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Gas Flow in a Formation-Well-Pipeline System

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Abstract. A model of the process of non-stationary gas flow in a formation-well-pipeline system is constructed. A boundary value problem of non-stationary gas flow in a formation-well-pipeline system involving a choke is solved allowing for the law of pressure change at the outlet of the pipeline.
Wellhead and bottomhole pressures are determined. Analytic expressions that allow to determine change in the volume of gas production per unit of time when connected to a drunk line are determined.
Key Words and Phrases: gas flow, pressure, simulation, Laplace transform, differential equation, pipeline.

1. Introduction

To determine operational indicators of gas wells with abnormally high formation pressures during gas connections and withdrawals from transport line, it is necessary to consider the gas flow in the formation-well-pipeline system. Connection and withdrawal of gas from the operating line leads to a violation of the operating mode of running well. Further, after a while the well goes into another steady state but with a different recovery. This raises the question of determining the influence of these connections or gas withdrawal on the existing regime [4].

The work [1-14] were devoted to this problem. The methods of conjugation of models for a formation and well are in the work [5, 6]. In this paper, joint solution of equations describing multiphase flows in a formation and well, are used.

In these works, filtration and gas flow in the conjugated formation-pipeline is not considered.

The work [5] was devoted to physic-mathematical formalization, development and software implementation of computational algorithms for simulating non-stationary three-phase flows in the conjugated formation-well system.

Installation of an electric drive centrifugal pump, that essentially differs from the considered formation-pipeline system.

The works [9-13] were devoted to the study of filtration of gas-liquid mixture and gassy fluid in one-dimensional and two-dimensional models.

In these works the researcher are conducted without taking into account the filtration process and fluid flow in the conjugated formation-well system.

As noted at the beginning, in reality the gas flow process occurs in the formation-pipeline system. Therefore, gas flow must be considered together allowing for the existent of a regularity choke, and this work is devoted to thus problem.

A strict solution of this problem in to take into account the interaction in the formation-pipeline system. At the same time it is necessary to consider and study the system of equation describing joint flow of gas-liquid mixture in the formation and in the wellbore and in the pipeline [2-5]

These are nonlinear differential equations and it is not possible to get theor exact solution. Therefore, carried out approximately with sufficient accuary for practice [1-2].

Problem solution and methods for solving it. Let us consider flat radial filtration of homogeneous gas in a homogeneous circular formation. We will solve the problem on the bais of the material balance.

Boundary and initial condition of gas filtration are of the form:

$$P|_{r=r_c} = P_{\text{c}}\left(t\right), t > 0, \tag{1.1}$$

$$\left.\frac{\partial P}{\partial r}\right|_{r=R_k} = 0, t > 0, \tag{1.2}$$

$$2\pi\, h\, r \frac{k}{\mu\, \beta} \frac{P_c\left(0\right) + P_c\left(T\right)}{2} \left.\frac{\partial P}{\partial r}\right|_{t=0} = G\ \left(r\right). \tag{1.3}$$

Then, we will look for the formation pressure allowing for boundary condition (1.1) and (1.2) in the form [1, 2]:

$$P = P_c\left(t\right) + A\left(t\right) f\left(r\right), \tag{1.4}$$

where $A\ (t)$ - is an unknown function dependent on time t, $f\ (r)$ - is a function dependents on the coordinate r and satisfying boundary conditions (1.1) and (1.2). We select the function $f\ (r)$ satisfying boundary conditions (1.1) and (1.2) as follows [1,2]:

$$f\left(r\right) = \ln\frac{r}{r_c} - \frac{r}{R_k} + \frac{r_c}{R_k}. \tag{1.5}$$

Taking the process isothermal, gas mass G_0 in the formation at every moment of time can bbe determined by the formula:

$$G_0 = \frac{2\pi\, m\, h}{\beta} \int_{r_c}^{R_k} P \cdot r dr, \tag{1.6}$$

where $\beta = \frac{P_{\text{atm}}}{\rho_{\text{atm}}}$, P_{atm} is atmospheric pressure, ρ_{atm}- is gas density at atmospheric pressure.

Gas inflow from the formation to the well per unit of time G may be determined by the formula:

$$G = -\frac{dG_0}{dt}. \tag{1.7}$$

Substituting expressions (1.4) and (1.5) in formula (1.6), we get:

$$G_0 = \frac{2\pi\, m\, h}{\beta}\left[P_c\left(t\right)\frac{R_k^2 - r_c^2}{2} + \frac{R_k^2}{2} D\, A\left(t\right)\right], \tag{1.8}$$

where $D = \ln\frac{R_k}{r_c} - \frac{7}{6} + \frac{1}{2}\left(\frac{r_c}{R_k}\right)^2 + \frac{r_c}{R_k} - \frac{1}{3}\left(\frac{r_c}{R_k}\right)^3$.
Substituting expressions (1.8) in formula (1.7) , we get:

$$G = -\frac{\pi\, m\, h\, R_k^2}{\beta}\left[\dot{P}_c\left(1 - \frac{r_c^2}{R_k^2}\right) + D\dot{A}\left(t\right)\right]. \tag{1.9}$$

On other hand, gas inflow into the well per unit of time can be determined by the formula [1]:

$$G = \frac{k\ \left(P_c(0) + P_c(T)\right)}{\mu\, \beta}\pi\, r_c h\left.\frac{\partial P}{\partial r}\right|_{r=r_c}, \tag{1.10}$$

where $P_c(T)$- is buttonhole pressure at the end of operation period.

Then substituting expression (1.4) in formula (1.10), we get:

$$G = \frac{k\ \left(P_c(0) + P_c(T)\right)}{\mu\, \beta}\pi\, h\, A\left(t\right)\left(1 - \frac{r_c}{R_k}\right). \tag{1.11}$$

Equating the expression (1.9) and (1.11), we get:

$$\dot{A} + \alpha\, A = -\frac{\dot{P}_c\left(t\right)}{D}. \tag{1.12}$$

The solution of differential equation (1.12) is of the form:

$$A = A_0 \exp\left(-\alpha t\right) - \frac{1}{D}\int_0^t \dot{P}_c\left(\tau\right)\exp\left[-\alpha\left(t - \tau\right)\right] d\tau, \tag{1.13}$$

where A_0-is a constant of integration determined from the initial condition (1.3), $\alpha = \frac{k\left(P_c(0)+P_c(T)\right)}{\mu\, m\, R_k^2\, D}$. Substituting the obtained expression in formula (1.5), we get pressure distribution field in the formation:

$$P = P_c\left(t\right) + \left(\ln\frac{r}{r_c} - \frac{r}{R_k} + \frac{r_c}{R_k}\right)\left[A_0 \exp\left(-\alpha t\right) - \frac{1}{D}\int_0^t \dot{P}_c\left(\tau\right)\exp\left[-\alpha\left(t - \tau\right)\right] d\tau\right]. \tag{1.14}$$

We now consider gas flow in a lifting pipeline. Gas flow in the pipe and continuity equation are described by I.A.Chorniy equations [17,18]:

$$\begin{aligned} -\frac{\partial P}{\partial x} &= \frac{\partial Q}{\partial t} + 2aQ, \\ -\frac{\partial P}{\partial t} &= c^2\frac{\partial Q}{\partial x}, \\ Q &= \rho v, \end{aligned} \tag{1.15}$$

were ρ-is gas density at the given pressure, υ- is gas flow rate averaged along the cross-section of the pipe.

Having differentiated the first equation of expression (1.15) with respect to time t, the second one with respect to x and subtracting one from another, we get:

$$\frac{\partial^2 Q}{\partial t^2} = c^2 \frac{\partial^2 Q}{\partial x^2} - 2a \frac{\partial Q}{\partial t}. \tag{1.16}$$

We represent the cross-sectional velocity of the gas column as the sum of two velocities:

$$\upsilon = \upsilon_e + \upsilon_r, \tag{1.17}$$

where υ_e - is the velocity of gas column as a solid body (portable velocity, the first letter from the french word 'entrainer'), υ_r is cross-sectional velocity of gas column from its compressibility (relative velocity, the first either of the English word relative).

Substituting expression (1.17) in formula

$$Q = \rho\,\upsilon = \rho\,\upsilon_e + \rho\,\upsilon_r \tag{1.18}$$

or

$$Q = u_e + u_r, \tag{1.19}$$

where $u_e = \rho\,\upsilon_e$, $u_r = \rho\,\upsilon_r$.
Then, substituting expression (1.19) in equation (1.16), we get [3]:

$$\frac{\partial^2 u_e}{\partial t^2} + \frac{\partial^2 u_r}{\partial t^2} = c^2 \frac{\partial^2 u_r}{\partial x^2} - 2a \left(\frac{\partial u_e}{\partial t} + \frac{\partial u_r}{\partial t} \right). \tag{1.20}$$

Since the equation (1.20) is linear, is decays into 2 equation

$$\frac{\partial^2 u_e}{\partial t^2} + 2a \frac{\partial u_e}{\partial t} = \frac{\dot{P}_c - \dot{P}_{\text{y}}}{l}, \tag{1.21}$$

where P_{y} is well head pressure.

$$\frac{\partial^2 u_r}{\partial t^2} = c^2 \frac{\partial^2 u_r}{\partial x^2} - 2a \frac{\partial u_r}{\partial t} + \frac{\dot{P}_{\text{y}} - \dot{P}_c}{l} \tag{1.22}$$

Having placed the origin of the coordinate axis $x$in the lower sections of the pipe and directed it upwards, for initial and boundary conditions we will have:

$$u_e|_{t=0} = \frac{G(0)}{f}, \tag{1.23}$$

$$\left. \frac{du_e}{dt} \right|_{t=0} = 0, \tag{1.24}$$

$$u_r|_{t=0} = 0, \tag{1.25}$$

$$\left.\frac{\partial u_r}{\partial t}\right|_{t=0} = 0, \tag{1.26}$$

$$u_r|_{x=0} = 0, \tag{1.27}$$

$$\left.\frac{\partial u_r}{\partial x}\right|_{x=l} = 0, \tag{1.28}$$

where f is pipe's flow area.

Applying the Laplace transform and taking into account the convolution theorem [19,20] allowing for initial condition (1.23) and (1.24), we have:

$$u_e = \frac{G(0)}{f} + \frac{1}{l}\int_0^t P_c(\tau)\exp\left[-2a(t-\tau)\right]d\tau - \frac{1}{l}\int_0^t P_{\text{y}}(\tau)\exp\left[-2a(t-\tau)\right]d\tau -$$

$$-\frac{1}{2al}\exp(-2at)\left[P_{\text{y}}(0) - P_c(0)\right] + \frac{1}{2al}\left[P_{\text{y}}(0) - P_c(0)\right]. \tag{1.29}$$

We will look for the solution of equation (1.22) allowing for boundary conditions (1.27) and (1.28) in the form [16-18]:

$$u_r = \sum_{i=1}^{n}\varphi_i(t)\left(1 - \cos\frac{i\pi x}{l}\right), \tag{1.30}$$

where $\varphi_i(t)$ is an unknown function dependent on time t, l is the pipe run depth. Substituting expresion (1.30) in equation (1.22), multiplying the both hand sides of the obtained expression by $\left(1 - \cos\frac{i\pi x}{l}\right)$ and integrating it form 0 to l we get the equation:

$$\ddot{\varphi}_i + 2a\dot{\varphi}_i + \frac{c^2 i^2 \pi^2}{3l^2}\varphi_i = \frac{2}{3l}\left(\dot{P}_{\text{y}} - \dot{P}_c\right). \tag{1.31}$$

Applying the Laplace transform and considering the conversion and convolution theorems [19,20], from equation (1.31) allowing for initial conditions (1.25) and (1.26), we get:

$$\varphi_i = \frac{2}{3l}\left[\int_0^t P_{\text{y}}(\tau)\exp\left[-a(t-\tau)\right]\cos\left[\omega_i(t-\tau)\right]d\tau - \frac{a}{\omega_i}\int_0^t P_{\text{y}}(\tau)\exp\left[-a(t-\tau)\right]\times\right.$$

$$\times\sin\left[\omega_i(t-\tau)\right]d\tau - \frac{P_{\text{y}}(0)}{\omega_i}\exp(-at)\sin(\omega\, t) - \int_0^t P_{\text{y}}(\tau)\exp\left[-a(t-\tau)\right]\cos\left[\omega_i(t-\tau)\right]d\tau +$$

$$\left. + \frac{a}{\omega_i}\int_0^t P_{\text{c}}(\tau)\exp\left[-a(t-\tau)\right]\sin\left[\omega_i(t-\tau)\right]d\tau + \frac{P_c(0)}{\omega_i}\exp(-at)\sin(\omega\, t)\right], \tag{1.32}$$

$$\omega_i^2 = \frac{c^2 i^2 \pi^2}{3l^2} - a^2.$$

From the continuity function allowing for boundary condition (1.27) and expressions (1.29), (1.30) and (1.32) we get the following integral equation:

$$G(0)\exp(-\alpha\, t) - \frac{k\ (P_c(0) + P_k)}{D\mu\beta}\pi\, h\int_0^t \dot{P}_{\text{c}}(\tau)\exp\left[-\alpha\ (t-\tau)\right]d\tau = G(0) + \frac{f}{2al}\left[P_{\text{y}}(0) - P_c(0)\right] +$$

$$+\frac{f}{l}\int_0^t P_c(\tau)\exp\left[-2a(t-\tau)\right]d\tau-\frac{f}{l}\int_0^t P_y(\tau)\exp\left[-2a(t-\tau)\right]d\tau-$$

$$-\frac{f}{2al}\exp(-2at)\left[P_y(0)-P_c(0)\right]. \tag{1.33}$$

Applying the Laplace transform and considering convolution and conversion theorems, from expression (1.33) we get:

$$P_c=P_c(0)\left[\exp(-\beta_1 t)\frac{2a-\beta_1}{\beta_2-\beta_1}+\exp(-\beta_2 t)\frac{2a-\beta_2}{\beta_1-\beta_2}\right]+\left[\frac{f\left(P_c(0)-P_y(0)\right)}{2alb}-\frac{G(0)}{b}\right]\times$$

$$\times\left[\frac{2\alpha\, a}{\beta_1\beta_2}+\frac{(\alpha-\beta_1)(2a-\beta_1)}{\beta_1(\beta_1-\beta_2)}\exp(-\beta_1 t)+\frac{(\alpha-\beta_2)\ (2a-\beta_2)}{\beta_2(\beta_2-\beta_1)}\exp(-\beta_2 t)\right]+$$

$$+\frac{f}{lb}\left[\frac{\alpha-\beta_1}{\beta_2-\beta_1}\int_0^t P_y(\tau)\exp\left[-\beta_1(t-\tau)\right]d\tau+\frac{\alpha-\beta_2}{\beta_1-\beta_2}\int_0^t P_y(\tau)\exp\left[-\beta_2(t-\tau)\right]d\tau\right]-$$

$$-\frac{f}{2alb}\left[P_c(0)-P(0)\right]\left(\frac{\alpha-\beta_1}{\beta_2-\beta_1}\exp(-\beta_1 t)+\frac{\alpha-\beta_2}{\beta_1-\beta_2}\exp(-\beta_2 t)\right)+$$

$$+\frac{G(0)}{b}\left[\exp(-\beta_1 t)\frac{2a-\beta_1}{\beta_2-\beta_1}+\exp(-\beta_2 t)\frac{2a-\beta_2}{\beta_1-\beta_2}\right], \tag{1.34}$$

where $b=\frac{k[P_k+P_c(0)]}{\beta\mu}\frac{\pi\, h}{D}$, β_1 and β_2 are the roots of the equation

$$s^2+\left(a+\frac{f}{bl}\right)s+\frac{f}{bl}\alpha=0. \tag{1.35}$$

From the continuity condition at the wellhead allowing for expression (1.29), (1.30), (1.32) and (1.33), applying the Laplace transform we have:

$$\begin{array}{c}\frac{G(0)}{fs}+\frac{1}{l}\frac{\bar{P}_c}{s(s+2a)}-\frac{1}{l}\frac{\bar{P}_y}{s(s+2a)}+\\ \sum_{i=1}^n\frac{2}{3l}\left[\frac{s\bar{P}_y}{(s+a)^2+\omega_i^2}-\frac{P_y(0)-P_c(0)}{(s+a)^2+\omega_i^2}-\frac{s\bar{P}_c}{(s+a)^2+\omega_i^2}\right]=\bar{G}_q\end{array}. \tag{1.36}$$

where $G(0)$ is gas inflow the well per unit of time at initial moment, G_q -is well productivity.

Passage of gas through the choke.

Passing through the choke, gas enters the pipeline. When gas pass through the choke is pressure significantly decreases.

In the first approximation, the dependence between gas consuption Q_{HB} and pressure drop between the inlet and outlet of the choke is accepted linear [4]:

$$Q_{HB}=\alpha_0(\mathrm{P_y}(t)-P_{HB}(t)). \tag{1.37}$$

where α_0 is a productivity coefficient.

After the Laplace transform expression (1.37) we get

$$\bar{Q}_{HB}=\alpha_0(\bar{\mathrm{P}}_y-\bar{P}_{HB}) \tag{1.38}$$

From the continuity equation we have

$$G_q|_{x=l} = Q_{HB}. \tag{1.39}$$

Substituting expressions (1.36) and (1.38) in formula (1.39) allowing for only one term of the series, in the first approximation we get the following integral Volterra typeb equation which we define $P_{HB}(t)$

$$\begin{array}{l} \frac{G(0)}{fs} + \frac{1}{l}\frac{\dot{\bar{P}}_c}{s(s+2a)} - \frac{1}{l}\frac{\dot{\bar{P}}_{\text{y}}}{s(s+2a)} + \\ \frac{2}{3l}\left[\frac{s\bar{P}_{\text{y}}}{(s+a)^2+\omega_1^2} - \frac{P_{\text{y}}(0)-P_c(0)}{(s+a)^2+\omega_1^2} - \frac{s\bar{P}_{\text{c}}}{(s+a)^2+\omega_1^2}\right] = \alpha_0(\bar{\text{P}}_{\text{y}} - \bar{P}_{HB}) \end{array} \tag{1.40}$$

and from (1.40) we have

$$\begin{array}{l} \bar{P}_{HB} = \bar{P}_y - \frac{G(0)}{f\alpha_0 s} - \frac{\dot{\bar{P}}_c}{l\alpha_0 s(s+2a)} + \frac{\dot{\bar{P}}_y}{l\alpha_0 s(s+2a)} - \\ -\frac{2}{3l\alpha_0}\left[\frac{s\bar{P}_y}{(s+a)^2-\omega_1^2} - \frac{P_y(0)-P_c(0)}{(s+a)^2-\omega_1^2}\frac{s\bar{P}_c}{(s+a)^2-\omega_1^2}\right] \end{array}. \tag{1.41}$$

Gas flow in the trunk pipeline.

We consider gas flow in the trunk pipeline. We locale the origin of the coordinate axisx_1 at the inlet of the pipeline and direct in the gas flow direction. Assume that at the moment $t = 0$ at the direction l_2 from the wellhead a pipeline with consumption G is connected to the trunk pipeline. The equation of gas flow in the pipeline will be of the form [9,10]

$$\frac{\partial^2 P}{\partial t^2} = c^2\frac{\partial^2 P}{\partial x_1^2} - 2a_1\frac{\partial P}{\partial t} - \frac{2a_1c^2G}{f_1}\delta(x_1 - l_2). \tag{1.42}$$

The initial and boundary conditions

$$\left.\frac{\partial P}{\partial t}\right|_{t=0} = -c^2\frac{G}{f_1}\delta\left(x_1 - l_2\right), 0 \le x \le l, \tag{1.43}$$

$$P(x_1, 0)|_{t=0} = P_{HB}(0) - 2a_1Q_1(0)x_1, 0 \le x \le l, \tag{1.44}$$

$$P|_{x_1=0} = P_{HB}(t), t > 0, \tag{1.45}$$

$$P|_{x_1=l_1} = P_2, t > 0, \tag{1.46}$$

where $\delta(x, l)$ is the Dirac's function.

Allowing for boundary condition (1.45) and (1.46), we will look for the solution of the equation (1.42) in the form:

$$P = P_{HB}(t) - \frac{P_{HB}(t) - P_2}{l_1}x_1 + \sum_{i=1}^{n}\varphi_{2i}\left(t\right)\left(\sin\frac{i\pi x_1}{l_1}\right), \tag{1.47}$$

where $\varphi_{2i}\left(t\right)$ is an unknown function dependent as time t, l_1 is pipeline's length.

Substituting expression (1.47) in formula (1.42) allowing for initial conditions (1.43), (1.44) similar to (1.31) in the case $P_2 = const$ we get a differential equation with respect to φ_{2i}. Having solved these equation and substituted the obtained result in the first equation of the system (1.15), after the solution we get

$$\begin{gathered} Q_1|_{x_1=0} = Q_1(0)e^{-2a_1 t} + \tfrac{1}{l_1}\int_0^t P_{Hm}(\tau)\exp\left[-2a_1(t-\tau)\right]d\tau - \\ - \sum_{i=1}^{n}\left(\tfrac{i\pi}{l_1}\right)\int_0^t \varphi_{2i}(\tau)\exp\left[-2a_1(t-\tau)\right]d\tau - \tfrac{P_2}{2a_1 l_1}\left(1-\exp\left(-2a_1 t\right)\right) \end{gathered}, \tag{1.48}$$

where P_2 is pressure at the pipeline's end

$$\begin{gathered} \varphi_{2i} = \tfrac{\varphi_{20}}{\xi_1-\xi_2}(\exp(\xi_1 t)(2a+\xi_1) - \exp(\xi_2 t)(2a+\xi_2)) + \dot{\varphi}_{20}\tfrac{\exp(\xi_1 t)-\exp(\xi_2 t)}{\xi_1-\xi_2} - \\ -\tfrac{2}{\pi i}\left(P_{HB}(t) + \tfrac{\xi_1(2a+\xi_1)\int_0^t P_{HB}(\tau)\exp(\xi_1(t-\tau))d\tau}{\xi_1-\xi_2} - \tfrac{\xi_2(2a+\xi_2)\int_0^t P_{HB}(\tau)\exp(\xi_2(t-\tau))d\tau}{\xi_1-\xi_2}\right) - \\ + \\ + \tfrac{2P_{HB}(0)}{\pi(\xi_1-\xi_2)}(\exp(\xi_1 t)(2a+\xi_1) - \exp(\xi_2 t)(2a+\xi_2)) + \tfrac{2\dot{P}_{HB}(0)}{\pi(\xi_1-\xi_2)}(\exp(\xi_1 t) - \exp(\xi_2 t)) - \\ - \tfrac{4a_1c^2}{l_1}\left[\tfrac{G}{f_1}\sin\left(\tfrac{\pi l_2}{l_1}\right)\right]\left[\tfrac{\xi_2(\exp(\xi_1 t)-1)-\xi_1(\exp(\xi_2 t)-1)}{\xi_1\xi_2(\xi_1-\xi_2)}\right], \end{gathered} \tag{1.49}$$

where ξ_1 and ξ_2 are the roots of the equation $s^2 + 2a_1 s + \frac{c^2\pi^2 i^2}{l_1^2} = 0$.

From the continuity condition $Q_{HB} = Q_1|_{x1=0}$ at the wellhead, allowing for expression (1.37) and (1.48) we get

$$\begin{gathered} \alpha_0(\mathrm{P_y}(t) - P_{HB}(t)) = Q_1(0)e^{-2a_1 t} + \tfrac{1}{l_1}\int_0^t P_{HB}(\tau)\exp\left[-2a_1(t-\tau)\right]d\tau - \\ - \sum_{i=1}^{n}\left(\tfrac{i\pi}{l_1}\right)\int_0^t \varphi_{2i}(\tau)\exp\left[-2a_1(t-\tau)\right]d\tau - \tfrac{P_2}{2a_1 l_1}\left(1-\exp\left(-2a_1 t\right)\right), \end{gathered} \tag{1.50}$$

where $_{HB}$ is pressure after the choke.

Applying the Lapace transformation, from the expression (1.50), in the first $i = 1$ define P_y

$$\begin{gathered} \bar{P}_{\mathrm{y}} = \tfrac{3\alpha_0 l^2 l_1 b(s+2a)(s+2a_1)(s+\beta_1)(s+\beta_2)(s-\xi_1)(s-\xi_2)((s+a)^2+\omega^2)}{A_1(s-j_1)(s-j_2)(s-j_3)(s-j_4)(s-j_5)(s-j_6)(s-j_7)} \times \\ \times\Big[-\tfrac{1}{l_1 l\alpha_0}\tfrac{P_c(0)}{s(s+2a_1)(s+2a)} + \tfrac{1}{l_1 l\alpha_0}\tfrac{\bar{P}1_c}{(s+2a_1)(s+2a)} + \tfrac{1}{l_1 l\alpha_0}\tfrac{P_y(0)}{s(s+2a_1)(s+2a)} - \\ - \tfrac{2}{3l_1 l\alpha_0(s+2a_1)}\tfrac{s\bar{P}1_{\mathrm{c}}}{\left((s+a)^2+\omega_1^2\right)} - \tfrac{2}{l_1 l\alpha_0}\tfrac{P_c(0)}{(s+2a)(s-\xi_1)(s-\xi_2)} + \tfrac{2}{l_1 l\alpha_0(s+2a)}\tfrac{s\bar{P}1_{\mathrm{c}}}{(s-\xi_1)(s-\xi_2)} + \\ + \tfrac{2}{l_1 l\alpha_0(s+2a)}\tfrac{P_{\mathrm{y}}(0)}{(s-\xi_1)(s-\xi_2)} - \tfrac{4}{3l_1 l\alpha_0(s-\xi_1)(s-\xi_2)}\tfrac{s^2\bar{P}1_{\mathrm{c}}}{\left((s+a)^2+\omega_1^2\right)} + \\ + \tfrac{\bar{P}1_c}{l(s+2a)} - \tfrac{P_c(0)}{ls(s+2a)} + \tfrac{P_y(0)}{ls(s+2a)} - \tfrac{2}{3l}\tfrac{s\bar{P}1_{\mathrm{c}}}{\left((s+a)^2+\omega_1^2\right)} + \tfrac{2G(0)a_1}{fs(s+2a_1)} - \\ - \tfrac{2}{3l_1 l\alpha_0(s+2a_1)}\tfrac{P_{\mathrm{y}}(0)-P_c(0)}{\left((s+a)^2+\omega_1^2\right)} + \tfrac{G(0)}{\alpha_0 l_1 fs(s+2a_1)} - \tfrac{4s}{3l_1 l\alpha_0(s-\xi_1)(s-\xi_2)}\tfrac{P_{\mathrm{y}}(0)-P_c(0)}{\left((s+a)^2+\omega_1^2\right)} + \\ + \tfrac{2G(0)}{l_1 f\alpha_0(s-\xi_1)(s-\xi_2)} + \tfrac{P_2}{l_1 s(s+2a_1)} + \left(\tfrac{\pi}{l_1}\right)\tfrac{\bar{\Phi}_1}{(s+2a_1)} - \sum_{i=1}^{n}\tfrac{2(P_{\mathrm{y}}(0)-P_c(0))}{3l\left((s+a)^2+\omega_1^2\right)}\Big] \end{gathered} \tag{1.51}$$

where, $A_1 = 5\alpha_0 l^2 b + \alpha_0 l_1 lb$, $P_y(0) = \frac{fP_c(0)-2\alpha G(0)l}{f}$

$$\begin{gathered} \bar{\Phi}_1 = \tfrac{\varphi_0(s+2a_1)}{(s-\xi_1)(s-\xi_2)} + \tfrac{\dot{\varphi}_0}{(s-\xi_1)(s-\xi_2)} + \tfrac{2P_{HB}(0)(s+2a_1)}{\pi(s-\xi_1)(s-\xi_2)} + \tfrac{2\dot{P}_{HB}(0)}{\pi(s-\xi_1)(s-\xi_2)} - \\ - \tfrac{4a_1c^2}{l_1 s(s-\xi_1)(s-\xi_2)}\tfrac{G}{f_1}\sin\left(\pi\tfrac{l_2}{l_1}\right) \end{gathered}$$

$$\bar{P}1_c = P_c(0)\left[\frac{2a-\beta_1}{\beta_2-\beta_1}\frac{1}{s+\beta_1}+\frac{2a-\beta_2}{\beta_1-\beta_2}\frac{1}{s+\beta_2}\right]+\left[\frac{f\,(P_c(0)-P_{\mathrm{y}}(0))}{2alb}-\frac{G(0)}{b}\right]\times$$
$$\times\left[\frac{2\alpha\,a}{\beta_1\beta_2 s}+\frac{(\alpha-\beta_1)(2a-\beta_1)}{\beta_1(\beta_1-\beta_2)}\frac{1}{s+\beta_1}+\frac{(\alpha-\beta_2)(2a-\beta_2)}{\beta_2(\beta_2-\beta_1)(s+\beta_2)}\right]+$$
$$-\frac{f}{2alb}\left[P_c(0)-P_{\mathrm{y}}(0)\right]\left(\frac{\alpha-\beta_1}{\beta_2-\beta_1}\frac{1}{s+\beta_1}+\frac{\alpha-\beta_2}{\beta_1-\beta_2}\frac{1}{s+\beta_2}\right)+$$
$$+\frac{G(0)}{b}\left[\frac{2a-\beta_1}{\beta_2-\beta_1}\frac{1}{s+\beta_1}+\frac{2a-\beta_2}{\beta_1-\beta_2}\frac{1}{s+\beta_2}\right]$$

$j_1, j_2, j_3, j_4, j_5, j_6, j_7$ - equation roots

$$\begin{aligned}
&(s-\xi_1)(s-\xi_2)(3\alpha_0 b\,l^2(s+2a)((s+a)^2+\omega^2)(s+\beta_1)(s+\beta_2)-\\
&-3f(s+\alpha)((s+a)^2+\omega^2)+3lb((s+a)^2+\omega^2)(s+\beta_1)(s+\beta_2)-\\
&-2s\,b\,l\,(s+2a)(s+\beta_1)(s+\beta_2)+2f\,s(s+2a)(s+\alpha))+\\
&+2s(s+2a_1)(3\alpha_0 b\,l^2(s+2a)((s+a)^2+\omega^2)(s+\beta_1)(s+\beta_2)-\\
&-3f(s+\alpha)((s+a)^2+\omega^2)+3lb((s+a)^2+\omega^2)(s+\beta_1)(s+\beta_2)-\\
&-2s\,b\,l\,(s+2a)(s+\beta_1)(s+\beta_2)+2f\,s(s+2a)(s+\alpha))-\\
&-\alpha_0 l_1(s+2a_1)(s-\xi_1)(s-\xi_2)(3f(s+\alpha)((s+a)^2+\omega^2)+\\
&+2slb\,(s+2a)(s+\beta_1)(s+\beta_2)-\\
&-3lb((s+a)^2+\omega^2)(s+\beta_1)(s+\beta_2)-2fs(s+2a)(s+\alpha))=0
\end{aligned} \tag{1.52}$$

Applying the Laplace transform and considering the convolution and conversion theorem, from the expressions (1.35), (1.52) and (1.49) allowing for numerical values of the system parametres

$$c=300m\cdot s^{-1}, \mu=10^{-5}Pa\cdot s; h=10m; k=5\cdot 10^{-14}m^2; \rho=0.668kq\cdot m^{-3}; l=1000m;$$
$$l_1=20000m; l_2=200m; P_c(0)=24\cdot 10^6 Pa; P_0=24\cdot 10^6 Pa; P_k(0)=25\cdot 10^6 Pa;$$
$$P_{atm}=10^5 Pa; P_c(T)=80\cdot 10^5 Pa; R_k=100m; \pi=3,14; a=10^{-3}s^{-1}; a_1=5\cdot 10^{-1}a^{-1};$$
$$d=6\cdot 10^{-2}m; d_1=28\cdot 10^{-2}m; d_2=28\cdot 10^{-2}m; r_c=7.5\cdot 10^{-2}m$$

Equation (1.52) takes the form

$$-8.53412065s^7-261.1447214s^6-554.8469569s^5-543.9719964s^4-463.0458386s^3-$$
$$-204.7252483s^2-0.4484496931s-2.7750954981\cdot 10^{-9}=0$$

and we get,

$$
\begin{aligned}
P_y = {} & 2.45999441 \cdot 10^6 + 2.47514867 \cdot 10^7 \exp(-6.18821549 \cdot 10^{-9} t) + \\
& +1.91734282 \cdot 10^5 \exp(-0.00220142208 t) + \\
& +1.77839541 \cdot 10^5 \exp(-0.8142343565 t) - \\
& -1.96218954 \cdot 10^7 \exp(-28.38548701 t) + \\
& +7.25584358 \cdot 10^5 \exp(-1.234873634\, t) + \\
& +1.343889923 \cdot 10^6 \exp(-0.08113866709 t) \cos(0.9108927519\ t) - \\
& -1.15848128 \cdot 10^5 \exp(-0.08113866709 t) \sin(0.9108927519\, t)
\end{aligned}
\tag{1.53}
$$

$$
\begin{aligned}
P_c = {} & 5.27444657 \cdot 10^7 \exp(-0.00000124\ t) - \\
& 4.27175414 \cdot 10^6 \exp(-19.3062\, t) - \\
& -4.14599944 \cdot 10^6 - \\
& -2.49989987 \cdot 10^7 \exp(-6.1883 \cdot 10^{-9} t) + \\
& 1.90978618 \cdot 10^5 \exp(-0.00220142208\, t) + \\
& +7.71152443 \cdot 10^5 \exp(-1.234873634 t) + \\
& 1.84709 \cdot 10^5 \exp(-0.8142343565\, t) - \\
& +1.3450358 \cdot 10^6 \exp(-0.08113866709 t) \cos(0.9108927519\ t) - \\
& -52009.6795 \exp(-0.08113866709 t) \sin(0.9108927519\ t) + \\
& +2.50329492 \cdot 10^7 \exp(-28.38648701\ t)
\end{aligned}
\tag{1.54}
$$

$$
\begin{aligned}
Q_1 = {} & -1.158837965 \exp(-0.1 t) \sin(0.5345914328\ t) + \\
& +0.1876109889 \exp(-0.1 t) \cos(0.5345914328\ t) + \\
& 0.0007131985852 \exp(-0.002201422081\ t) + \\
& +0.003766347582 \exp(-1.234873634 t) + \\
& +0.09856122154 \exp(-28.38648701\ t) + \\
& +6.821282984 \exp(-6.1883 \cdot 10^{-9} t) + \\
& +0.001212016763 \exp(-0.8142343565\, t) - \\
& -3.426454683 \exp(-0.000001240642\, t) - \\
& -0.0139214703 \exp(-19.30619548\, t) - \\
& -3.645959724 \exp(-0.2 t) + \\
& +0.0003152671 \exp(-0.08113866709 t) \cos(0.9108927519\ t) + \\
& +0.000346236 \exp(-0.08113866709 t) \cos(0.9108927519\ t) \\
& +6.160089261
\end{aligned}
\tag{1.55}
$$

Conclusion. The result of numerical calculation are in Fig. 2-6. In Fig. 2 and Fig.5 for small values of time, in Fig.3 and Fig.4, Fig.6 for great values of time.

As can be seen from Fig. 2-4, when connected to the trunk line the well head and bottom hole pressure at first increases and after same tie it drops and stabilized approaching to in steady-state.

It is given from Fig.5 and Fig.6 that after connected to the trunk line the well productivity at first fluctuate and then begin to drop tending to its steady state value.

A model of the process of non-stationary gas flow in a formation-pipeline system during connections and fluid withdrawal from the operating transport line, is constructed. Analytic expression allowing to determine well productivity and also well head and buttonhole pressure change at the outlet of the pipeline are obtained.

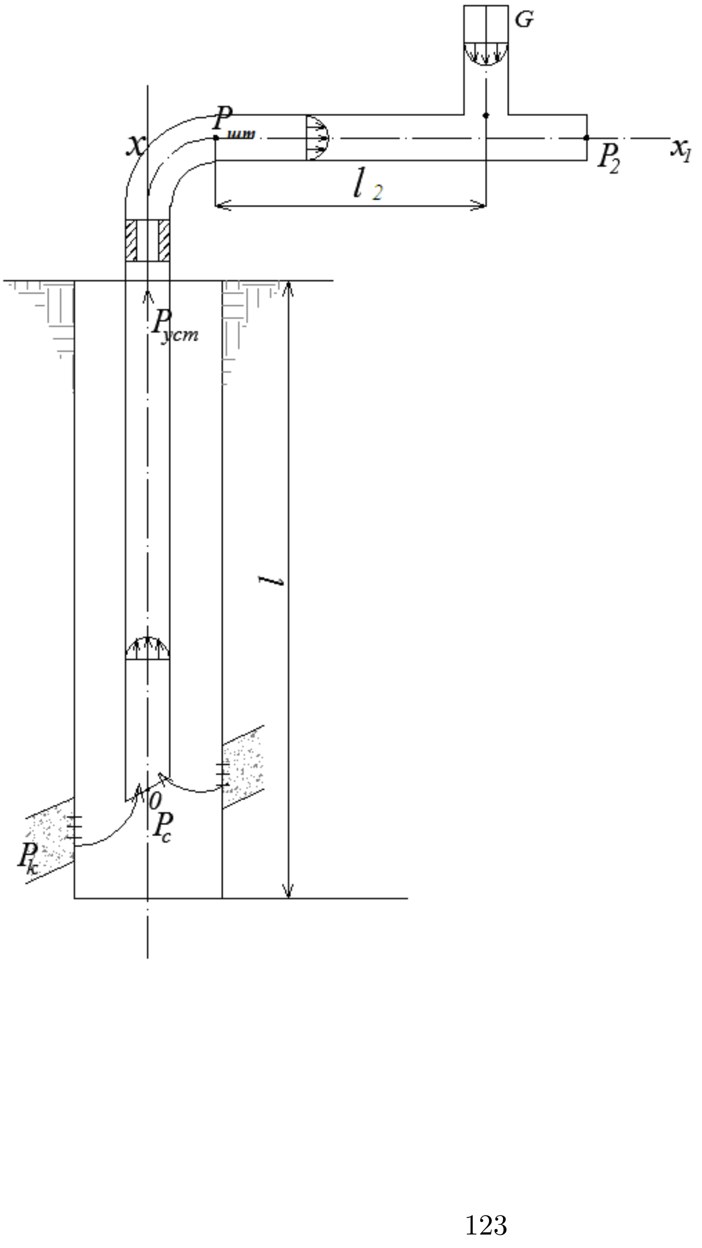


Fig.1

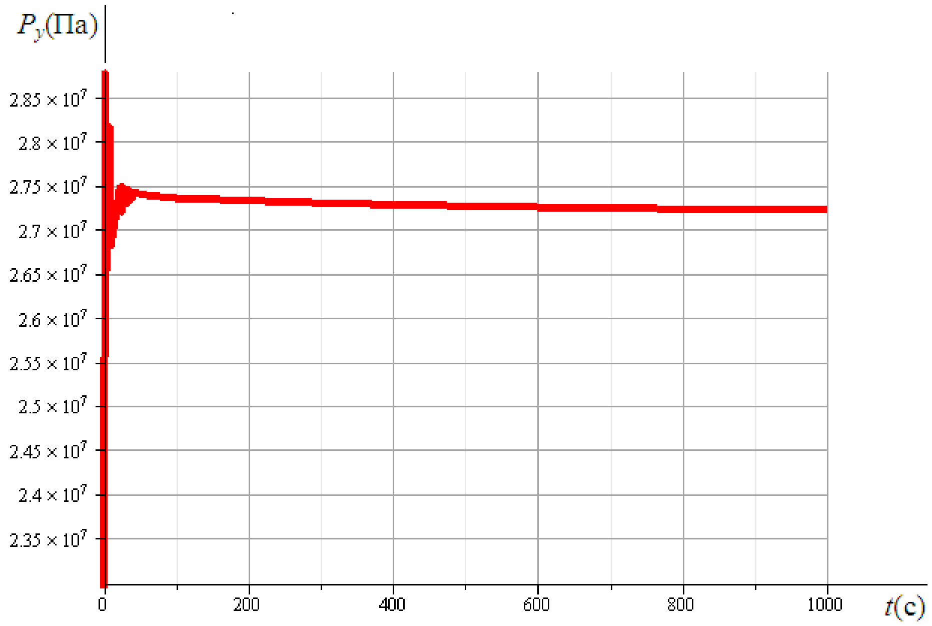


Fig.2 $t = 1000c$

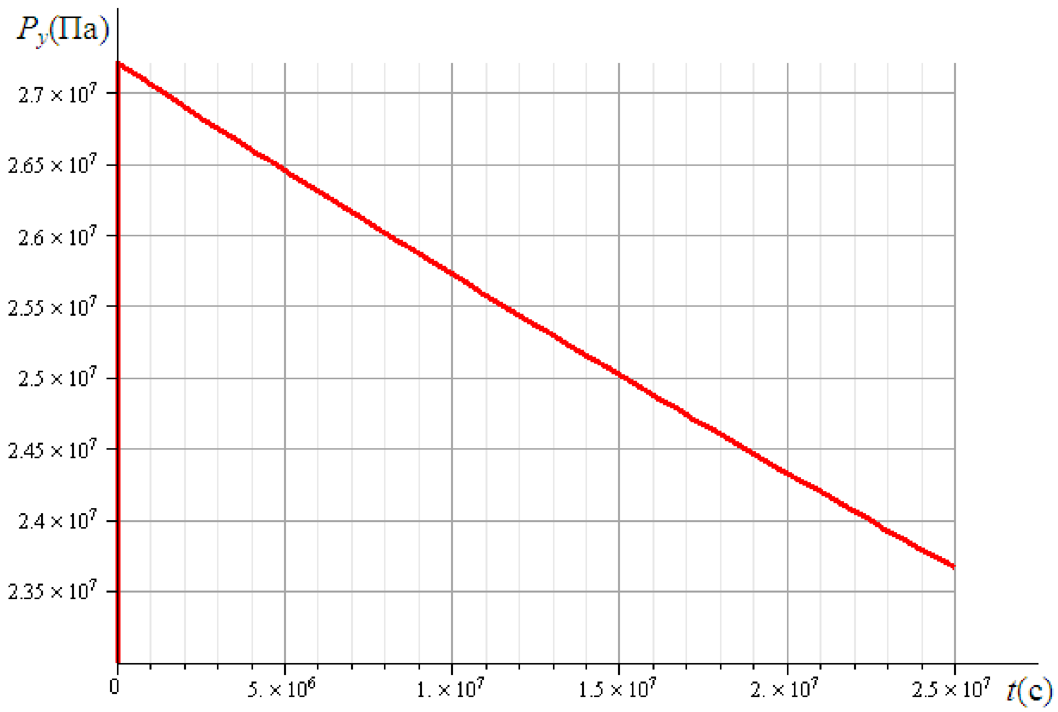
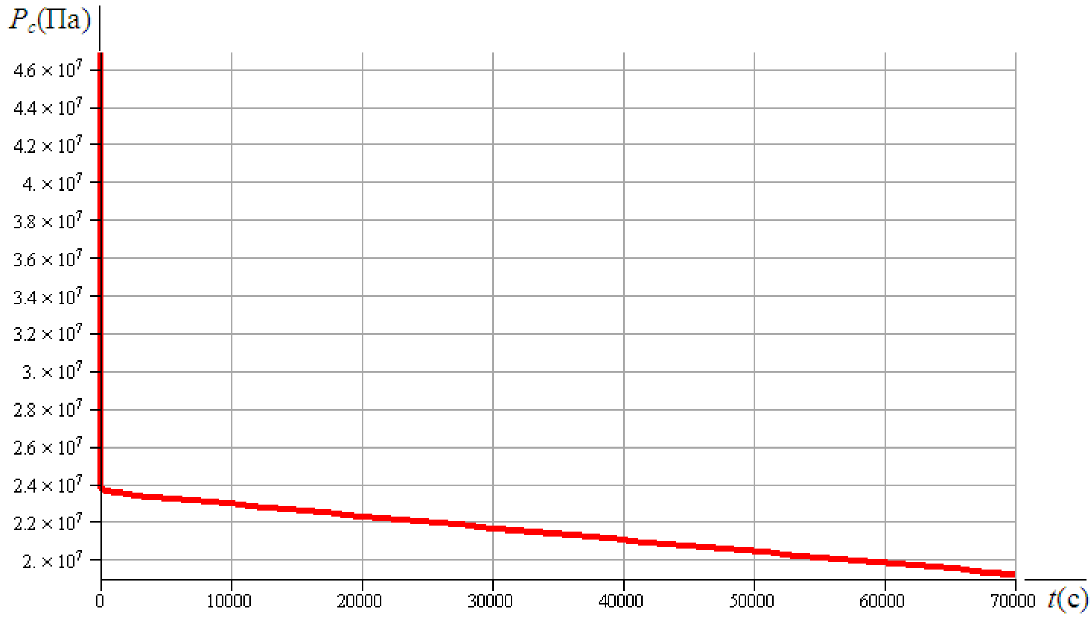
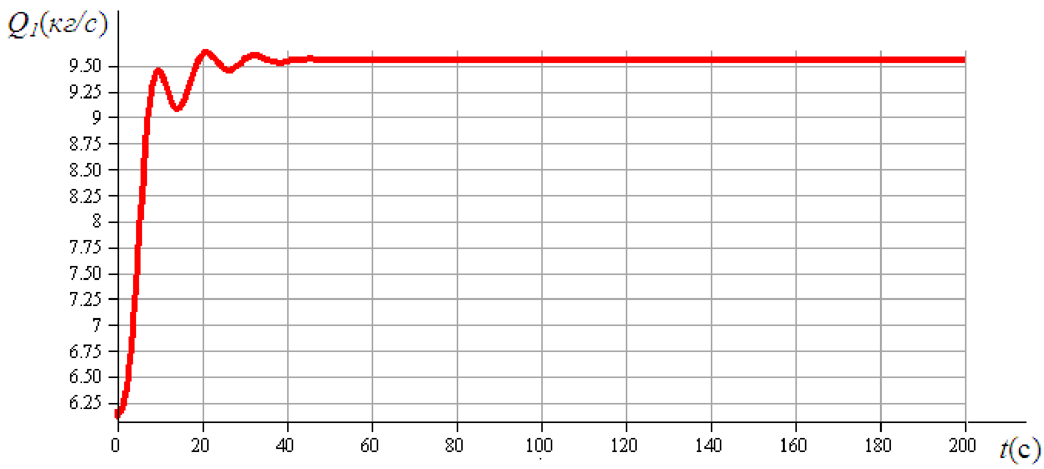


Fig.3 $t = 2.5 \cdot 10^7 c = 289 day$



P_c(Па)
4.6 × 10^7
4.4 × 10^7
4.2 × 10^7
4. × 10^7
3.8 × 10^7
3.6 × 10^7
3.4 × 10^7
3.2 × 10^7
3. × 10^7
2.8 × 10^7
2.6 × 10^7
2.4 × 10^7
2.2 × 10^7
2. × 10^7
0
10000
20000
30000
40000
50000
60000
70000
t(c)

Fig.4



Q_1(кг/с)
9.50
9.25
9
8.75
8.50
8.25
8
7.75
7.50
7.25
7
6.75
6.50
6.25
0
20
40
60
80
100
120
140
160
180
200
t(c)

Fig.5

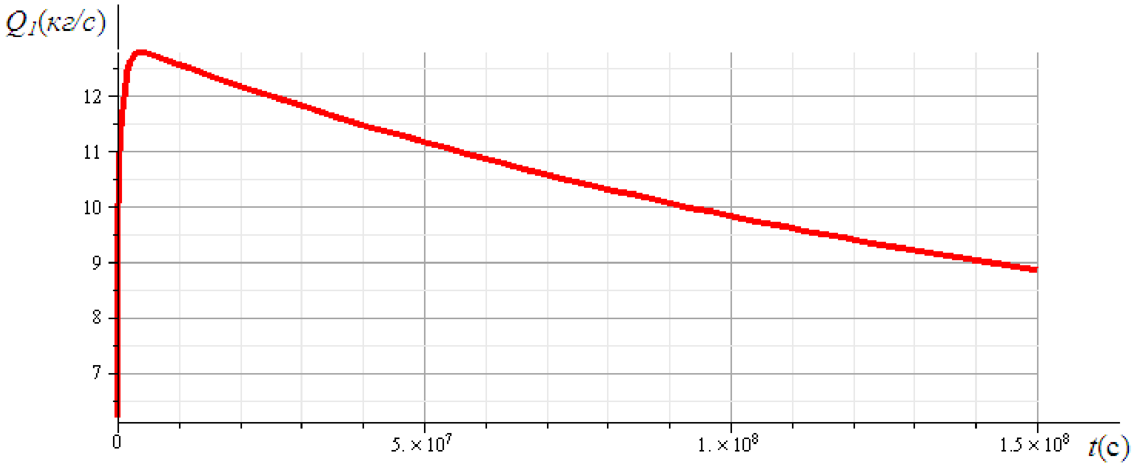


Fig.6

Denotations P-is pressure at any point of the formation, Pa; P_k-is pressure on the formation's contour, Pa;

P_c- is buttonhole pressure, Pa; P_y - is wellhead pressure, Pa;

φ - is a time-dependent function, Pa; f- is flow area of the column of lifting pipes, m^2 ; f_1-is flow are of the transport pipeline, m^2 ;h-formation's power, m;a, a_1 - is a resistant factor, c^{-1};r-is a coordinate, m;l-is a lifting pipe run depth, m;l_1- is the length of the transport pipeline, m; r_c-is well radius, m;R_k- is a radius of formation's contour, m;T - is well's operation period, s;τ, t-is time, s;μ - is dynamical viscosity of gas, Pas;m - is porosity of formation's rock , k - is formation's permeability factor m_2,ρ-is gas density, kg/m^3;s - is gas propagation speed in gas, m/s;x-is a coordinate; $\beta, \theta, \Phi_1, P1_c, D, \alpha$–are denotations.

Indices: atm—- atmospheric; k - contour; c - well; y —- wellhead; e —- portable (the first letter from the French word "entrainer") r – relative (the first letter of the English word relative)

Sketched inscriptions

Fig.1-Calculation scheme

Fig.2-3-Dynamics of wellhead pressure

Fig.4- Dynamics of buttonhole pressure change

Fig.5 Dynamics of well productivity at the initial period

Fig.6-Dynamic of well productivity

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