A. Lusternik and commented in their works [3], [4]. Now this method is known as "Vishik-Lusternik method". After developing this method, the number of works is growing rapidly. As present, there are numerous works devoted to each classical type singularly perturbed differential equations. But, the number of works on non-classical type singularly perturbed differential equations is few. M.I. Vishik and L.A. Lusternik for the first time constructed the first terms of asymptotic expansion of the solution of a boundary value problem for a non-classical type one-characteristics differential equation stated in a bounded domain. In this paper, when constructing the first term of the asymptotic expansion, very strong conditions were imposed on the function in the right-hand side of the considered equation. In the paper [5] constructing internal layer type functions M.M. Sabzaliyev rejected these strong conditions and constructed the first terms of the asymptotics of this problem. In the paper [5], the complete asymptotics of the solution of this problem was also constructed. The wide information related to the studies on non-classical type singularly perturbed differential equations can be found in M.M.Sabzaliyev and I.M.Sabzaliyeva's monograph [6].

2. Problem statement and the first iterative process.

On the infinite half-strip $P_+ = \{(x, y)|0 \leq x \leq 1,\ 0 \leq y < +\infty\}$ consider the following boundary value problem:

$$L_\varepsilon u \equiv \varepsilon^2 \frac{\partial}{\partial x}(\Delta u) - \varepsilon \Delta u + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = f(x, y), \tag{2.1}$$

$$u|_{x=0} = u|_{x=1} = 0, \frac{\partial u}{\partial x|}_{x=1} = 0, (0 \leq y < +\infty), \tag{2.2}$$

$$u|_{y=0} = 0, \lim_{y\to\infty} u = 0, (0 \leq x \leq 1), \tag{2.3}$$

here $\varepsilon > 0$ is a small parameter, $\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is a Laplace operator, $f(x, y)$ is a given function.

Our goal is to construct complete asymptotics of the solution of boundary value problem (2.1)-(2.3) with respect to the small parameter. For constructing the asymptotics we conduct iterative processes. In the first iterative processes looking for the approximate solution of differential equation (2.1) in the form of

$$W = W_0 + \varepsilon W_1 + ... + \varepsilon^n W_n \tag{2.4}$$

for determining the unknown function $W_i; i = 0, 1, ..., n$, we obtain the following boundary value problems:

$$\frac{\partial W_0}{dx} + \frac{\partial W_0}{dy} + W_0 = f(x, y), W_0|_{x=0}, W_0|_{y=0} = 0, \tag{2.5}$$

$$\frac{\partial W_i}{dx} + \frac{\partial W_i}{dy} + W_i = f_i(x, y), W_i|_{x=0}, W_i|_{y=0} = 0; i = 1, 2, ..., n. \tag{2.6}$$

Here the functions $f_i(x, y)$; $i = 1, 2, ..., n$ are determined by the formula

$$f_1 = \Delta W_0, f_j = \Delta W_{j-1} - \frac{\partial}{\partial x}(\Delta W_{j-2}); j = 2, 3, ..., n \tag{2.7}$$

The following proposition is valid.

Lemma 2.1. *Assume that the function $f(x, y)$ is included into the space $C^{2n+3}(P_+)$ and in the domain P_+ satisfies the following conditions:*

$$\frac{\partial^i f(x, y)}{\partial x^{i_1} \partial y^{i_2}}\,|_{y=x} = 0; i = i_1 + i_2; i = 0, 1, ..., 2n + 3; (0 \leq x \leq 1), \tag{2.8}$$

$$\left|\frac{\partial^i f(x,y)}{\partial x^{i_1}\partial y^{i_2}}\right| \leq C^{(1)}_{i_1 i_2} exp(-\lambda y); i = i_1 + i_2; i = 0,1,...,2n+3; C^{(1)}_{i_1 i_2} > 0, \lambda > 0. \quad (2.9)$$

Then the function $W_0(x,y)$ being the solution of boundary value problem (5) is included in space $C^{2n+3}(P_+)$ and satisfies the following conditions:

$$\frac{\partial^i W_0(x,y)}{\partial x^{i_1}\partial y^{i_2}} \mid_{y=x} = 0; i = i_1 + i_2; i = 0,1,...,2n+3; (0 \leq x \leq 1), \quad (2.10)$$

$$\left|\frac{\partial^i W_0(x,y)}{\partial x^{i_1}\partial y^{i_2}}\right| \leq C^{(2)}_{i_1 i_2} exp(-\lambda y); i = i_1 + i_2; i = 0,1,...,2n+3; C^{(2)}_{i_1 i_2} > 0, \lambda > 0. \quad (2.11)$$

Proof. For $f(x,y) \in C^{2n+3}(P_+)$ and if the condition (8) is satisfied, the proof of $W_0(x,y) \in C^{2n+3}(P_+)$ and of the condition (2.10) is given in [7], p.108-109.

When the function $f(x,y)$ satisfies the condition (2.9), in order to prove that the function $W_0(x,y)$ being the solution of boundary value problem (2.5) satisfies the condition (2.11), at first we note that the solution of boundary value problem (2.5) is determined by the following formula:

$$W_0(x,y) = \begin{cases} \int\limits_0^x f(\xi, \xi + y - x) exp[-(x-\xi)] d\xi, & 0 \leq x < y < +\infty, \\ \int\limits_0^y f(x - y + \xi, \xi) exp[-(y-\xi)] d\xi, & 0 \leq y < x \leq 1. \end{cases} \quad (2.12)$$

From (2.12) we obtain that at rather large values of the variable y the function $W_0(x,y)$ is determined by the formula

$$W_0(x,y) = \int\limits_0^x f(\xi, \xi + y - x) exp[-(x-\xi)] d\xi \quad (2.13)$$

At first we show that for the case $i = 0$ the condition (2.11) is satisfied. In the case $i = 0$, using the condition (2.9), from the formula (2.13) we get:

$$\mid W_0(x,y) \mid \leq C_0 \int\limits_0^x exp[-\lambda(\xi + y - x) d\xi \leq \frac{C_0[exp(\lambda) + 1]}{\lambda} exp(-\lambda y)$$

In the last inequality, denoting $C_0^{(2)} = \frac{C_0[exp(\lambda)+1]}{\lambda}$ we get

$$\mid W_0(x,y) \mid \leq C_0^2 exp(-\lambda y) \quad (2.14)$$

The last inequality shows that the condition (2.11) is satisfied for the function $W_0(x,y)$ itself.

To show that the condition (2.11) is satisfied for the derivatives of the function $W_0(x,y)$, at first by the mathematical induction method we prove the validity of the following formula for the derivative of the functions

$$\frac{\partial^k W_0(x,y)}{\partial x^{k_1}\partial y^{k_2}} = -\frac{\partial^{k_1+k_2-1} W_0(x,y)}{\partial x^{k_1-1}\partial y^{k_2}}+$$

$$+(-1)^{k_1}\int\limits_0^x exp[-(x-\xi)]\sum_{i=1}^{k_1} C_{k_1-1}^{i-1} f_{II}^{(i+k_2)}(\xi,\xi+y-x)d\xi+$$

$$+\sum_{i=0}^{k_1-1}\sum_{i_1+i_2=i} b_{i_1 i_2}\frac{\partial^{i+k_2} f(x,y)}{\partial x^{i_1}\partial y^{i_2+k_2}}, 0\le x<y<+\infty \tag{2.15}$$

Here $b_{i_1 i_2}$ are certain known numbers.

Using the inequalities (2.9), (2.14) and formula (2.15), we prove that the condition (2.11) is satisfied for the derivatives of the functions $W_0(x,y)$ as well. The lemma is proved. □

The functions $W_1, W_2, ..., W_n$ included in expansion (2.4) of the function W are sequentially determined from Lemma 2.1 as the solutions of boundary value problems (2.6). The last $W_1, W_2, ..., W_n$ functions included in expansion (2.4) will enter into the space $C^3(P_+)$.

Thus, in the first iterative process we constructed the function $W=\sum\limits_{i=0}^{n}\varepsilon^i W_i$ satisfying the following boundary conditions:

$$W|_{x=0}=0, W|_{y=0}=0, \lim_{y\to+\infty} W=0 \tag{2.16}$$

The constructed function W need not satisfy the boundary conditions on $x=1$ in (2.2). To provide the satisfaction of these lost boundary conditions we must conduct the second iterative process and construct boundary layer type functions.

3. The second iterative process

To conduct the second iterative process, at first near the boundary $x=1$ we must write a new expansion of the operator L_ε. For that near the boundary $x=1$ we make the substitution $1-x=\varepsilon\tau, y=y$. In such a substitution, the expansion of the operator L_ε in the new coordinates (τ,y) is as follows:

$$L_{\varepsilon,1}V \equiv \varepsilon^{-1}\left\{-\left(\frac{\partial^3 V}{\partial\tau^3}+\frac{\partial^2 V}{\partial\tau^2}+\frac{\partial V}{\partial\tau}\right)+\right.$$

$$\left.+\varepsilon\left(\frac{\partial V}{\partial y}+V\right)-\varepsilon^2\left(\frac{\partial^2 V}{\partial y^2}+\frac{\partial^3 V}{\partial\tau\partial y^2}\right)\right\}. \tag{3.1}$$

Looking for a layer type function near boundary $x = 1$ in the form

$$V = V_0 + \varepsilon V_1 + ... + \varepsilon^{n+1} V_{n+1} \tag{3.2}$$

as an approximation solution of the equation

$$L_{\varepsilon,1} V = 0, \tag{3.3}$$

for determining the unknown functions $V_j; j = 0, 1, ..., n+1$ included in the right hand side of inequality (3.2), we obtain the following differential equations:

$$\frac{\partial^3 V_0}{\partial \tau^3} + \frac{\partial^2 V_0}{\partial \tau^2} + \frac{\partial V_0}{\partial \tau} = 0, \tag{3.4}$$

$$\frac{\partial^3 V_1}{\partial \tau^3} + \frac{\partial^2 V_1}{\partial \tau^2} + \frac{\partial V_1}{\partial \tau} = \frac{\partial V_0}{\partial y} + V_0, \tag{3.5}$$

$$\frac{\partial^3 V_k}{\partial \tau^3} + \frac{\partial^2 V_k}{\partial \tau^2} + \frac{\partial V_k}{\partial \tau} = \frac{\partial V_{k-1}}{\partial y} + V_{k-1} - \frac{\partial^2 V_{k-2}}{\partial y^2} - \frac{\partial^3 V_{k-2}}{\partial \tau \partial y^2}, k = 2, 3, ..., n+1. \tag{3.6}$$

The boundary condition for differential equations (3.4)-(3.6) are found from the requirement that the sum $W + V$ satisfies the following boundary conditions:

$$(W + V) \mid_{x=1} = 0, \frac{\partial}{\partial x}(W + V) \mid_{x=1} = 0 \tag{3.7}$$

Considering the expansion (2.4) of the function W, expansion (3.2) of the function V in the equalities (3.7), making the grouping with respect to the same power of a small parameter, we get the following boundary conditions:

$$V_i \mid_{\tau=0} = -W_i \mid_{x=1}; i = 0, 1, ..., n+1; V_{n+1} \mid_{\tau=0} = 0. \tag{3.8}$$

$$\frac{\partial V_0}{\partial \tau} \mid_{\tau=0} = 0, \frac{\partial V_j}{\partial \tau} \mid_{\tau=0} = \frac{\partial W_{j-1}}{\partial x} \mid_{x=1}; j = 1, 2, ..., n+1 \tag{3.9}$$

Thus, the function V_0 is a boundary ayer type solution of differential equation (3.4) satisfying the boundary conditions

$$V_0 \mid_{\tau=0} = -W_0 \mid_{x=1}, \frac{\partial V_0}{\partial \tau} \mid_{\tau=0} = 0. \tag{3.10}$$

The characteristic function corresponding to ordinary differential equations (3.4)-(3.6) has two complex roots with a negative real part. We denote these roots by $\lambda_{1,2} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$. Note that the number of the lost boundary conditions near the boundary $x = 1$ is also two. This facts shows that the boundary value problem (2.1)-(2.3) regularly degenerates near the boundary $x = 1$.

The boundary layer type solution of the differential equation (3.4) satisfying the boundary conditions (3.10) is determined by the formula

$$V_0(\tau, y) = \frac{W_0(1, y)}{\lambda_1 - \lambda 2}(\lambda_2 e^{\lambda_1 \tau} - \lambda_1 e^{\lambda_2 \tau}). \tag{3.11}$$

From the condition (2.11), formula (3.11) we get that the function $V_0(\tau, y)$ satisfies the condition

$$\lim_{y \to +\infty} V_0(\tau, y) = 0 \tag{3.12}$$

as well.

As the function V_0 is known, we can construct the function V_1. The function V_1 is a boundary layer type solution of differential equation (3.5) satisfying the following boundary conditions obtained from (3.8), (3.9)

$$V_1|_{\tau=0} = -W_1|_{x=1}; \frac{\partial V_1}{\partial \tau}|_{\tau=0} = \frac{\partial W_0}{\partial x}|_{x=1} \tag{3.13}$$

It is easy to show that the function V_1 is determined by the following formula:

$$V_1(\tau, y) = [a_{10}(y) + a_{11}(y)\tau]e^{\lambda_1 \tau} + [b_{10}(y) + b_{11}(y)\tau]e^{\lambda_2 \tau}. \tag{3.14}$$

The functions $a_{10}(y), a_{11}(y), b_{10}(y), b_{11}(y)$ are known functions expressed by the functions $W_0(1, y), \frac{\partial W_0(1,y)}{\partial x}, \frac{\partial W_0(1,y)}{\partial y}, W_1(1, y)$ in the homogeneous form. Hence and from formula (3.14) we get that for the function $V_1(\tau, y)$ the condition

$$\lim_{y \to +\infty} V_1(\tau, y) = 0 \tag{3.15}$$

is also satisfied.

The construction other functions $V_2, V_3, ..., V_{n+1}$ of the function V included in expansion (3.2) is based on the following proposition.

Lemma 3.1. *The boundary layer type solutions of differential equations* (3.4)-(3.6) *satisfying appropriate boundary conditions in inequalities* (3.8), (3.9) *are determined by the following formula:*

$$V_k(\tau, y) = e^{\lambda_1 \tau} \sum_{j=0}^{k} a_{kj}(y)\tau^j + e^{\lambda_2 \tau} \sum_{j=0}^{k} b_{kj}(y)\tau^j;\ k = 0, 1, ..., n+1. \tag{3.16}$$

Here the functions $a_{kj}(y), b_{kj}(y); j = 0, 1, ..., k$ are such known function expressed in the homogeneous form by means of the functions $\frac{\partial^r W_i(1,y)}{\partial x^{r_1} \partial y^{r_2}}; r = r_1 + r_2; r + i \leq k; r_1 = 0; 1$ when all the functions $\frac{\partial^r W_i(1,y)}{\partial x^{r_1} \partial y^{r_2}}$ vanish, all the functions $a_{kj}(y), b_{kj}(y)$ also vanish.

Proof. The functions $V_0(\tau, y), V_1(\tau, y)$ have been already constructed and they are in the form of (3.16).

Assume that the statement of Lemma 3.1 is valid for $k = 0, 1, ..., m-1$. Prove that then the function $V_m(\tau, y)$ is determined by the formula in the form of (3.16). From inequalities (3.6), (3.8), (3.9) we get that the function $V_m(\tau, y)$ is a boundary layer type solution of the following problem:

$$\frac{\partial^3 V_m}{\partial \tau^3} + \frac{\partial^2 V_m}{\partial \tau^2} + \frac{\partial V_m}{\partial \tau} = f_m(\tau, y), \tag{3.17}$$

$$V_m|_{\tau=0} = -W_m|_{x=1}, \frac{\partial V_m}{\partial \tau}|_{\tau=0} = \frac{\partial W_{m-1}}{\partial x}|_{x=1}. \tag{3.18}$$

According to (3.6) and our assumption, the right hand side of differential equation (3.17) is in the following form:

$$f_m(\tau, y) = e^{\lambda_1 \tau} \sum_{j=0}^{m-1} C_{mj}(y)\tau^j + e^{\lambda_2 \tau} \sum_{j=0}^{m-1} d_{mj}(y)\tau^j. \tag{3.19}$$

Then it is clear that the differential equation (3.17) has a particular solution in the form of

$$\overline{V_m}(\tau, y) = e^{\lambda_1 \tau} \sum_{i=0}^{m-1} P_{mi}(y)\tau^{i+1} + e^{\lambda_2 \tau} \sum_{i=0}^{m-1} q_{mi}(y)\tau^{i+1}. \tag{3.20}$$

Looking for a boundary layer type solution of problem (3.17), (3.18) in the form of $V_m = \overline{V_m} + \overline{\overline{V}}_m$, then $\overline{\overline{V}}_m$ is a boundary layer type solution of the following problem:

$$\frac{\partial^3 \overline{\overline{V}}_m}{\partial \tau^3} + \frac{\partial^2 \overline{\overline{V}}_m}{\partial \tau^2} + \frac{\partial \overline{\overline{V}}_m}{\partial \tau} = 0 \tag{3.21}$$

$$\overline{\overline{V}}_m|_{\tau=0} = \varphi(y), \frac{\partial \overline{\overline{V}}_m}{\partial \tau}|_{\tau=0} = \psi(y). \tag{3.22}$$

Here the functions $\varphi(y), \psi(y)$ are known functions determined by the following formulas:

$$\varphi(y) = -W_m|_{x=1} - \overline{V}_m|_{\tau=0}, \psi(y) = \frac{\partial W_{m-1}}{\partial x}|_{x=1} - \frac{\partial \overline{V}_m}{\partial x}|_{x=1}. \tag{3.23}$$

The boundary layer type solution of problem (3.21), (3.22) is determined by the formula

$$\overline{\overline{V}}_m(\tau, y) = \frac{\psi(y) - \lambda_2 \varphi(y)}{\lambda_1 - \lambda_2} e^{\lambda_1 \tau} + \frac{\lambda_1 \varphi(y) - \psi(y)}{\lambda_1 - \lambda_2} e^{\lambda_1 \tau} \tag{3.24}$$

From the formulas (3.20) and (3.24) we obtain that for the function $V_m(\tau, y)$ being the sum of the functions $\overline{V_m}, \overline{\overline{V}}_m$ for $m = k$ the formula (3.16) is valid.

Lemma 3.1 is proved. □

Multiplying all the functions V_j by the smoothing functions, we retain all the previous denotations $V_j; j = 0, 1, ..., n+1$ for the obtained functions.

At the expense of the smoothing functions the functions $V_j; j = 0, 1, ..., n+1$ vanish for $x = 0$. Hence from the first boundary condition in (2.16) and (3.7) we get that the sum $W + V$ satisfies the boundary conditions

$$(W+V)|_{x=0} = 0, (W+V)|_{x=1} = 0, \frac{\partial}{\partial x}(W+V)|_{x=1} = 0. \tag{3.25}$$

According to the second boundary condition in (2.16), in order the sum $W + V$ satisfy the boundary condition

$$(W+V)|_{y=0} = 0,$$

$$V|_{y=0} = 0. \tag{3.26}$$

If for $y = 0$ all the functions $V_j; j = 0, 1, ..., n+1$ vanish, according to (3.2) $V|_{y=0} = 0$. According to the formula (3.16) where the functions V_j are determined, if

$$\frac{\partial^r W_i(1,0)}{\partial x^{r_1} \partial y^{r_2}} = 0; r = r_1 + r_2; r + i \leq n+1; i = 0, 1, ..., n, \tag{3.27}$$

then the conditions $V_j|_{y=0} = 0; j = 0, 1, ..., n+1$ are satisfied. Using formula (2.12) we can verify that when the function $f(x, y)$ satisfies the condition

$$\frac{\partial^k f(1,0)}{\partial x^{k_1} \partial y^{k_2}} = 0; k = k_1 + k_2; k = 0, 1, ..., n, \tag{3.28}$$

then the conditions (3.27), and so the condition (3.26) are satisfied.

From the formula (3.16) where the functions V_j are determined, these functions satisfy the condition $\lim\limits_{y \to +\infty} V_j = 0; j = 0, 1, ..., n+1$.

Hence, from (3.2) and from the third equality in (2.16) we get that the sum $W + V$ in addition to conditions (3.25), (3.26) satisfies the condition

$$\lim_{y \to +\infty} (W+V) = 0 \tag{3.29}$$

as well.

Denoting the structured sum by $\widetilde{u} = W + V$, the difference of the function u being the solution of boundary value problem (2.1)-(2.3) and the function $\widetilde{u}$ by

$$u - \widetilde{u} = \varepsilon^{n+1} z, \tag{3.30}$$

for the solution of the considered problem we get the following asymptotic expansion:

$$u = \sum_{i=o}^{n} \varepsilon^i W_i + \sum_{j=o}^{n+1} \varepsilon^j V_j + \varepsilon^{n+1} z. \tag{3.31}$$

$\varepsilon^{n+1} z$ in the expansion (3.31) is a residual term. To justify mathematically the conducted operations, we should estimate the function z.

4. Estimating the residual term and the obtained main result

Acting on the both hand sides of the equality (3.30) by the appropriate expansions of the operator L_ε and considering the equations obtained in the iterative process, we get that the function z is a solution of the following differential equation

$$L_\varepsilon z = H(\varepsilon, x, y). \tag{4.1}$$

Here the function $H(\varepsilon, x, y)$ is a function continuous in the whole domain P_+ from the semi-interval $[0, \varepsilon_0)$ of the small parameter ε and exponentially decreasing with respect to the variable y provided $y \to +\infty$, at each value of x from the interval $[0, 1]$.

From equalities (2.2), (2.3), (3.25), (3.26) (3.29) and (3.30) we get that the function z satisfies the following boundary conditions:

$$z|_{x=0} = z|_{x=1} = 0, \frac{\partial z}{\partial x}|_{x=1} = 0, (0 \le y < +\infty), \tag{4.2}$$

$$z|_{y=0} = 0, \lim_{y \to +\infty} z = 0, (0 \le x \le 1). \tag{4.3}$$

The following proposition is valid.

Lemma 4.1. *For the function z being the solution of boundary value problem* (4.1)-(4.3) *the following estimation is valid:*

$$\varepsilon^2 \| \frac{\partial z}{\partial x}|_{x=0} \|^2_{L_2(0,+\infty)} + \varepsilon \left[\| \frac{\partial z}{\partial x} \|^2_{L_2(P_+)} + \| \frac{\partial z}{\partial y} \|^2_{L_2(P_+)} \right] +$$

$$+ C_1 \|z\|^2_{L_2(P_+)} \le C_2. \tag{4.4}$$

Here the constants $c_1 > 0, c_2 > 0$ are independent of ε.

Proof. Multiplying the both hand sides of the equality (4.1) by the function z and integrating by parts the obtained terms considering the boundary conditions (4.2), (4.3), after some transformations we get the estimation (4.4). The results obtained in the paper can be generalized in the form of the following theorem.

Theorem 4.2. *Assume that the function $f(x, y)$ is included in the space $C^{2n+3}(P_+)$ and satisfies the conditions* (2.8), (2.9), (3.28)*. Then the asymptotic expansion of the solution of boundary value problem* (2.1)-(2.3) *with respect to a small parameter is in the form of* (3.31)*. Here the functions W_i are determined in the first iterative process, the functions V_j are boundary layer type functions near $x = 1$ and are determined in the second iterative process, $\varepsilon^{n+1} z$ is a residual term and the estimation* (4.4) *is valid for the function z.*

□

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Received 12 February 2021
Accepted 14 July 2021