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The Study of Rainbow Dynamic Coloring in Corona Product Graphs

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Abstract. Let *G* be a connected graph that is simple, non-trivial, and finite. Its vertices have the coloring *N*. Rainbow dynamic coloring is a dynamic coloring of a graph that requires a minimum number of colors such that each pair of vertices is connected by at least one path whose internal vertices have different colors. In this work, we determine the rainbow dynamic coloring of corona product graphs such as $K_{2,3}$ with a path graph, $K_{2,3}$ with a complete graph, a path with $K_{2,3}$ graph, and $K_{2,3}$ with wheel graph.

Key Words and Phrases: Rainbow dynamic coloring, dynamic coloring, rainbow vertex coloring, corona product

2010 Mathematics Subject Classifications: 05C15, 05C69, 05C76.

1. Introduction

Graph theory in mathematics is the study of graphs, which are mathematical structures that represent pairwise connections between objects. A graph G consists of a vertex set V(G) and an edge set E(G). The name "graph coloring" comes from the mapcoloring system. Vertices and edges are given labels. A particular instance of graph labeling in graph theory is graph coloring, which is the process of assigning labels, traditionally referred to as "colors" to graph elements. Within a graph, no two neighboring vertices, neighboring edges, or neighboring regions have the same color scheme. In addition to its theoretical issues, graph coloring has several practical applications in clustering, data mining, image capturing, networking, image segmentation, resource allocation, process scheduling, etc. In 2001 [1] Bruce Montgomery presented dynamic coloring of the set of vertex such that each vertex of degree at least two of its neighbors receives at least two different colors. In 2010, [2] Krivelevich and Yuster presented the

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idea of rainbow vertex coloring. A rainbow vertex connection number, rvc(G) of a connected graph, is the minimum number of colors required to color its vertices. Every pair of vertices is connected by at least one path whose internal vertices have distinct colors. A rainbow dynamic coloring of a graph is not just a theoretical concept but a practical one. It is a dynamic coloring, and a minimum number of colors is required such that every pair of vertices is connected by at least one path whose internal vertices have different colors. The minimum k for which k-vertex coloring exists is called the rainbow dynamic coloring of G, denoted by rdyc(G) [6], [7], [8], [11], [12].

1.1. Definition

Consider two graphs, A and B. One copy of A and |V(A)| copies of B are taken, and each vertex of a copy of B is joined to a corresponding vertex of A, to give the corona product of A and B, represented as $A \circ B$. [3], [9]. Note: a = 1, 2, 3, ...n where computation is performed modulo 'a'

2. Results

Proposition 1. $rdyc(K_2 \circ K_{2,3}) = 3$

Theorem 1. Let G be equal to $K_n \circ K_{2,3}$. Then, rdyc(G) = n for $n \ge 3$

Proof. Let $V(K_n) = \{v_a : 1 \le a \le n\}$ and let $(K_{2,3})_a$ be the vertex set of 'a' copies of $(K_{2,3})$ i.e $V\{(K_{2,3})_a\} = \{u_{ab} : 1 \le a \le n, 1 \le b \le 5\}$. Every vertex of K_n is adjacent to every vertex of a copy of $K_{2,3}$ according to the definition of the corona product, that is, for $1 \le a \le n$, the vertex v_a of $V(K_n)$ is adjacent to the vertices of the set $\{u_{ab} : 1 \le b \le 5\}$ in the a^{th} copy of $K_{2,3}$. Let $Q(K_n \circ K_{2,3})$ be $\{Q_1 \cup Q_2 \cup Q_3\}$ where Q_1 be the edge set of K_n , Q_2 be the edge set of $K_{2,3}$ and $Q_3 = \{(e_q)_a = (v_a, u_{ab}); 1 \le a \le n, 1 \le q \le 5$ and $1 \le b \le 5\}$. The vertices of $K_n \circ K_{2,3}$ are assigned a rainbow dynamic coloring in the following manner.

Assign color *a* to vertex v_a of K_n for $1 \le a \le n$ and color |a+1| to vertices of $(K_{2,3})_a$ for $1 \le b \le 3$ and |a+2| for $4 \le b \le 5$. This color assignment indicates that

$$rdyc(K_n \circ K_{2,3}) \le n \tag{1}$$

To prove $rdyc(K_n \circ K_{2,3}) \ge n$. We assume that $rdyc(K_n \circ K_{2,3}) = n - 1$. The vertices of $K_n \circ K_{2,3}$ must be assigned n - 1 colors for rdyc. We will assign K_n , n - 1 colors. We observe that at least two neighboring vertices of $K_n \circ K_{2,3}$ have the same color assigned to them, and at least one path is not rdyc. This contradicts the assumption. As a result

$$rdyc(K_n \circ K_{2,3}) \ge n \tag{2}$$

It is evident from (1) and (2) that $rdyc(K_n \circ K_{2,3}) = n$



Proposition 2. $rdyc(P_2 \circ K_{2,3}) = 3$

Theorem 2. Let G be equal to $P_n \circ K_{2,3}$. Then, rdyc(G) = n for $n \ge 3$

Proof. Let $V(P_n) = \{v_a : 1 \le a \le n\}$ and let $(K_{2,3})_a$ be the vertex set of a' copies of $(K_{2,3})$ i.e $V\{(K_{2,3})_a\} = \{u_{ab} : 1 \le a \le n, 1 \le b \le 5\}$. Every vertex of P_n is adjacent to every vertex of a copy of $K_{2,3}$ according to the definition of the corona product, that is, for $1 \le a \le n$, the vertex v_a of $V(P_n)$ is adjacent to the vertices of the set $\{u_{ab} : 1 \le b \le 5\}$ in the a^{th} copy of $K_{2,3}$. Let $Q(P_n \circ K_{2,3})$ be $\{Q_1 \cup Q_2 \cup Q_3\}$ where Q_1 be the edge set of P_n , Q_2 be the edge set of $K_{2,3}$ and $Q_3 = \{(e_q)_a = (v_a, u_{ab}); 1 \le a \le n, 1 \le q \le 5$ and $1 \le b \le 5\}$. The vertices of $P_n \circ K_{2,3}$ are assigned a rainbow dynamic coloring in the following manner.

Assign color *a* to vertex v_a of P_n for $1 \le a \le n$ and color |a+1| to vertices of $(K_{2,3})_a$ for $1 \le b \le 3$ and |a+2| for $4 \le b \le 5$. This color assignment indicates that

$$rdyc(P_n \circ K_{2,3}) \le n \tag{3}$$

To prove $rdyc(P_n \circ K_{2,3}) \ge n$. We assume that $rdyc(P_n \circ K_{2,3}) = n - 1$. The vertices of $P_n \circ K_{2,3}$ must be assigned n - 1 colors for rdyc. We will assign P_n , n - 1 colors. We observe that at least two neighboring vertices of $P_n \circ K_{2,3}$ have the same color assigned to them and at least one path is not rdyc. This contradicts the assumption. As a result

$$rdyc(P_n \circ K_{2,3}) \ge n \tag{4}$$

It is evident from (3) and (4) that $rdyc(P_n \circ K_{2,3}) = n$



Figure 2: $P_4 \circ K_{2,3}$

Theorem 3. Let G be equal to $K_{2,3} \circ P_n$. Then, rdyc(G) = 5 for $n \ge 2$

Proof. Let $V(K_{2,3}) = \{v_a : 1 \le a \le 5\}$ and let $(P_n)_a$ be the vertex set of 'a' copies of (P_n) i.e $V\{(P_n)_a\} = \{u_{ab} : 1 \le a \le 5, 1 \le b \le n\}$. Every vertex of $K_{2,3}$ is adjacent to every vertex of a copy of P_n according to the definition of the corona product, that is, for $1 \le a \le 5$, the vertex v_a of $V(K_{2,3})$ is adjacent to the vertices of the set $\{u_{ab} : 1 \le b \le n\}$ in the a^{th} copy of P_n . Let $Q(K_{2,3} \circ P_n)$ be $\{Q_1 \cup Q_2 \cup Q_3\}$ where Q_1 be the edge set of $K_{2,3}$, Q_2 be the edge set of P_n and $Q_3 = \{(e_q)_a = (v_a, u_{ab}); 1 \le a \le 5, 1 \le q \le n$ and $1 \le b \le n\}$. The vertices of $K_{2,3} \circ P_n$ are assigned a rainbow dynamic coloring in the following manner.

Assign color *a* to vertex $K_{2,3}$ for $1 \le a \le 5$ and color $\{|a+1|, |a+2|\}$ and the same pattern is followed till the end vertex to vertices of $(P_n)_a$. This color assignment indicates that

$$rdyc(K_{2,3} \circ P_n) \le 5 \tag{5}$$

To prove $rdyc(K_{2,3} \circ P_n) \ge 5$. We assume that $rdyc(K_{2,3} \circ P_n) = 4$. The vertices of $K_{2,3} \circ P_n$ must be assigned 4 colors for rdyc. We will assign $K_{2,3}$, 4 colors. We observe that at least one path of $K_{2,3} \circ P_n$ is not rdyc. This contradicts the assumption. As a result

$$rdyc(K_{2,3} \circ P_n) \ge 5 \tag{6}$$

It is evident from (5) and (6) that $rdyc(K_{2,3} \circ P_n) = 5$

Figure 3: $K_{2,3} \circ P_4$

Theorem 4. Let G be equal to $K_{2,3} \circ W_{1,n}$. Then, rdyc(G) = 5 for $n \ge 2$

Proof. Let $V(K_{2,3}) = \{v_a : 1 \le a \le 5\}$ and let $(W_{1,n})_a$ be the set of vertices of 'a' copies of $(W_{1,n})_a$ ($W_{1,n})_a$ contains the *n*-cycle, $(C_n)_a = \{u_{a1}, u_{a2}, u_{a3}, u_{a4}, ...$

..., $u_{a(n+1)}$ and an additional vertex u_{a0} that connects to each of $(C_n)_a$ of $(W_{1,n})_a$. Every vertex of $K_{2,3}$ is adjacent to every vertex of a copy of $W_{1,n}$ according to the definition of the corona product, that is, for $1 \le a \le 5$, the vertex v_a of $V(K_{2,3})$ is adjacent to the vertices of the set $\{u_{ab} : 1 \le b \le n\}$ in the a^{th} copy of $W_{1,n}$. Let $Q(K_{2,3} \circ W_{1,n})$ be $\{Q_1 \cup Q_2 \cup Q_3\}$ where Q_1 be the edge set of $K_{2,3}$, Q_2 be the edge set of $W_{1,n}$ and $Q_3 = \{(e_q)_a = (v_a, u_{ab}); 1 \le a \le 5, 1 \le q \le n+1 \text{ and } 0 \le b \le n\}$. The vertices of $K_{2,3} \circ W_{1,n}$ are assigned a rainbow dynamic coloring in the following manner.

Case 1: *n* is even, Assign color *a* to vertex $K_{2,3}$ for $1 \le a \le 5$, color |a+1| to the vertex u_{a0} of $(W_{1,n})_a$ and color $\{|a+2|, |a+3|\}$ the same pattern is followed till the end vertex to $(C_n)_a$ of $(W_{1,n})_a$.

Case 2: *n* is odd, Assign color *a* to vertex $K_{2,3}$ for $1 \le a \le 5$, color |a+1| to the vertex u_{a0} of $(W_{1,n})_a$ and color $\{|a+2|, |a+3|\}$ the same pattern is followed with the end vertex as |a+4| to $(C_n)_a$ of $(W_{1,n})_a$. This color assignment indicates that

$$rdyc(K_{2,3} \circ W_{1,n}) \le 5 \tag{7}$$

To prove $rdyc(K_{2,3} \circ W_{1,n}) \ge 5$. We assume that $rdyc(K_{2,3} \circ W_{1,n}) = 4$. The vertices of $K_{2,3} \circ W_{1,n}$ must be assigned 4 colors for rdyc. We will assign $K_{2,3}$, 4 colors. We observe that at least one path of $K_{2,3} \circ W_{1,n}$ is not rdyc. This contradicts the assumption. As a result

$$rdyc(K_{2,3} \circ W_{1,n}) \ge 5 \tag{8}$$

It is evident from (7) and (8) that $rdyc(K_{2,3} \circ W_{1,n}) = 5$

Figure 4: $K_{2,3} \circ W_{1,3}$

3. Discussions

From Proposition 1,2 it is observed that the $rdyc(K_2 \circ K_{2,3})=3=rdyc(P_2 \circ K_{2,3})$ for n = 2 as the P_2 and K_2 graphs are the same. The results of Theorem 3,4 are obtained for $n \ge 2$ and $rdyc(K_{2,3} \circ W_{1,n}) = 5 = rdyc(K_{2,3} \circ P_n)$ for $n \ge 2$.

4. Conclusions

In this paper, we explored the concept of rainbow dynamic coloring for several corona product graphs, including combinations of $(K_n \circ k_{2,3})$, $(P_n \circ k_{2,3})$, $(K_{2,3} \circ P_n)$ and $(K_{2,3} \circ W_{1,n})$. We also describe the general problems that motivated this research. Today, computer networks in which certain links connect nodes are widespread. This network creates a graph. Graphs are used in computer networks to create a network of nodes and facilitate efficient packet routing. Some examples include finding the shortest routes between nodes, analyzing network traffic to identify the fastest root, and determining the most economical route. [4], [5], [10], [13], [14],

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