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Existence and Uniqueness Results for the First-Order Nonlinear Implicit Differential Equations with Three-Point Boundary Conditions

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Abstract. This study focuses on investigating the existence and uniqueness of solutions for a system of nonlinear first-order implicit ordinary differential equations with three-point boundary conditions. Initially, the considered problem is reduced to an equivalent integral equation using Green's function. Subsequently, the Banach contraction mapping theorem is employed to establish the main result for the given problem. Finally, a numerical example is presented to demonstrate our main results.

Key Words and Phrases: nonlocal boundary conditions, existence, uniqueness, fixed point, implicit nonlinear differential equations.

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1. Introduction and Problem Statement

Differential equations are prevalent in real-life situations, particularly in modeling complex systems where relationships are not necessarily linear. Some examples of their role include chemical reactions, biological systems, economics, engineering systems, physics, and neural networks [1, 2].

There has been significant interest in researching boundary value problems for nonlinear systems of ordinary differential equations with boundary conditions [3-29], as well as nonlinear implicit differential equations, which have been intensively discussed by several researchers and in the literature [30, 31, 32, 33]. However, as far as the author is aware, almost nothing is known about the existence and uniqueness of the solutions for first-order nonlinear implicit differential equations with three-point boundary conditions. S. Heikkilä considered this problem using discontinuous boundary conditions [30]. In that work, the author established discontinuous implicit differential equations. Viorel Barbu and Abgelo Favini investigated the existence of an implicit nonlinear differential equation for the Cauchy problem [31]. Inspired and motivated by the above works, we study the existence and uniqueness of solutions for first-order nonlinear implicit differential equations with nonlocal conditions. The Banach contraction mapping principle is used to prove the existence and uniqueness of theorem. Additionally, a crucial aspect of this line of study involves finding the Green function in a practical manner.

In this research, we set out to establish the existence and uniqueness of the system of nonlinear implicit differential equations of the type

$$\dot{x}(t) = f\left(t, x(t), \dot{x}(t)\right) \text{ for } t \in [0, T], \tag{1}$$

subject to three-point boundary conditions

$$Ax(0) + Bx(t_1) + Cx(T) = d, \tag{2}$$

where A, B, C are constant square matrices of order n such that $\det N \neq 0$, $N = A + B + C$; $f : [0, T] \times R^n \times R^n \to R^n$ is a given function, $d \in R^n$is a given vector, and t_1 satisfies the condition $0 < t_1 < T$.

We denote by $C\left([0, T]; R^n\right)$ the Banach space of all continuous functions $x(t)$from $[0, T]$ to R^n with the norm

$$\|x\| = \max\left\{|x(t)| : t \in [0, T]\right\}$$

where $|\cdot|$ is the norm in the space R^n.

This paper is structured as follows. Section 2 addresses the definitions and lemmas, which are the key tools for our main results. Section 3 describes the theorem on the existence and uniqueness of the solution of problem (1)-(2) established under sufficient conditions on the nonlinear terms. In Section 4, the given example is verified to show the legitimacy and applicability of the proposed technique.

2. Preliminaries

We define the solution of problem (1)-(2) as follows:

Definition 2.1. A function $x \in C\left([0, T],\ R^n\right)$ is said to be a solution to problem (1)-(2) if $\dot{x}(t) = f\left(t, x(t), \dot{x}(t)\right)$ for each $t \in [0, T]$, and boundary conditions (2) are satisfied.

For simplification, we can consider the following problem:

$$\dot{x} = y(t), \quad t \in [0, T], \tag{3}$$

$$Ax(0) + Bx(t_1) + Cx(T) = d. \tag{4}$$

Lemma 2.1. Let $y \in C\left([0, T] \times R^n \times R^n, R^n\right)$. Then, the unique solution $x \in C\left([0, T], R^n\right)$ of the boundary value problem for differential equation (3) with boundary conditions (4) is given by

$$x(t) = D + \int_0^T G(t, \tau) y(\tau) d\tau, \tag{5}$$

for $t \in [0, T],$ where

$$G(t, \tau) = \begin{cases} G_1(t, \tau), & 0 \le t \le t_1 \\ G_2(t, \tau), & t_1 < t \le T \end{cases},$$

$$D = N^{-1}d,$$

with

$$G_1(t,\tau) = \begin{cases} N^{-1}A, & 0 \le \tau \le t, \\ -N^{-1}(B+C)\ , & t < \tau \le t_1, \\ -N^{-1}C, & t_1 < \tau \le T, \end{cases}$$

and

$$G_2(t,\tau) = \begin{cases} N^{-1}A, & 0 \le \tau \le t_1, \\ N^{-1}(A+B)\ , & t_1 < \tau \le t, \\ -N^{-1}C\,, & t < \tau \le T. \end{cases}$$

Proof. Assuming that $x(t)$ is a solution of boundary-value problem (3)-(4), then for $t \in [0,T]$

$$x(t) = x(0) + \int_0^t y(\tau)d\tau. \tag{6}$$

When formula (6) satisfies condition (4), we obtain

$$(A+B+C)\,x(0) = d - B\int_0^{t_1} y(t)dt - C\int_0^T y(t)dt. \tag{7}$$

Let us denote $N = A+B+C$, and from equality (7), we determine $x(0)$ as follows:

$$x\,(0) = N^{-1}d - N^{-1}B\int_0^{t_1} y(t)dt - N^{-1}C\int_0^T y(t)dt. \tag{8}$$

By substituting the value $x(0)$ determined from equality (8) into (6), we obtain

$$x(t) = N^{-1}d - N^{-1}B\int_0^{t_1} y(t)dt - N^{-1}C\int_0^T y(t)dt + \int_0^t y(\tau)d\tau. \tag{9}$$

Now, consider that $t \in [0,t_1]$. Then, we can rewrite equality (9) as follows:

$$x(t) = N^{-1}d - N^{-1}B\int_0^t y(\tau)d\tau - N^{-1}B\int_t^{t_1} y(\tau)d\tau - N^{-1}C\int_0^t y(\tau)d\tau$$

$$-N^{-1}C\int_t^{t_1} y(\tau)d\tau - N^{-1}C\int_{t_1}^T y(t)dt + \int_0^t y(\tau)d\tau.$$

In the above formula, by grouping similar terms and then simplifying, we obtain

$$x(t) = N^{-1}d + \int_0^t \left(E - N^{-1}B - N^{-1}C\right) y(\tau)d\tau$$

$$+\int_t^{t_1} \left(-N^{-1}B - N^{-1}C\right) y(\tau)d\tau + \int_{t_1}^T \left(-N^{-1}C\right) y(t)dt$$

$$= N^{-1}d + \int_0^t N^{-1}A\, y(\tau)d\tau - \int_t^{t_1} N^{-1}\,(B+C)\, y(\tau)d\tau$$

$$- \int_{t_1}^T N^{-1}Cy(t)dt. \tag{10}$$

Let us define the new function as follows:

$$G_1(t,\tau) = \begin{cases} N^{-1}A, & 0 \le \tau \le t, \\ -N^{-1}(B+C), & t < \tau \le t_1, \\ -N^{-1}C, & t_1 < \tau \le T. \end{cases}$$

Using the above equality as in (10), we obtain the following result:

$$x(t) = D + \int_0^T G_1(t,\tau)y(\tau)d\tau.$$

For this case, $t \in (t_1, T]$ we can write equality (9) as follows:

$$x(t) = N^{-1}d - N^{-1}B\int_0^{t_1} y(t)dt - N^{-1}C\int_0^{t_1} y(t)dt-$$

$$-N^{-1}C\left(\int_{t_1}^t y(\tau)d\tau + \int_t^T y(\tau)d\tau\right) + \int_0^{t_1} y(t)dt + \int_{t_1}^t y(\tau)d\tau$$

$$= N^{-1}d + \left(E - N^{-1}B - N^{-1}C\right)\int_0^{t_1} y(t)dt + \left(E - N^{-1}C\right)\int_{t_1}^t y(\tau)d\tau-$$

$$-N^{-1}C\int_t^T y(\tau)d\tau = N^{-1}d + N^{-1}A\int_0^{t_1} y(t)dt$$

$$N^{-1}(A+B)\int_{t_1}^t y(\tau)d\tau - N^{-1}C\int_t^T y(\tau)d\tau.$$

Hence, we introduce the new function

$$G_2(t,\tau) = \begin{cases} N^{-1}A, & 0 \le \tau \le t_1, \\ N^{-1}(A+B), & t_1 < \tau \le t, \\ -N^{-1}C, & t < \tau \le T. \end{cases}$$

Thus, for each $t \in (t_1, T]$, we have

$$x(t) = D + \int_0^T G_2(t,\tau)y(\tau)d\tau.$$

We deduce that the solution of boundary-value problem (3)-(4) is in the form

$$x(t) = D + \int_0^T G(t,\tau)y(\tau)d\tau. \tag{11}$$

Consequently, we have successfully completed the proof.

Lemma 2.2. Assume that $f \in C[0,T] \times R^n \times R^n$. Then, the function $x(t)$ is a solution of the boundary-value problem (1)-(2) if and only if $x(t)$is a solution of the integral equation

$$x(t) = D + \int_0^T G(t,\tau) f(\tau, x(\tau), \dot{x}(\tau)) d\tau. \tag{12}$$

Proof. Let $x(t)$ be a solution of the boundary-value problem (1)-(2). Then, in the same way as in Lemma 2.1, we can prove that it is also a solution of the integral equation (12). Obviously, the solution of the integral equation (12) satisfies the boundary-value problem (1)-(2).

This completes the proof of Lemma 2.2.

3. Main results

Let P be an operator such that $P : C([0,T]R^n) \to C([0,T]R^n)$ as

$$(Px)(t) = D + \int_0^T G(t,\tau) f(\tau, x(\tau), \dot{x}(\tau))\, d\tau.$$

Problem (1)-(2) equivalent to fixed-point problem. Hence, problem (1)-(2) has a solution if and only if the operator P has a fixed point.

We now present the existence and uniqueness results for nonlinear problem (1)-(2) by applying the Banach fixed-point theorem.

Theorem. We assume that the following assumptions hold:

(H1) There exist constants $M_1 > 0$, $M_2 \in (0,\, 1)$ such that

$$|f(t, x_1, x_2) - f(t, y_1, y_2)| \le M_1 |x_1 - y_1| + M_2 |x_2 - y_2|$$

for each $t \in [0,T]$ and all $x_1, x_2, y_1, y_2 \in R^n$.

(H2)

$$L = \frac{STM_1}{1 - M_2} < 1, \tag{13}$$

where

$$S = \max_{[0,T]\times[0,T]} \|G(t,\tau)\|.$$

Then, boundary-value problem (1)-(2) has a unique solution on$[0,T]$.

Proof. We denote $\max_{[0,T]} |f(t,0,0)| = M_f$ and choose $r \ge \frac{\|D\| + M_f TS\cdot \frac{2-M_2}{1-M_2}}{1-L}$. We show that $PB_r \subset B_r$, where

$$B_r = \{x \in C([0,T]; R^n) : \|x\| \le r\}.$$

It is clear that

$$\left|f\left(t, x(t), x'(t)\right)\right| = \left|f\left(t, x(t), x'(t)\right) - f(t,\, 0,\, 0) + f(t,\, 0,\, 0)\right| \le$$

$$\leq \left|f\left(t,x(t),x'(t)\right)-f(t,\,0,\,0)\right|+|f(t,\,0,\,0)|\leq$$
$$\leq M_1\,|x(t)|+M_2\,\left|x'(t)\right|+M_f=M_1\,|x(t)|+M_2\left|f\left(t,x(t),x'(t)\right)\right|+M_f.$$

From here, we obtain

$$(1-M_2)\left|f\left(t,x(t),x'(t)\right)\right|\leq M_1\,|x(t)|+M_f.$$

Therefore,

$$\left|f\left(t,x(t),x'(t)\right)\right|\leq \frac{M_1}{1-M_2}\,|x(t)|+\frac{M_f}{1-M_2}. \tag{14}$$

However, for $x\in B_r$, we have

$$|Px(t)|\leq |D|+\int_0^T |G(t,\tau)|\left(\left|f\left(\tau,x(\tau),x'(\tau)\right)-f(\tau,0,0)\right|+|f(\tau,0,0)|\right)d\tau$$

$$\leq |D|+S\int_0^T\left(M_1\,|x(t)|+M_2\left|x'(t)\right|+M_f\right)dt.$$

We can rewrite the above inequality by using (14) as (15):

$$|Px(t)|\leq |D|+S\int_0^T\left(M_1\,|x(t)|+M_2\left(\frac{M_1}{1-M_2}\,|x(t)|+\frac{M_f}{1-M_2}\right)+M_f\right)dt=$$

$$=|D|+SM_fT+\frac{TSM_f}{1-M_2}+S\int_0^T\left(M_1+\frac{M_1M_2}{1-M_2}\right)|x(t)|\,dt\leq$$

$$\leq \|D\|+SM_fT\left(1+\frac{1}{1-M_2}\right)+ST\left(M_1+\frac{M_1M_2}{1-M_2}\right)\|x\|\leq$$

$$\leq \|D\|+SM_fT\cdot\frac{2-M_2}{1-M_2}+ST\frac{M_1}{1-M_2}r\leq r. \tag{15}$$

Thus, we obtain $P:B_r\to B_r$.
Since

$$|\dot{x}(t)-\dot{y}(t)|=\left|f\left(t,x(t),x'(t)\right)-f\left(t,y(t),\dot{y}(t)\right)\right|\leq$$
$$\leq M_1\,|x(t)-y(t)|+M_2\,|\dot{x}(t)-\dot{y}(t)|\,.$$

We obtain

$$|\dot{x}(t)-\dot{y}(t)|\leq\frac{M_1}{1-M_2}\,|x(t)-y(t)|\,. \tag{16}$$

For any $x,y\in B_r$, it is true that

$$|Px-Py|\leq\int_0^T\left|G(t,s)(f(s,x(s),x'(s))-f(s,y(s),y'(s))\right|ds\leq$$

$$\leq S\int_0^T\left(M_1\,|x(t)-y(t)|+M_2\left|x'(t)-y'(t)\right|\right)dt. \tag{17}$$

When we substitute (16) in (17), we have a true statement such that

$$|Px - Py| \leq S \int_0^T \left(M_1 \left| x(t) - y(t) \right| + \frac{M_1 M_2}{1 - M_2} \left| x(t) - y(t) \right| \right) dt \leq$$

$$\leq TS \left(M_1 \left\| x - y \right\| + \frac{M_1 M_2}{1 - M_2} \left\| x - y \right\| \right) =$$

$$= TS \frac{M_1}{1 - M_2} \left\| x - y \right\|$$

or

$$\left\| Px - Py \right\| \leq L \left\| x - y \right\|. \tag{18}$$

It is clearly shown that inequality (18) holds under condition (13). Consequently, boundary-value problem (1)-(2) has a unique solution.

4. Example

In this section, we provide an example to illustrate the main results obtained in this paper.

Example. Let us consider the system of implicit differential equations given by the three-point boundary conditions as follows:

$$\begin{cases} \dot{x}_1 = \alpha x_1 + 0.1 \sin x_2 \\ \dot{x}_2 = \beta \sin x_1 \end{cases}, \quad t \in [0,\, 2], \tag{19}$$

$$x_1(0) + \frac{1}{2} x_2(1) - \frac{1}{2} x_2(2) = 1. \tag{20}$$

We can rewrite problem (19)-(20) in the equivalent form:

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(1) \\ x_2(1) \end{pmatrix} + \begin{pmatrix} 0 & -\frac{1}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(2) \\ x_2(2) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Obviously,

$$N = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\frac{1}{2} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

matrix N is invertible, and $N^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Condition (H1) holds with $G_{\max} \leq 1.5$ and $M_2 = 0.1$, and condition (13) is satisfied. Hence,

$$L = \frac{G_{\max} T M_1}{1 - M_2} = \frac{1.5 \cdot 2 \cdot \max(\alpha,\, \beta)}{1 - 0.1} < 1 \tag{21}$$

Inequality (21) implies that $\max(\alpha,\, \beta) < 0.3$; by Theorem, we can guarantee the existence of the unique solution of the boundary-value problem (19)-(20) on [0,2].

Conclusion. The boundary conditions considered in this paper are sufficiently general and can be used extensively for a wide class of problems. In this work, the existence and uniqueness of the solutions for first order nonlinear implicit differential equations with three-point boundary conditions are generated under sufficient conditions. Note that, given here, methods can be used in similar multipoint problems for ordinary differential equations as follows:

$\dot{x}(t) = f(t, x(t), x'(t))$ for $t \in [0, T]$,

$$\sum_{j=0}^{m} L_j x(t_j) = \alpha.$$

Here, $0 = t_0 < t_1 ... < t_{m-1} < t_m = T$; $L_j \in R^{n \times n}$ are given matrices; $\alpha \in R^n$ is a given vector,

$$\det N \neq 0, \quad N = \sum_{j=0}^{m} L_j.$$

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