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$(\omega, c)-$periodic solution for an impulsive system of differential equations with the quadrate of Gerasimov–Caputo fractional operator and maxima

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Abstract. Existence and uniqueness of $(\omega, c)-$periodic solution of boundary value problem for a nonlinear system of ordinary differential equations with the quadrate of Gerasimov–Caputo operator, impulsive effects and maxima are studied. Problem is reduced to solvability of the complex system of nonlinear functional integral equations. The method of contracted mapping is used in the proof of one-valued solvability of nonlinear functional integral equations. Some estimates are obtained for the $(\omega, c)-$periodic solution of the problem.

Key Words and Phrases: Impulsive system of differential equations, quadrate of Gerasimov–Caputo operator, $(\omega, c)-$periodic solution, contracted mapping, existence and uniqueness.

2010 Mathematics Subject Classifications: Primary 34B37, 34C25, 34B15

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1. Introduction

The dynamics of evolving processes sometimes undergoes abrupt changes. Often such short-term perturbations are interpreted as impulses. That is, we actually have the dynamic systems with impulsive actions. So, we have to consider differential equations, the solutions of which are functions with first kind discontinuities at times. Impulsive differential and integro-differential equations have applications in biological, chemical and physical sciences, ecology, biotechnology, industrial robotic, pharmacokinetics, optimal control, etc. [1, 2, 3, 4, 5]. In particular, some problems with impulsive effects appear in biophysics at micro- and nano-scales [6, 7, 8, 9, 10]. A lot of publications of studying on differential equations with impulsive effects are appearing [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

Fractional calculus plays an important role in the mathematical modeling of many problems in scientific and engineering disciplines [25, 26, 27, 28]. In the works [29, 30, 31, 32, 33, 34, 35, 36, 37], the problems of applying of the fractional calculations to the theory of differential equations are considered. In the works [38, 39, 40], the periodic solutions of impulsive differential equations are studied. We note that the theory of $(\omega, c)-$periodic differential systems are important part of the theory of differential equations. Therefore,

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this direction is rapidly developing. A large number of papers are published in this field in serious scientific journals (see, for examples, the works [41, 42, 43], where are studied the problems of unique existence of the $(\omega, c)-$periodic solution for impulsive differential equations.

In our present paper on the interval $\Omega \equiv [0, \omega] \setminus \{t_i\}$ for $i = 1, 2, ..., p$ we consider the questions of existence and uniqueness of the $(\omega, c)-$periodic solutions of the nonlinear system of impulsive differential equations with the quadrate of Gerasimov–Caputo operator and maxima

$$[{}_C D_{0t}^{\alpha}]^2 \, x(t) = f\left(t, x(t), \max\left\{x(\tau)\,|\tau \in [t-h, t]\right\}\right), \tag{1}$$

where $0 < h = \text{const}$ is delay.

The equation (1) we study with $(\omega, c)-$periodic conditions

$$x(\omega) = c \cdot x(0), \tag{2}$$

$${}_C D_{0t}^{\alpha} x(\omega) = c \cdot {}_C D_{0t}^{\alpha} x(0) \tag{3}$$

and nonlinear impulsive conditions

$$x\left(t_i^+\right) - x\left(t_i^-\right) = F_i\left(x\left(t_i\right)\right), \qquad i = 1, 2, ..., p, \tag{4}$$

$${}_C D_{0t}^{\alpha} x\left(t_i^+\right) - {}_C D_{0t}^{\alpha} x\left(t_i^-\right) = G_i\left(x\left(t_i\right)\right), \qquad i = 1, 2, ..., p, \tag{5}$$

where ${}_C D_{0t}^{\alpha}$ is the Gerasimov–Caputo α-order fractional derivative for a function $x(t)$ and defined by

$${}_C D_{0t}^{\alpha} x(t) = I_{0t}^{1-\alpha} x'(t) = \frac{1}{\Gamma(1-\alpha)} \int\limits_0^t \frac{x'(s)\,ds}{(t-s)^{\alpha}}, \;\; t \in (0, \omega),$$

$$I_{0t}^{\alpha} x(t) = \frac{1}{\Gamma(\alpha)} \int\limits_0^t (t-s)^{\alpha-1} x(s) ds, \quad 0 < \alpha \le 1,$$

$\Gamma(\alpha)$ is the Gamma-function, $0 = t_0 < t_1 < ... < t_p < t_{p+1} = \omega$, $x \in \mathbb{X}$, $\mathbb{X}$ is the closed bounded domain in the space $\mathbb{R}^n$, $f \in \mathbb{R}^n$, $x\left(t_i^+\right) = \lim\limits_{\nu \to 0^+} x\left(t_i + \nu\right)$, $x\left(t_i^-\right) = \lim\limits_{\nu \to 0^-} x\left(t_i - \nu\right)$.

We note that this paper is further development of the work [44] to the case of quadrate of fractional operator. In addition, we note that the works [38, 39, 40] are not particular cases of the present paper. Because in our work we assume that $c \neq 1$.

By $C\left([0, \omega], \mathbb{R}^n\right)$ is denoted the Banach space, which consists continuous vector functions $x(t)$, defined on the segment $[0, \omega]$, with the norm

$$\| \, x(t) \, \| = \sqrt{\sum_{j=1}^n \max_{0 \le t \le \omega} |\, x_j(t) \,|}.$$

By $PC\left([0,\omega],\mathbb{R}^n\right)$ is denoted the linear vector space

$$PC\left([0,\omega],\mathbb{R}^n\right)=\Big\{x:[0,\omega]\to\mathbb{R}^n;\ x(t)\in C\left(\left(t_i,t_{i+1}\right],\mathbb{R}^n\right),\ i=1,...,p\Big\},$$

where $x\left(t_i^+\right)$ and $x\left(t_i^-\right)$ $(i=0,1,...,p)$ exist and are bounded; $x\left(t_i^-\right)=x\left(t_i\right)$. Note, that the linear vector space $PC\left([0,\omega],\mathbb{R}^n\right)$ is Banach space with the norm

$$\parallel x(t)\parallel_{PC}=\max\Big\{\parallel x(t)\parallel_{C(t_i,t_{i+1}]},\ i=1,2,...,p\Big\}.$$

We use also the vector space $BD\left([0,\omega],\mathbb{R}^n\right)$, which is Banach space with the following norm

$$\parallel x(t)\parallel_{BD}=\parallel x(t)\parallel_{PC}+h\cdot\big\|\,x'(t)\,\big\|_{PC},$$

where $0<h=\text{const}$ is delay given by equation (1).

Formulation of problem. To find the $(\omega,c)-$periodic function $x(t)\in BD\left([0,\omega],\mathbb{R}^n\right)$, which for all $t\in\Omega$ satisfies the system of differential equations (1) for $0<\alpha\le 1$, $(\omega,c)-$periodic conditions (2), (3) and for $t=t_i,\ \ i=1,2,...,p,\ \ 0<t_1<t_2<...<t_p<\omega$ satisfies the nonlinear impulsive conditions (4), (5).

2. Reduction to functional integral equation

Let the function $x(t)\in BD\left([0,\omega],\mathbb{R}^n\right)$ is a solution of the $(\omega,c)-$periodic boundary value problem (1)–(5). Then, by virtue of fractional operators, after integration on the intervals $\left(0,t_1\right],\ \left(t_1,t_2\right],\ \ldots,\ \left(t_p,t_{p+1}\right]$ we have:

$$I_{0\,t_1}^{\alpha}{}_CD_{0\,t_1}^{\alpha}\left[D_{0\,t_1}^{\alpha}x(t)\right]=I_{0\,t_1}^{\alpha}I_{0\,t_1}^{1-\alpha}\left[D_{0\,t_1}^{\alpha}x(t)\right]'=I_{0\,t_1}^{1}\left[D_{0\,t_1}^{\alpha}x(t)\right]'=$$

$$=\int\limits_0^{t_1}\left[D_{0\,t_1}^{\alpha}x(s)\right]'ds=D_{0\,t_1}^{\alpha}x(t_1^-)-D_{0\,t_1}^{\alpha}x(0^+),\tag{6}$$

$$I_{0t_1}^{\alpha}f\left(t,x(t),y(t)\right)=\frac{1}{\Gamma(\alpha)}\int\limits_0^{t_1}(t_1-s)^{\alpha-1}f\left(s,x(s),y(s)\right)ds,\tag{7}$$

$$\frac{1}{\Gamma(\alpha)}\int\limits_{t_1}^{t_2}(t_2-s)^{\alpha-1}f\left(s,x(s),y(s)\right)ds=D_{t_1t_2}^{\alpha}x(t_2^-)-D_{t_1t_2}^{\alpha}x(t_1^+),\tag{8}$$

$$\vdots$$

$$\frac{1}{\Gamma(\alpha)}\int\limits_{t_p}^{\omega}(T-s)^{\alpha-1}f\left(s,x(s),y(s)\right)ds=D_{t_pt_{p+1}}^{\alpha}x(t_{p+1}^-)-D_{t_pt_{p+1}}^{\alpha}x(t_p^+),\tag{9}$$

where $f\left(t,x(t),y(t)\right)=f\left(t,x(t),\max\left\{x(\tau)\,|\tau\in[t-h,t]\right\}\right).$

From the formulas (6)–(9) and $D^{\alpha}_{0t_1}x(0^+) = D^{\alpha}_{0t_1}x(0)$, $D^{\alpha}_{t_pt_{p+1}}x(t^-_{p+1}) = D^{\alpha}_{t_pt_{p+1}}x(t)$, on the interval $(0,\omega]$ we have

$$\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1} f\left(s, x(s), y(s)\right) ds = -{}_C D^{\alpha}_{0t_1}x(0) - \left[{}_C D^{\alpha}_{t_1t_2}x\left(t_1^+\right) -_C D^{\alpha}_{0t_1}x\left(t_1\right)\right] -$$

$$-\left[{}_C D^{\alpha}_{t_2t_3}x\left(t_2^+\right) -_C D^{\alpha}_{t_1t_2}x\left(t_2\right)\right] - \ \ldots \ - \left[{}_C D^{\alpha}_{t_pt_{p+1}}x\left(t_p^+\right) -_C D^{\alpha}_{t_{p-1}t_p}x\left(t_p\right)\right] +_C D^{\alpha}_{t_pt_{p+1}}x(t).$$

Taking into account the impulsive condition (5) in the last equality, we obtain

$$ {}_C D^{\alpha}_{0t}x(t) =_C D^{\alpha}_{0t}x(0) + \sum_{0<t_i<t} G_i\left(x\left(t_i\right)\right) + \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1} f\left(s, x(s), y(s)\right) ds. \tag{10}$$

Hence, we have

$$ {}_C D^{\alpha}_{0t}x(\omega) =_C D^{\alpha}_{0t}x(0) + \sum_{0<t_i<\omega} G_i\left(x\left(t_i\right)\right) + \frac{1}{\Gamma(\alpha)}\int_0^{\omega} (\omega-s)^{\alpha-1} f\left(s, x(s), y(s)\right) ds. \tag{11}$$

Let the function $x(t) \in BD\left([0,\omega], \mathbb{R}^n\right)$ in (10), satisfies the boundary value condition (3). Then from (11) we have

$$ {}_C D^{\alpha}_{0t}x(0) = \frac{1}{c-1}\sum_{0<t_i<\omega} G_i\left(x\left(t_i\right)\right) + \frac{1}{(c-1)\Gamma(\alpha)}\int_0^{\omega} (\omega-s)^{\alpha-1} f\left(s, x(s), y(s)\right) ds. \tag{12}$$

By virtue of (12), from (10) we obtain the functional differential equation

$$ {}_C D^{\alpha}_{0t}x(t) = \frac{1}{c-1}\sum_{0<t_i<\omega} G_i\left(x\left(t_i\right)\right) + \sum_{0<t_i<t} G_i\left(x\left(t_i\right)\right) +$$

$$+\frac{1}{(c-1)\Gamma(\alpha)}\int_0^{\omega} (\omega-s)^{\alpha-1} f\left(s, x(s), \max\left\{x(\tau)\left|\tau \in [s-h,s]\right.\right\}\right) ds +$$

$$+\frac{1}{\Gamma(\alpha)}\int_0^{t} (t-s)^{\alpha-1} f\left(s, x(s), \max\left\{x(\tau)\left|\tau \in [s-h,s]\right.\right\}\right) ds. \tag{13}$$

Repeating the process above and using conditions (2) and (4), from (13) we arrive at the following system of functional integral equations

$$x(t) = \frac{1}{c-1}\sum_{0<t_i<\omega} F_i\left(x\left(t_i\right)\right) + \sum_{0<t_i<t} F_i\left(x\left(t_i\right)\right) +$$

$$+\frac{1}{(c-1)\Gamma(\alpha)}\frac{t^\alpha}{\alpha}\sum_{0<t_i<\omega} G_i\left(x\left(t_i\right)\right)+\frac{1}{\Gamma(\alpha)}\int\limits_0^t (t-s)^{\alpha-1}\sum_{0<t_i<s} G_i\left(x\left(t_i\right)\right)ds+$$

$$+\frac{1}{(c-1)^2\Gamma(\alpha)}\frac{\omega^\alpha}{\alpha}\sum_{0<t_i<\omega} G_i\left(x\left(t_i\right)\right)+\frac{1}{(c-1)\Gamma(\alpha)}\int\limits_0^\omega (\omega-s)^{\alpha-1}\sum_{0<t_i<s} G_i\left(x\left(t_i\right)\right)ds+$$

$$+\frac{1}{(c-1)^2\Gamma^2(\alpha)}\int\limits_0^\omega (\omega-s)^{\alpha-1}\int\limits_0^\omega (\omega-\theta)^{\alpha-1} f\left(\theta, x(\theta), \max\left\{x(\tau)\left|\tau\in[\theta-h,\theta]\right.\right\}\right)d\theta ds+$$

$$+\frac{1}{(c-1)\Gamma^2(\alpha)}\int\limits_0^\omega (\omega-s)^{\alpha-1}\int\limits_0^s (s-\theta)^{\alpha-1} f\left(\theta, x(\theta), \max\left\{x(\tau)\left|\tau\in[\theta-h,\theta]\right.\right\}\right)d\theta ds+$$

$$+\frac{1}{(c-1)\Gamma^2(\alpha)}\int\limits_0^t (t-s)^{\alpha-1}\int\limits_0^\omega (\omega-\theta)^{\alpha-1} f\left(\theta, x(\theta), \max\left\{x(\tau)\left|\tau\in[\theta-h,\theta]\right.\right\}\right)d\theta ds+$$

$$+\frac{1}{\Gamma^2(\alpha)}\int\limits_0^t (t-s)^{\alpha-1}\int\limits_0^s (s-\theta)^{\alpha-1} f\left(\theta, x(\theta), \max\left\{x(\tau)\left|\tau\in[\theta-h,\theta]\right.\right\}\right)d\theta ds. \tag{14}$$

Lemma 2.1. *The following equalities are true:*

$$\int\limits_0^t (t-s)^{\alpha-1}\int\limits_0^s (s-\theta)^{\alpha-1} f(\theta)d\theta ds=\frac{\Gamma^2(\alpha)}{\Gamma(2\alpha)}\int\limits_0^t (t-\theta)^{2\alpha-1} f(\theta)d\theta, \tag{15}$$

$$\int\limits_0^\omega (\omega-s)^{\alpha-1}\int\limits_0^s (s-\theta)^{\alpha-1} f(\theta)d\theta ds=\frac{\Gamma^2(\alpha)}{\Gamma(2\alpha)}\int\limits_0^\omega (\omega-\theta)^{2\alpha-1} f(\theta)d\theta. \tag{16}$$

Proof. The left-hand side of (15) we rewrite as

$$\int\limits_0^t (t-s)^{\alpha-1}\int\limits_0^s (s-\theta)^{\alpha-1} f(\theta)d\theta ds=\int\limits_0^t f(\theta)\int\limits_\theta^t (t-s)^{\alpha-1}(s-\theta)^{\alpha-1}dsd\theta. \tag{17}$$

It is easy to calculate the internal integral on the right-hand side of (17):

$$\int\limits_\theta^t (t-s)^{\alpha-1}(s-\theta)^{\alpha-1}ds=(t-\theta)^{2\alpha-1}\int\limits_0^1 \tau^{\alpha-1}(1-\tau)^{\alpha-1}d\tau=\mathrm{B}(\alpha,\alpha)(t-\theta)^{2\alpha-1}. \tag{18}$$

Taking (18) and $\mathrm{B}(\alpha,\alpha)=\frac{\Gamma^2(\alpha)}{\Gamma(2\alpha)}$ into account, from (17) we obtain (15). The proof of (16) is similar. The Lemma 2.1 is proved. □

Lemma 2.2. *The following equalities are true:*

$$\int_0^t (t-s)^{\alpha-1} \int_0^\omega (\omega-\theta)^{\alpha-1} f(\theta) d\theta ds = \frac{t^\alpha}{\alpha} \int_0^\omega (\omega-\theta)^{\alpha-1} f(\theta) d\theta, \tag{19}$$

$$\int_0^\omega (\omega-s)^{\alpha-1} \int_0^\omega (\omega-\theta)^{\alpha-1} f(\theta) d\theta ds = \frac{\omega^\alpha}{\alpha} \int_0^\omega (\omega-\theta)^{\alpha-1} f(\theta) d\theta. \tag{20}$$

Proof. The left-hand side of (19) we rewrite as:

$$\int_0^t (t-s)^{\alpha-1} \int_0^\omega (\omega-\theta)^{\alpha-1} f(\theta) d\theta ds = \int_0^t (t-s)^{\alpha-1} ds \int_0^\omega (\omega-\theta)^{\alpha-1} f(\theta) d\theta. \tag{21}$$

We calculate the first integral on the right-hand side of (21):

$$\int_0^t (t-s)^{\alpha-1} ds = -\frac{(t-s)^\alpha}{\alpha}\bigg|_{s=0}^{s=t} = \frac{t^\alpha}{\alpha}. \tag{22}$$

By virtue of (22), from (21) we obtain (19). The proof of (20) is similar. The Lemma 2.2 is proved. $\square$

Applying the Lemmas 2.1 and 2.2, from (14) we obtain

$$x(t) = J(t;x) \equiv \frac{1}{c-1} \sum_{0<t_i<\omega} F_i\left(x\left(t_i\right)\right) + \sum_{0<t_i<t} F_i\left(x\left(t_i\right)\right) +$$

$$+\chi(t) \sum_{0<t_i<\omega} G_i\left(x\left(t_i\right)\right) + \int_0^\omega K_1(t,s) \sum_{0<t_i<s} G_i\left(x\left(t_i\right)\right) ds +$$

$$+ \int_0^\omega K_2(t,s) f\left(s, x(s), \max\left\{x(\tau) \,|\tau \in [s-h, s]\right\}\right) ds, \tag{23}$$

where

$$\chi(t) = \frac{(c-1)t^\alpha + \omega^\alpha}{\alpha(c-1)^2\Gamma(\alpha)}, \quad K_1(t,s) = \begin{cases} \frac{(\omega-s)^{\alpha-1}}{(c-1)\Gamma(\alpha)}, & t < s \leq \omega, \\ \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} + \frac{(\omega-s)^{\alpha-1}}{(c-1)\Gamma(\alpha)}, & 0 \leq s < t, \end{cases}$$

$$K_2(t,s) = \begin{cases} \frac{(\omega-s)^{2\alpha-1}}{(c-1)\Gamma(2\alpha)} + \frac{\omega^\alpha + (c-1)t^\alpha}{\alpha(c-1)^2\Gamma^2(\alpha)}(\omega-s)^{\alpha-1}, & t < s \leq \omega, \\ \frac{(t-s)^{2\alpha-1}}{\Gamma(2\alpha)} + \frac{(\omega-s)^{2\alpha-1}}{(c-1)\Gamma(2\alpha)} + \frac{\omega^\alpha + (c-1)t^\alpha}{\alpha(c-1)^2\Gamma^2(\alpha)}(\omega-s)^{\alpha-1}, & 0 \leq s < t. \end{cases}$$

Lemma 2.3. *For the equation* (23) *is true the following estimate*

$$\| J(t;x) \|_{PC} \leq M_2 p \frac{1+|c-1|}{|c-1|} + M_3 p \frac{[1+|c-1|]^2 \omega^\alpha}{\alpha(c-1)^2 \Gamma(\alpha)} +$$

$$+ M_1 \frac{\alpha |c-1|^2 \, \Gamma^2(\alpha) + \alpha |c-1| \, \Gamma^2(\alpha) \, + 2\, \Gamma(2\alpha)}{2\alpha^2 |c-1|^2 \, \Gamma^2(\alpha) \, \Gamma(2\alpha)} \omega^{2\alpha}, \tag{24}$$

where $M_1 = \| f \|, \ \ M_2 = \max\limits_{1 \leq i \leq p} \| F_i \|, \ \ M_3 = \max\limits_{1 \leq i \leq p} \| G_i \|.$

Proof. From the equation (23) we obtain

$$\| J(t;x) \|_{PC} \leq \frac{1+|c-1|}{|c-1|} \sum_{0<t_i<\omega} \|F_i\| +$$

$$+ \left[\max_{t \in \Omega} | \chi(t) | + \int\limits_0^\omega K_1(t,s) ds \right] \sum_{0<t_i<\omega} \|G_i\| + \| f \| \int\limits_0^\omega K_2(t,s) ds \leq$$

$$\leq p \frac{1+|c-1|}{|c-1|} \max_{1 \leq i \leq p} \| F_i \| + p \frac{[1+|c-1|]^2 \omega^\alpha}{\alpha(c-1)^2 \Gamma(\alpha)} \max_{1 \leq i \leq p} \| G_i \| +$$

$$+ \frac{\alpha |c-1|^2 \, \Gamma^2(\alpha) + \alpha |c-1| \, \Gamma^2(\alpha) \, + 2\, \Gamma(2\alpha)}{2\alpha^2 |c-1|^2 \, \Gamma^2(\alpha) \, \Gamma(2\alpha)} \omega^{2\alpha} \, \| f \|. \tag{25}$$

From the estimate (25) follows (24). Lemma 2.3 is proved. □

Lemma 2.4. *Let the following conditions to be met: there exist constants* P_i, Q_i, $i = 1, 2, 3$, *such that*

1). $\| f\,(t, x(t), y(t)) \| \leq P_1 \, \| x(t) \| + Q_1 < \infty$;

2). $\max\limits_{1 \leq i \leq p} \| F_i\,(x(t_i)) \| \leq P_2 \max\limits_{1 \leq i \leq p} \| x(t_i) \| + Q_2 < \infty$;

3). $\max\limits_{1 \leq i \leq p} \| G_i\,(x(t_i)) \| \leq P_3 \max\limits_{1 \leq i \leq p} \| x(t_i) \| + Q_3 < \infty$.

Then the following estimate is true

$$\| x(t) \|_{PC} \leq \frac{\mu}{1-\nu}, \tag{26}$$

where

$$\mu = p \left[Q_2 \frac{1+|c-1|}{|c-1|} + Q_3 \frac{[1+|c-1|]^2 \omega^\alpha}{\alpha(c-1)^2 \Gamma(\alpha)} \right] +$$

$$+ Q_1 \frac{\alpha |c-1|^2 \, \Gamma^2(\alpha) + \alpha |c-1| \, \Gamma^2(\alpha) \, + 2\, \Gamma(2\alpha)}{2\alpha^2 |c-1|^2 \, \Gamma^2(\alpha) \, \Gamma(2\alpha)} \omega^{2\alpha},$$

$$\nu = p \left[P_2 \frac{1+|c-1|}{|c-1|} + P_3 \frac{[1+|c-1|]^2 \omega^\alpha}{\alpha(c-1)^2 \Gamma(\alpha)} \right] +$$

$$+ P_1 \frac{\alpha |c-1|^2 \, \Gamma^2(\alpha) + \alpha |c-1| \, \Gamma^2(\alpha) \, + 2\, \Gamma(2\alpha)}{2\alpha^2 |c-1|^2 \, \Gamma^2(\alpha) \, \Gamma(2\alpha)} \omega^{2\alpha} < 1.$$

Proof. Similar to the proof of the Lemma 2.3 above, we obtain

$$\| J(t;x) \|_{PC} \leq \frac{1+|c-1|}{|c-1|} \sum_{0<t_i<\omega} \|F_i\| +$$

$$+ \left[\max_{t\in\Omega} |\chi(t)| + \int_0^\omega K_1(t,s)ds \right] \sum_{0<t_i<\omega} \|G_i\| + \| f \| \int_0^\omega K_2(t,s)ds \leq$$

$$\leq p\frac{1+|c-1|}{|c-1|} \left[P_2 \max_{1\leq i\leq p} \| x(t_i) \| + Q_2 \right] + p\frac{[1+|c-1|]^2\omega^\alpha}{\alpha(c-1)^2\Gamma(\alpha)} \left[P_3 \max_{1\leq i\leq p} \| x(t_i) \| + Q_3 \right] +$$

$$+ \frac{\alpha|c-1|^2\,\Gamma^2(\alpha) + \alpha|c-1|\,\Gamma^2(\alpha) + 2\,\Gamma(2\alpha)}{2\alpha^2|c-1|^2\,\Gamma^2(\alpha)\,\Gamma(2\alpha)} \omega^{2\alpha} \left[P_1 \| x(t) \| + Q_1 \right] \leq$$

$$\leq p \left[P_2 \frac{1+|c-1|}{|c-1|} + P_3 \frac{[1+|c-1|]^2\omega^\alpha}{\alpha(c-1)^2\Gamma(\alpha)} \right] \max_{1\leq i\leq p} \| x(t_i) \| +$$

$$+ P_1 \frac{\alpha|c-1|^2\,\Gamma^2(\alpha) + \alpha|c-1|\,\Gamma^2(\alpha) + 2\,\Gamma(2\alpha)}{2\alpha^2|c-1|^2\,\Gamma^2(\alpha)\,\Gamma(2\alpha)} \omega^{2\alpha} \| x(t) \| +$$

$$+ p\,Q_2 \frac{1+|c-1|}{|c-1|} + p\,Q_3 \frac{[1+|c-1|]^2\omega^\alpha}{\alpha(c-1)^2\Gamma(\alpha)} +$$

$$+ Q_1 \frac{\alpha|c-1|^2\,\Gamma^2(\alpha) + \alpha|c-1|\,\Gamma^2(\alpha) + 2\,\Gamma(2\alpha)}{2\alpha^2|c-1|^2\,\Gamma^2(\alpha)\,\Gamma(2\alpha)} \omega^{2\alpha}. \tag{27}$$

From (27) we derive the estimate (26). The Lemma 2.4 is proved. □

Lemma 2.5 ([33]). *For the difference of two functions with maxima there holds the following estimate*

$$\| \max \{ x(\tau) \,|\, \tau \in [t-h,t] \} - \max \{ y(\tau) \,|\, \tau \in [t-h,t] \} \| \leq$$

$$\leq \| x(t) - y(t) \| + 2h \left\| \frac{\partial}{\partial t} [x(t) - y(t)] \right\|,$$

where $0 < h = \mathrm{const}$.

3. Main result

Theorem 3.1. *Let* $f : \Omega \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ *be a continuous function and* $f(t+\omega, cx, cy) = c\,f(t,x,y)$*. Assume that there exist positive quantities* M_1*,* M_2*,* M_3*,* L_1*,* L_{2i}*,* L_{3i} *such that for all* $t \in \Omega$ *are fulfilled the following conditions:*

1. For the positive integer p, there hold $F_i = F_{i+p}$*,* $G_i = G_{i+p}$*,* $t_{i+p} = t_i + \omega$*;*

2. $\| f(t, x(t), y(t)) \| \leq M_1 < \infty$*;*

3. $\max_{1\leq i\leq p} \| F_i(x(t_i)) \| \leq M_2 < \infty$*,* $\max_{1\leq i\leq p} \| G_i(x(t_i)) \| \leq M_3 < \infty$*;*

4. $\| f(t, x_1, y_1) - f(t, x_2, y_2) \| \leq L_1 \left[\| x_1 - x_2 \| + \| y_1 - y_2 \| \right]$*;*

5. $\| F_i(t,x_1) - F_i(t,x_2) \| \le L_{2i} \| x_1 - x_2 \|$;
6. $\| G_i(t,x_1) - G_i(t,x_2) \| \le L_{3i} \| x_1 - x_2 \|$;
7. The radius of the inscribed ball in X is greater than

$$M_2 p \frac{1+|c-1|}{|c-1|} + M_3 p \frac{[1+|c-1|]^2 \omega^\alpha}{\alpha (c-1)^2 \Gamma(\alpha)} +$$

$$+ M_1 \frac{\alpha |c-1|^2 \Gamma^2(\alpha) + \alpha |c-1| \Gamma^2(\alpha) + 2\Gamma(2\alpha)}{2\alpha^2 |c-1|^2 \Gamma^2(\alpha) \Gamma(2\alpha)} \omega^{2\alpha};$$

8. $\rho < 1$, *where* $\rho = \beta_1 + h\beta_2$ *and* β_1, β_2 *are defined from* (31) *and* (35) *below.*
Then the problem (1)–(5) *has a unique* $(\omega, c)-$*periodic solution for all* $t \in \Omega$.

Proof. The theorem we proof by the method of contacting mapping. According to the theorem condition, we have

$$f(t+\omega, x(t+\omega), y(t+\omega)) = f(t+\omega, \, c\,x(t), \, c\,y(t)) = c\,f(t, x(t), y(t)).$$

We differentiate (23):

$$x'(t) = J(t; x') \equiv \chi'(t) \sum_{0<t_i<\omega} G_i(x(t_i)) + \int_0^\omega K_1'(t,s) \sum_{0<t_i<s} G_i(x(t_i))\, ds +$$

$$+ \int_0^\omega K_2'(t,s) f(s, x(s), \max\{x(\tau) \,|\tau \in [s-h, s]\})\, ds, \tag{28}$$

where

$$\chi'(t) = \frac{t^{\alpha-1}}{(c-1)\Gamma(\alpha)}, \quad K_1'(t,s) = \begin{cases} 0, & t < s \le \omega, \\ \frac{(\alpha-1)(t-s)^{\alpha-2}}{\Gamma(\alpha)}, & 0 \le s < t, \end{cases}$$

$$K_2'(t,s) = \begin{cases} \frac{t^{\alpha-1}}{(c-1)\Gamma^2(\alpha)} (\omega - s)^{\alpha-1}, & t < s \le \omega, \\ \frac{(2\alpha-1)(t-s)^{2\alpha-2}}{\Gamma(2\alpha)} + \frac{t^{\alpha-1}}{(c-1)\Gamma^2(\alpha)} (\omega-s)^{\alpha-1}, & 0 \le s < t. \end{cases}$$

For the difference of two operators in (23), we have estimate

$$\|J(t;x) - J(t;y)\| \le \frac{1+|c-1|}{|c-1|} \sum_{i=1}^p \| F_i(x(t_i)) - F_i(y(t_i)) \| +$$

$$+ \frac{[1+|c-1|]^2 \omega^\alpha}{\alpha(c-1)^2 \Gamma(\alpha)} \sum_{i=1}^p \| G_i(x(t_i)) - G_i(y(t_i)) \| +$$

$$+ \frac{\alpha|c-1|^2 \Gamma^2(\alpha) + \alpha|c-1| \Gamma^2(\alpha) + 2\Gamma(2\alpha)}{2\alpha^2 |c-1|^2 \Gamma^2(\alpha) \Gamma(2\alpha)} \omega^{2\alpha} \times$$

$$\times \| f(t, x(t), \max\{x(\tau) \,|\tau \in [t-h, t]\}) - f(t, y(t), \max\{y(\tau) \,|\tau \in [t-h,t]\}) \|.$$

Hence, using conditions of the theorem, we have

$$\|J(t;x)-J(t;y)\|\leq\frac{1+|c-1|}{|c-1|}\sum_{i=1}^{p}L_{2i}\,\|\,x\,(t_i)-y\,(t_i)\,\|+$$

$$+\frac{[1+|c-1|]^2\omega^{\alpha}}{\alpha(c-1)^2\Gamma(\alpha)}\sum_{i=1}^{p}L_{3i}\,\|\,x\,(t_i)-y\,(t_i)\,\|+$$

$$+L_1\frac{\alpha|c-1|^2\,\Gamma^2(\alpha)+\alpha|c-1|\,\Gamma^2(\alpha)\,+2\,\Gamma(2\alpha)}{2\alpha^2|c-1|^2\,\Gamma^2(\alpha)\,\Gamma(2\alpha)}\omega^{2\alpha}\times$$

$$\times\big[\,\|\,x(t)-y(t)\,\|+\|\max\,\{x(\tau)\,|\tau\in[t-h,t]\}-\max\,\{y(\tau)\,|\tau\in[t-h,t]\}\,\|\,\big]. \qquad (29)$$

By virtue of estimate given in the Lemma 2.5, from (29) we obtain that

$$\|J(t;x)-J(t;y)\|\leq\frac{1+|c-1|}{|c-1|}\sum_{i=1}^{p}L_{2i}\,\|\,x\,(t_i)-y\,(t_i)\,\|+$$

$$+\frac{[1+|c-1|]^2\omega^{\alpha}}{\alpha(c-1)^2\Gamma(\alpha)}\sum_{i=1}^{p}L_{3i}\,\|\,x\,(t_i)-y\,(t_i)\,\|+$$

$$+2L_1\frac{\alpha|c-1|^2\,\Gamma^2(\alpha)+\alpha|c-1|\,\Gamma^2(\alpha)\,+2\,\Gamma(2\alpha)}{2\alpha^2|c-1|^2\,\Gamma^2(\alpha)\,\Gamma(2\alpha)}\omega^{2\alpha}\times$$

$$\times\big[\,\|\,x(t)-y(t)\,\|+h\,\left\|\,x'(t)-y'(t)\,\right\|\,\big]\leq\beta_1\,\|\,x(t)-y(t)\,\|+\gamma_1 h\,\left\|\,x'(t)-y'(t)\,\right\|, \qquad (30)$$

where

$$\beta_1=\frac{1+|c-1|}{|c-1|}\sum_{i=1}^{p}L_{2i}+\frac{[1+|c-1|]^2\omega^{\alpha}}{\alpha(c-1)^2\Gamma(\alpha)}\sum_{i=1}^{p}L_{3i}+$$

$$+L_1\frac{\alpha|c-1|^2\,\Gamma^2(\alpha)+\alpha|c-1|\,\Gamma^2(\alpha)\,+2\,\Gamma(2\alpha)}{\alpha^2|c-1|^2\,\Gamma^2(\alpha)\,\Gamma(2\alpha)}\omega^{2\alpha}, \qquad (31)$$

$$\gamma_1=L_1\frac{\alpha|c-1|^2\,\Gamma^2(\alpha)+\alpha|c-1|\,\Gamma^2(\alpha)\,+2\,\Gamma(2\alpha)}{\alpha^2|c-1|^2\,\Gamma^2(\alpha)\,\Gamma(2\alpha)}\omega^{2\alpha}. \qquad (32)$$

From (31) and (32) clear that $\beta_1>\gamma_1$. So, the estimate (30) one can rewrite as

$$\|J(t;x)-J(t;y)\|_{PC}\leq\beta_1\big[\,\|\,x(t)-y(t)\,\|_{PC}+h\,\left\|\,x'(t)-y'(t)\,\right\|_{PC}\big]. \qquad (33)$$

Now for the difference of two operators in (28), similarly, we have estimate

$$\left\|J(t;x')-J(t;y')\right\|\leq\frac{\omega^{\alpha-1}}{|c-1|\Gamma(\alpha)}\sum_{i=1}^{p}L_{3i}\,\|\,x\,(t_i)-y\,(t_i)\,\|+$$

$$+2L_1\frac{\alpha|c-1|\,\Gamma^2(\alpha)\,+\,\Gamma(2\alpha)}{\alpha|c-1|\,\Gamma^2(\alpha)\,\Gamma(2\alpha)}\omega^{2\alpha-1}\times$$

$$\times \big[\, \| \, x(t) - y(t) \, \| + h \, \left\| \, x'(t) - y'(t) \, \right\| \, \big] \le \beta_2 \, \| \, x(t) - y(t) \, \| + \gamma_2 h \, \left\| \, x'(t) - y'(t) \, \right\|, \quad (34)$$

where

$$\beta_2 = \frac{\omega^{\alpha-1}}{|c-1|\Gamma(\alpha)} \sum_{i=1}^{p} L_{3i} + 2L_1 \frac{\alpha|c-1|\,\Gamma^2(\alpha) + \Gamma(2\alpha)}{\alpha|c-1|\,\Gamma^2(\alpha)\,\Gamma(2\alpha)} \omega^{2\alpha-1}, \quad (35)$$

$$\gamma_2 = 2L_1 \frac{\alpha|c-1|\,\Gamma^2(\alpha) + \Gamma(2\alpha)}{\alpha|c-1|\,\Gamma^2(\alpha)\,\Gamma(2\alpha)} \omega^{2\alpha-1}. \quad (36)$$

From (35) and (36) clear that $\beta_2 > \gamma_2$. So, the estimate (34) one can rewrite as

$$\left\| J(t;x') - J(t;y') \right\|_{PC} \le \beta_2 \big[\, \| \, x(t) - y(t) \, \|_{PC} + h \, \left\| \, x'(t) - y'(t) \, \right\|_{PC} \big]. \quad (37)$$

We multiply both sides of (37) to h term by term. Then, adding the estimates (33) and (37) term by term, we obtain that

$$\| J(t;x) - J(t;y) \|_{BD} \le \rho \cdot \| \, x(t) - y(t) \, \|_{BD} \,, \quad (38)$$

where $\rho = \beta_1 + h\beta_2$.

According to the last condition of the theorem $\rho < 1$, so right-hand side of (23) as an operator is contraction mapping. From the estimates (24), (26) and (38) implies that there exists a unique fixed point $x(t)$, satisfying equation (1), $(\omega, c)-$periodic conditions (2), (3) and impulsive conditions (4), (5). The theorem is proved. $\square$

4. Conclusion

The theory of differential equations plays an important role in solving applied problems of sciences and technology. Especially, periodic and almost periodic boundary value problems for differential equations with impulsive actions have many applications in mathematical physics, mechanics and technology, in particular in nanotechnology.

In this paper, we investigated the questions of $(\omega, c)-$periodic solvability of the system of impulsive differential equations (1) with the quadrate of Gerasimov–Caputo operator, $(\omega, c)-$periodic boundary value conditions (2), (3) and impulsive conditions (4), (5) for $t = t_i$, $i = 1, 2, \ldots, p$, $0 < t_1 < t_2 < \cdots < t_p < T$. The nonlinear right-hand side of this equation consists the construction of maxima. The questions of existence and uniqueness of the $(\omega, c)-$periodic solution of the problem (1)–(5) are studied. The problem we reduce to the $(\omega, c)-$periodic solvability of the system of nonlinear functional integral equations (23). The estimates (24) and (26) are obtained for $(\omega, c)-$periodic solutions of the problem (1)–(5).

The results obtained in this work will allow us in the future to investigate another kind of $(\omega, c)-$periodic problems for the equations of mathematical physics with impulsive actions. We hope that our work will stimulate the study of various kind of $(\omega, c)-$periodic direct and inverse boundary value problems for impulsive ordinary and partial differential and integro-differential equations and results of investigations find the applications in mechanics, technology and in nanotechnology.

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