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Free Convection Flow from a Dissipative Vertical Cone: A Group Method Approach to Non-uniform Surface Heat Flux

Bapuji Pullepu, Thirupathy Maheshwaran, Shyam Sundar Santra,
Dumitru Baleanu

Abstract. The group transformation in this study is created to simulate the laminar free convective incompressible flow through a vertical cone with viscous dissipation and a non-uniform surface flux ($q_w(x) = ax^m$) that varies as a distance power function. Where m represents the variable surface heat flux power law exponent, and a represents a constant. The governing partial differential equations are transformed into ordinary differential equations using group theory and solved numerically with a Runge–Kutta shooting technique. The analysis focuses on the roles of viscous dissipation, Prandtl number, and the non-uniform surface heating parameter. The results reveal that viscous dissipation enhances both velocity and temperature profiles by increasing buoyancy forces. Variations in the surface heating parameter further redistribute the flow and temperature gradients. The findings demonstrate the effectiveness of the group method in reducing complex transport problems and provide insights relevant to cone-shaped engineering systems such as heat exchangers and chemical reactors.

Key Words and Phrases: Free convection, Heat flux, R-K method, Vertical cone, Viscous dissipation.

2010 Mathematics Subject Classifications: 76D05,80A20,76R10

1. Introduction

In fluid mechanics, the free convection process is vital in our environment. Due to excessive acceleration or operation at high rotational speeds, significant heat losses in free convection have been reported in a variety of equipment. Thermal dissipation effects are stronger in weak gravitational fields with huge process scales, i.e., velocities are higher in many applications of industry, such as electronic cooling and drying. Moran and Gaggiali [1-2] investigated the one-parameter group transformation with a similarity approach. A methodological approach is showcased to decrease the number of independent variables in systems that typically include a collection of PDE (partial differential equations) as well as auxiliary conditions. These processes are often referred to as similarity analyses in engineering. A major simplification of group theory methods was established by Moran and Gaggioli. Kassem [3] discussed the free convective flow across a continuously moving vertical plate exposed to a constant flux of heat at different velocities. A vertical cone with a variable surface temperature that changes with the distance from the tip along the flow of the cone was explained by Herring and Grosh [4].

Numerical solutions of the efficient boundary layer equation with a Prandtl value of 0.7 were found for both isothermal and linear surface temperatures. A vertical permeable cone with a non-uniform heat flow was used to study surface convection by Paul and Hossain [5]. Heat transfer from a vertical circular cone submerged in a thermally stratified medium having uniform surface temperature and heat flux was examined by Hossain et al. [6] with non-similarity solutions to the laminar natural convection flow. Free convection from a vertically rotating cone having constant heat flow along the wall has been addressed by Kumari and Pop [7]. To examine the viscous dissipation impact in steady-state free convection flow through a non-isothermal vertical cone, Kannan et al. [8] employed a group approach. In the situation of constant heat flow as well as mass flux, Hassanien et al. [9] represented mixed convection along a wedge surrounded by a fluid saturated with a porous material by using the impact of variable viscosity along with thermal conductivity. Kameswaran et al. [10] observed the mass transport and heat convection in a chemically reactive and dissipative magneto-nano fluid flow with soret diffusion effects. The phenomena investigated by Vajravelu and Hadjinicolaou [11] examine the transfer of heat characteristics in a linearly expanding continuous surface with variable wall temperature in the existence of viscous dissipation. To represent both the wall skin friction as well as wall temperature distributions, Lin [12] derived the similarity solution for laminar free convection of the right circular cone under constant heat flow circumstances for Pr = 0.72, 1, 2, 4, 6, 8, 10, 100. Free convection over a vertical frustum of a cone with uniform heat flux was studied by Na and Chiou [13].

The governing differential equations were solved by combining the finite difference approach with the quasi-linearization technique, and provided a broad variety of transverse curvature parameter values for Prandtl numbers from 0.1 to 100. A theoretical analysis of suction or injection interactions from a vertical cone with constant surface heat flux is solved by Pop and Watanabe [14] through the iterative difference-differential technique. When the Prandtl number is 0.72, the numerical calculations are observed for various suction/injection parameter values. Rama Subba Reddy et al. [15] implemented a numerical solution for laminar-free convection from a vertical cone frustum in power-law fluid flow with constant heat flux. Watanabe [16] concentrated his theoretical investigation on the effects of injection or suction over a vertical cone while maintaining the constant wall temperature. Schlichting [17] delves more into the topic of the boundary layer theory. In order to investigate mass as well as heat transfer in a porous medium, Queeny and Singh [18] looked at free convection over a vertical surface. A constant two-dimensional laminar incompressible flow around a sphere with viscous dissipation and heat production has been analysed by Raihanul Haque et al. [19]. It is assumed that the relationship between temperature and thermal conductivity is linear.

The main PDEs are transformed into locally non-similar partial differential forms by utilizing suitable transformations. The Keller box method and the implicit finite difference methodology have been utilized to numerically solve the modified boundary layer equations. Visual representations of numerical findings for the rate of heat transfer along with important physical parameters are provided. Shekar Saranya et al. [20] conducted an analysis on the flow as well as the transfer of heat of a hybrid ferrofluid experiencing free convection from a heated spinning vertical cone by utilizing the IPS (Iterative Power Series) technique. Palani and Kim [21] investigated the characteristics of natural convection heat transfer under varying surface heat flux while also analyzing the effects of thermal radiation as well as a magnetic field. Bapuji Pullepu and Immanuel [22] studied the unsteady natural convective flow from a vertical cone by examining the various impacts of an isothermal temperature and non-uniform concentration. Eldabe et al. [23] utilized the finite difference method to derive the numerical solution for the flow of gyrotactic microorganisms in a non-Darcian micropolar fluid containing different nanoparticles.

This current study simplifies the governing boundary conditions and PDE into an ODE with suitable boundary conditions. The group method approach has not gained much attention in the literature concerning the vertical cone surface. Free convective flow from a vertical cone with viscous dissipation and the non-uniform heat flux surface condition has been analysed using the differential equations generated by the group method technique. The dimensionless boundary layer equation is solved using the Runge-Kutta approach. The present findings are compared to those of Watanabe and Pop [14], Hossain and Paul [5], and others to ensure the correctness of the numerical outcomes, and they are found to be highly consistent.

2. Mathematical Analysis

The axis-symmetric free convective incompressible viscous flow over a vertical cone with viscous dissipation under the non-uniform surface heat flux condition is examined in this paper. The surface of the cone and the embedded fluid are considered to be at rest at the same temperature T'_∞. The coordinate system has been depicted in Fig 1. x represents the wall surface along the cone and y indicates the distance perpendicular to the cone wall. where r is the cone's local radius and ϕ is the half angle. Thermal buoyancy effect affects an upward flow $T'_w > T'_\infty$ with a heat flux $q_w(x) = ax^m$. The fluid properties stay constant with the exception of fluctuations in density and buoyancy force.

The governing equations of the boundary layer with Boussinesq approximation for continuity, momentum, and energy are shown below

$$(ru)_x + (rv)_y = 0 \tag{1}$$

$$uu_x + vu_y = g\beta(T' - T'_\infty)\cos\phi + vu_{yy'} \tag{2}$$

$$uT'_x + vT'_y = \alpha T'_{yy} + \frac{\mu}{\rho C_p}(u_y)^2 \tag{3}$$

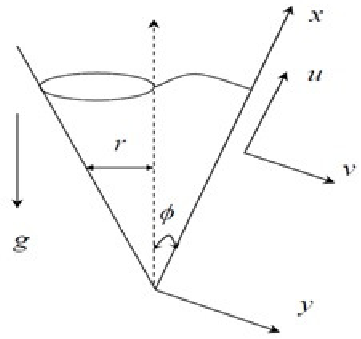

Figure 1: Model of physical system

The basic as well as boundary conditions have been given as:

$$\begin{aligned} &u(x,0)=v(x,0)=0, T'_y=\frac{-q_w(x)}{k}\ y=0 \\ &u(0,y)=v(0,y)=0, T'(0,\infty)=T'_\infty\ x=0 \\ &u(x,y), v(x,y)\to 0, T'(x,y)\to T'_\infty\ y\to\infty \end{aligned} \tag{4}$$

Local skin friction τ_x and Local Nusselt numbers Nu_x have been provided by

$$\tau_x=\mu(u_y)_{y=0}, Nu_x=\frac{x}{T'_w-T'_\infty}(-T'_y)_{y=0} \tag{5a}$$

Introducing the suitable non-dimensional quantities

$$\begin{aligned} &X=\frac{x}{L}, Y=\frac{y}{L}(G_{r_L})^{1/5}, R=\frac{r}{L} \text{ where } r=x\sin\varphi \\ &U=\frac{uL}{\nu}(G_{r_L})^{-2/5}, V=\frac{vL}{\nu}(G_{r_L})^{-1/5}, T=\frac{(T'-T'_\infty)(G_{r_L})^{1/5}}{q_w(L)/k} \\ &G_{r_L}=\frac{g\beta q_w(L)L^4\cos\phi}{\nu^2 k}, Pr=\frac{\nu}{\alpha}, \epsilon=\frac{g\beta L}{C_P} \end{aligned} \tag{6}$$

Where ϵ is the viscous dissipation parameter and Pr is the Prandtl number. By using equ (5), the non-dimensional form of equations (1) to (3) are as follows:

$$(RU)_X+(RV)_Y=0 \tag{7}$$

$$UU_X+VU_Y=T+U_{YY} \tag{8}$$

$$UT_X+VT_Y=\frac{1}{Pr}T_{YY}+\epsilon(U_Y)^2 \tag{9}$$

The non-dimensional boundary conditions:

$$\begin{aligned} &U(X,0)=V(X,0)=0, T_Y=-X^m,\ \text{at } Y=0\\ &U(0,Y)=V(0,Y)=0, T(0,Y)=0,\ \text{at } X=0\\ &U(X,Y), V(X,Y)\to 0, T(X,Y)\to 0 \text{ as } Y\to\infty \end{aligned} \tag{10}$$

From equation (5a), the local dimensionless skin friction τ_x and Nusselt numbers Nu_x becomes

$$\tau_x = Gr_L^{3/5}(U_Y)_{Y=0}, Nu_x = \frac{Gr_L^{1/5}}{T_{Y=0}}X^{m+1} \tag{11a}$$

Similarity variables are as follows

$$L = XRQ(X,Y), T = T(X,Y) \tag{12a}$$

We explain the work of the flow by reducing the number of conditions from 3 to 2

$$U = \frac{1}{R}\zeta_Y,\ \text{and } V = \frac{1}{R}\zeta_X \tag{13a}$$

Condition (6) is met by a stream function, while (7) and (8) are transformed into corresponding conditions.

$$\zeta_Y\left(\frac{1}{R}\zeta_Y\right) - \frac{1}{R}\zeta_X\zeta_{YY} = RT + \zeta_{YYY} \tag{14}$$

$$\frac{1}{R}(\zeta_Y T_X - \zeta_X T_Y) - \frac{1}{Pr}T_{YY} + \epsilon\frac{1}{R^2}(\zeta_{YY})^2 \tag{15}$$

Boundary condition (9) communicated as:

$$\begin{aligned} &\lim_{Y\to 0}\zeta_Y = 0, \lim_{Y\to 0}\zeta_X = 0, \lim_{Y\to 0}T_Y = -X^m\\ &\lim_{Y\to\infty}\zeta_Y = 0, \lim_{Y\to\infty}T = 0 \end{aligned} \tag{16}$$

3. Problem Formulation in Groups

The solution technique involves utilizing a 1-parameter group transformation on the PDE (10) to (11). Two independent variables are decreased by one in this transformation, causing differential equations (10) to (11) to become an ODE with only one independent variable, known as the similarity variable.
$G : \bar{P} = C^p(a)P + k^p(a)$.

4. The group's systematic formulation

Class of one-parameters 'a' with group G initiated

$$G = \begin{cases} C^X(a)X + k^X(a)\ y = C^Y(a)Y + k^Y(a)\\ \bar{\zeta} = C^X(a)\zeta + k^X(a)\ r = C^R(a)R + k^R(a)\\ \bar{T} = C^T(a)X + k^T(a) \end{cases} \tag{17}$$

5. Transformation

The differential equations are transformed by obtaining the derivatives transformation from G via chain rule operations

$$\left.\begin{aligned}\bar{P}_i &= \left(C^P/C^i\right) P_i \\ \bar{P}_{ii} &= \left(C^P/C^iC^j\right) P_{ij}\end{aligned}\right\} \tag{18}$$

here Pi's for ζ, R, T.
Equations (10) & (11) are transformed invariantly under (13) and (14).

$$\bar{\zeta}_Y\left(\frac{1}{r}\bar{\zeta}_Y\right)_x - \frac{1}{r}\bar{\zeta}_x\bar{\zeta}_{yy} - r\bar{T} - \bar{\zeta}_{yyy} = H_1(a)\left[X_Y\left(\frac{1}{R}\zeta_Y\right)_X - \frac{1}{R}\zeta_X\zeta_{YY} - RT - \zeta_{YYY}\right] \tag{19}$$

$$\begin{aligned}&\frac{1}{r}[\bar{\zeta}_y - \bar{T}_x - \bar{T}_y\bar{\zeta}_x] - \frac{1}{Pr}\bar{T}_{yy} - \epsilon\frac{1}{r^2}(\bar{\zeta}_{yy})^2 \\ &\quad = H_2(a)\left[\frac{1}{R}(\zeta_Y T_X - \zeta_X T_Y) - \frac{1}{Pr}T_{YY} - \epsilon\frac{(C^X)^2}{(C^R R)^2}\frac{1}{(C^Y)^2C^T}(\zeta_{YY})^2\right]\end{aligned} \tag{20}$$

where $H_1(a), H_2(a)$ are functions (or) may be constants

$$\begin{aligned}&\frac{(C^\zeta)^2}{C^R C^X (C^Y)^2}\left[\frac{1}{R}(\zeta_Y\zeta_{XY} - \frac{1}{R^2}(\zeta_Y)^2 R_x - \frac{1}{R}\zeta_X\zeta_{YY}\right] - RC^RC^TT - \frac{C^\zeta}{(C^Y)^3}\zeta_{YYY} + I_1(a) \\ &\quad = H_1(a)\left[\zeta_Y\left(\frac{1}{R}\zeta_Y\right)_X - \frac{1}{R}\zeta_X\zeta_{YY} - RT - \zeta_{YY}\right]\end{aligned} \tag{21}$$

$$\begin{aligned}&\frac{C^\zeta C^T}{C^R C^X C^Y}\frac{1}{R}(\zeta_Y T_X - \zeta_X T_Y) - \frac{1}{Pr}\frac{C^T}{(C^Y)^2}T_{YY} - \epsilon\frac{(C^\zeta)^2}{(C^R)^2(C^Y)^4}(\zeta_{YY})^2 + I_2(a) \\ &\quad = H_2(a)\left[\frac{1}{R}(\zeta_Y T_X - \zeta_X T_Y) - \frac{1}{Pr}T_{YY} - \epsilon\frac{(C^\zeta)^2}{(C^R R)^2}\frac{1}{(C^Y)^2C^T}(\zeta_{YY})^2\right]\end{aligned} \tag{22}$$

where

$$\begin{aligned}I_1(a) &= \sum_1^\infty \binom{-1}{n}\left(\frac{k^R}{C^R R}\right)^n \frac{(C^\zeta)^2}{C^R C^X (C^Y)^2}\frac{1}{R}(\zeta_Y\zeta_{XY} - \zeta_X\zeta_{YY}) - \sum_1^\infty \binom{-2}{n}\left(\frac{k^R}{C^R R}\right)^n \\ &\times \frac{(C^\zeta)^2}{C^R C^X (C^Y)^2}\frac{1}{R^2}(\zeta_Y)^2 R_X - C^R k^R R - k^R C^T T - k^R k^T\end{aligned} \tag{23}$$

$$\begin{aligned}I_2(a) &= \sum_1^\infty \binom{-1}{n}\left(\frac{k^R}{C^R R}\right)^n \frac{C^\zeta C^T}{C^R C^X C^Y}\frac{1}{R}(\zeta_Y T_X - \zeta_X T_Y) \\ &- \frac{\epsilon}{(C^R R)^2}\sum_1^\infty \binom{-2}{n}\left(\frac{k^R}{C^R R}\right)^n \times \frac{(C^\zeta)^2}{(C^Y)^4}(\zeta_{YY})^2\end{aligned} \tag{24}$$

Invariance of equations (17) and (18)
$\Rightarrow I_1(a) = I_2(a) = 0$
The above equations are satisfied by substitution.

$$k^R = k^T = k^Y = 0 \tag{25}$$

$$\frac{(C^\zeta)^2}{C^R C^X (C^Y)^2} = \frac{C^\zeta}{(C^Y)^3} = \frac{C^\zeta}{C^Y} = 0 \tag{26}$$

$$\frac{C^\zeta C^T}{C^R C^X C^Y} = k^T = k^Y = 0 \tag{27}$$

These yields

$$C^X = (C^Y)^2, C^R = \frac{1}{(C^Y)^2}, C^\zeta = C^Y \tag{28}$$

Limit equations (19) & (20) are also invariant.

$$k^R = k^T = 0, C^T = 1 \tag{29}$$

Finally, a one boundary exhaustive G that varies invariantly, conditions (17) & (18), and the most extreme conditions (19) & (20).
We get G from the above-mentioned conditions

$$G = \begin{cases} x = (C^y)^2 X + k^X \\ y = C^Y Y \\ r = \frac{R}{(C^y)^2} \\ \bar{\zeta} = C^y \zeta + k^\zeta \\ \bar{T} = T \end{cases} \tag{30}$$

6. Group transformation of the boundary layer flow equations

To solve this problem, we will use group techniques to transform it into an ODE with 1-independent variable. Therefore, we must continue our investigation in order to attain an absolute invariants complete collection.
If $\mu = \mu(X, Y)$ independent variables X and Y absolute invariants are present, then

$$F_j(X, Y, \varphi, R, T) = Q_j(\mu(X, Y)), \ j = 1, 2, 3 \tag{31}$$

In group theory, A function $F_j(X, Y, \varphi, R, T)$ is an absolute invariant of a one-parameter group if it satisfies the following first-order linear differential equation

$$\sum_{i=1}^{5} (A_i P_i + B_i) \frac{\partial F}{\partial P_i} = 0, P_i = X, Y, \zeta, R, T \text{ where } A_i = \frac{\partial C^{p_i}}{\partial a}(a_0), B_i = \frac{\partial k^{p_i}}{\partial a}(a_0) \tag{32}$$

Here, a_0 denotes the value thathas been produceby the group's identity element.
Since $k^R = k^T = k^Y = 0$
From equation (23) and utilizing (22), we attain
$B_2 = \frac{\partial k^Y}{\partial a}(a_0) = 0, B_4 = \frac{\partial k^R}{\partial a}(a_0) = 0, B_5 = \frac{\partial k^T}{\partial a}(a_0) = 0$
$B_3 = \frac{\partial k^\zeta}{\partial a}(a_0) = 0,$ (i.e.) $B_2 = B_3 = B_4 = B_5 = 0$
By satisfying the first-order linear PDE, $\mu(X, Y)$ is an invariant by equation (22)

$$(A_1X + B_1)\frac{\partial \mu}{\partial X} + A_2Y\frac{\partial \mu}{\partial Y} = 0 \tag{33}$$

From the above equation, we get

$$\frac{\partial \mu}{\partial X} = 0 \tag{34}$$

Therefore eqn (30)

$$\Rightarrow \mu = Y \tag{35}$$

Likewise, not altering the analysis of ζ, R, T dependent variables

$$\zeta(X, Y) = \Gamma_1(X)Q(\mu), R(X, Y) = \Gamma_2(X)E(\mu), T(X, Y) = T(\mu) \tag{36}$$

Here, functions $\Gamma_1(X), \Gamma_2(X), Q(\mu)$ & $E(\mu)$ have been calculated.
Since $R(X, Y)$ is independent of Y.

$$R(X, Y) = R_0\Gamma_2(X) \tag{37}$$

7. The Formation of ODE

The determined form of independent as well as dependent absolute invariants is used to derive ODE as the general analysis progresses. The $\mu = \mu(X, Y)$ absolute invariant takes the form provided in condition (31) Substitute equation (32) into equation (17) and after dividing it by $\Gamma_1(X)$, we obtain

$$\Rightarrow \frac{1}{R}\zeta_Y\zeta_{XY} - \frac{1}{R^2}(\zeta_Y)^2R_X - \frac{1}{R}\zeta_X\zeta_{YY} - RT - \zeta_{YYY} = 0 \tag{38}$$

$$Q''' + \frac{1}{R_0\Gamma_2}QQ''\frac{\partial \Gamma_1}{\partial X} - \left(\frac{1}{R_0\Gamma_2}\frac{\partial \Gamma_1}{\partial X} - \frac{\Gamma_1}{R_0\Gamma_2^2}\frac{\partial \Gamma_2}{\partial X}\right)Q'^2 + \frac{R_0\Gamma_2\Gamma_3}{\Gamma_1}T = 0 \tag{39}$$

Also, by substituting conditions (26) to (28) in the condition (13) we obtain

$$\frac{1}{R}(\zeta_YT_X - \zeta_XT_Y) - \frac{1}{Pr}T_{YY} + \epsilon\frac{1}{R^2}(\zeta_{YY})^2 = 0 \tag{40}$$
$$\frac{1}{R_0\Gamma_2}\frac{\partial \Gamma_1}{\partial X}Q(\mu)T' + \frac{1}{Pr}T'' - \epsilon\frac{\Gamma_1^2}{R_0^2\Gamma_2^2}(Q''(\mu)^2) = 0$$

$$C_1 = \frac{1}{R_0\Gamma_2}\frac{\partial \Gamma_1}{\partial X}, C_2 = \frac{\Gamma_1}{R_0\Gamma_2^2}\frac{\partial \Gamma_2}{\partial X}, C_3 = \frac{R_0\Gamma_2\Gamma_3}{\Gamma_1} \text{ are the random coefficients} \tag{41}$$

By using the above documentation of condition (36), conditions (34) & (35) are reduced to

$$Q''' + C_1 QQ'' - (C_1 - C_2)Q'^2 + C_3 T = 0 \tag{42}$$

$$C_1 QT' + \frac{1}{Pr}T'' - \epsilon\frac{Q''^2}{C_3^2} = 0 \tag{43}$$

Corresponding boundary conditions are

$$Q'(0) = 0, T'(0) = -X^m, Q'(\infty) = T(\infty) = 0 \tag{44}$$

Case (i) put $C_1 = \frac{3}{4}, C_2 = \frac{1}{4}$ & $C_3 = 1$ in eqns (37) & (38) we attain

$$Q''' + \frac{3}{4}QQ'' - \frac{1}{2}Q'^2 + T = 0 \tag{45}$$

$$\frac{3}{4}QT' + \frac{1}{Pr}T'' - \epsilon Q''^2 = 0 \text{ (or) } T'' + \frac{3}{4}PrQT' - Pr\epsilon Q''^2 = 0 \tag{46}$$

Case (ii) put $C_1 = \frac{7}{4}, C_2 = \frac{5}{4}$ & $C_3 = 1$ in eqns (37) & (38) we attain

$$Q''' + \frac{7}{4}QQ'' - \frac{1}{2}Q'^2 + T = 0 \tag{47}$$

$$\frac{7}{4}QT' + \frac{1}{Pr}T'' - \epsilon Q''^2 = 0 \text{ (or) } T'' + \frac{7}{4}PrQT' - Pr\epsilon Q''^2 = 0 \tag{48}$$

With boundary conditions:

$$Q'(0) = 0, T'(0) = -X^m, Q'(\infty) = T(\infty) = 0 \tag{49}$$

Utilizing equations (9b) & (9c) into equation (9a), the dimensionless τ_x and Nu_x becomes

$$\tau_x = XQ''(0)Gr_l^{3/5},\ Nu_x = \frac{X^{m+1}}{T(0)}Gr_l^{1/5} \tag{50}$$

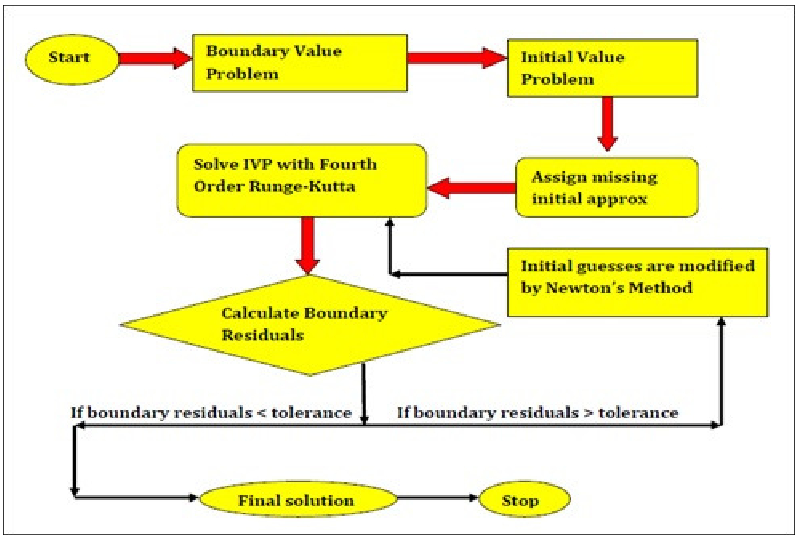

Figure 2: Flow chart of the shooting method

8. Results and Discussion

The fourth-order Runge-Kutta approach is used to solve equations (40) to (43) by using the boundary conditions (44) numerically. The current results were compared with available data in the literature to verify our numerical results. Table 1 shows the numerical values of temperature (T) and for various Prandtl values, which have been compared with the available results of Palani and Kim [21] and are found to be in excellent agreement. It is also important to note that the current findings are consistent with those of Pop and Watanabe [14].

Table 1: Skin friction and temperature values are compared with the results of Palani and Kim [21] with the absence of a magnetic field and m=0

	Local skin friction		Temperature	
	Palani and Kim [21] results	Present results	Palani and Kim [21] results	Present results
Pr	τ_X	τ_X	T	T
0.72	1.2240 1.2180∗	1.2205	1.7996 1.7870∗	1.7883
1	1.0797	1.0789	1.6325	1.6302
2	0.8193	0.8172	1.3532	1.3576
4	0.6473	0.6465	1.1480	1.1401
6	0.5462	0.5471	1.0501	1.0502
8	0.4895	0.4798	0.9806	0.9863
10	0.4494	0.4489	0.9290	0.9260
100	0.1839	0.1812	0.5571	0.5505

∗ Shows the values found in Pop and Watanabe [14] study when the suction injection is 0.

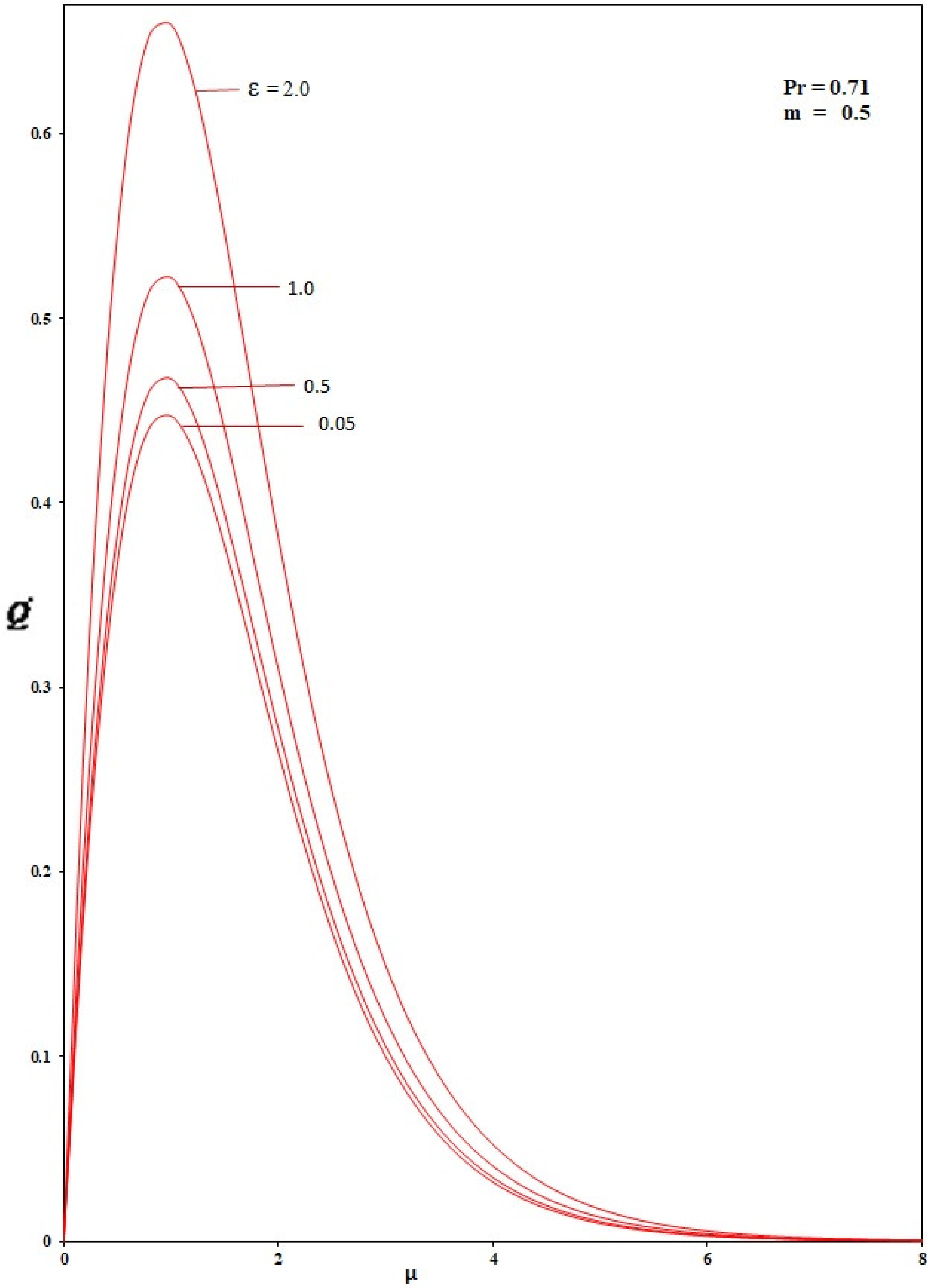

Figure 3: Velocity profile for various ϵ values.

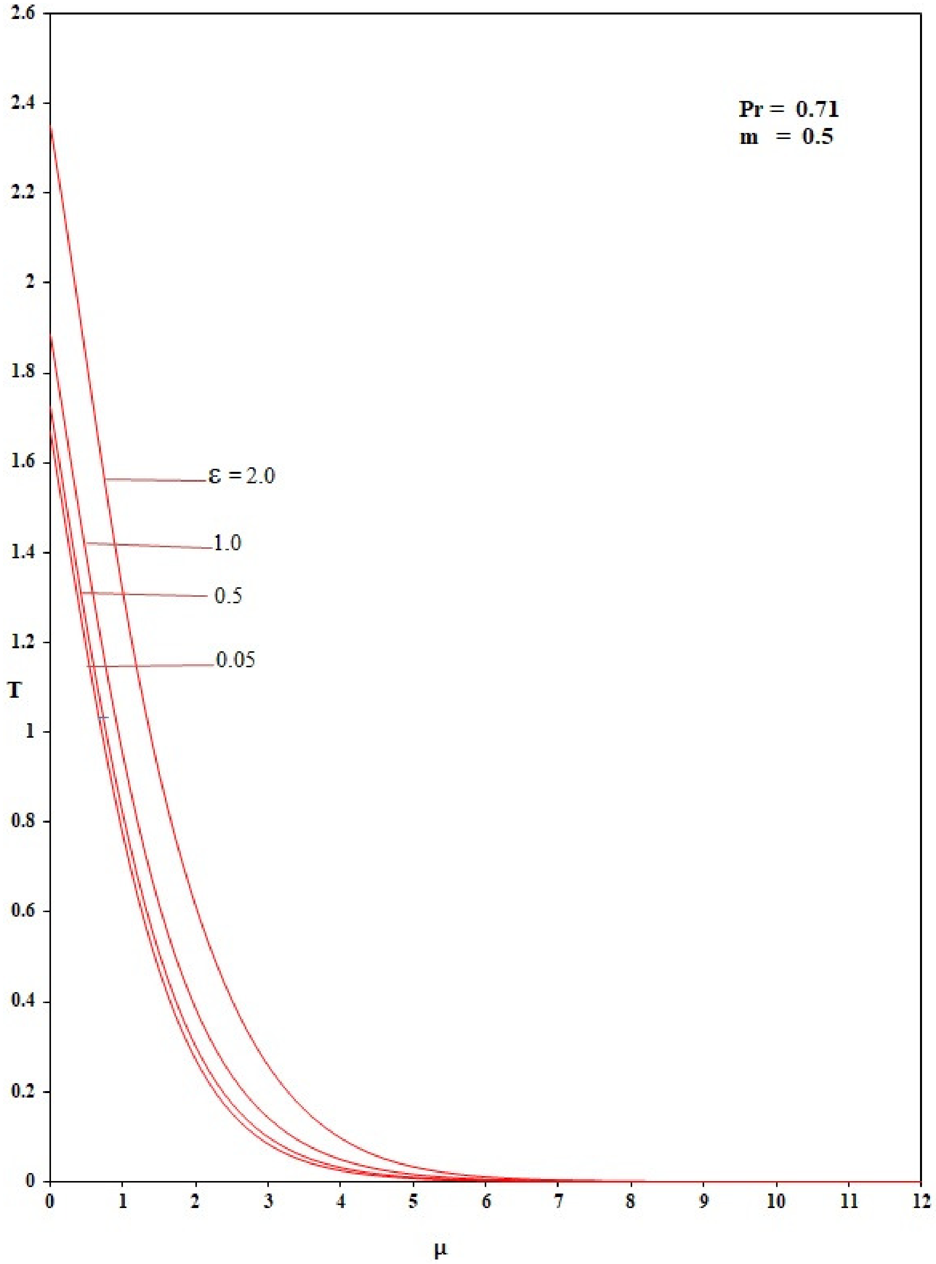

Figure 4: Temperature profile for various ϵ values.

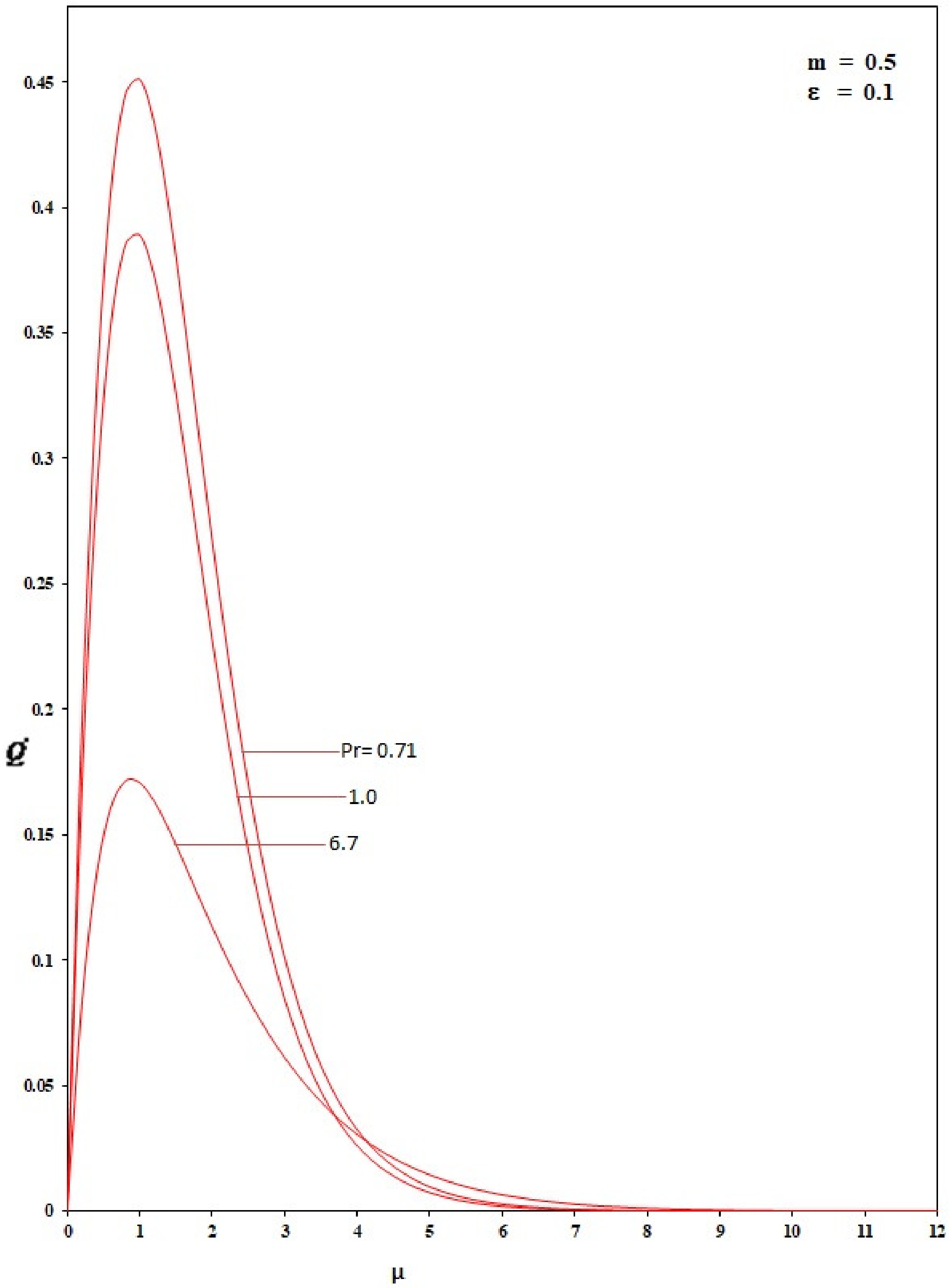

Figure 5: Velocity profile for various Pr values.

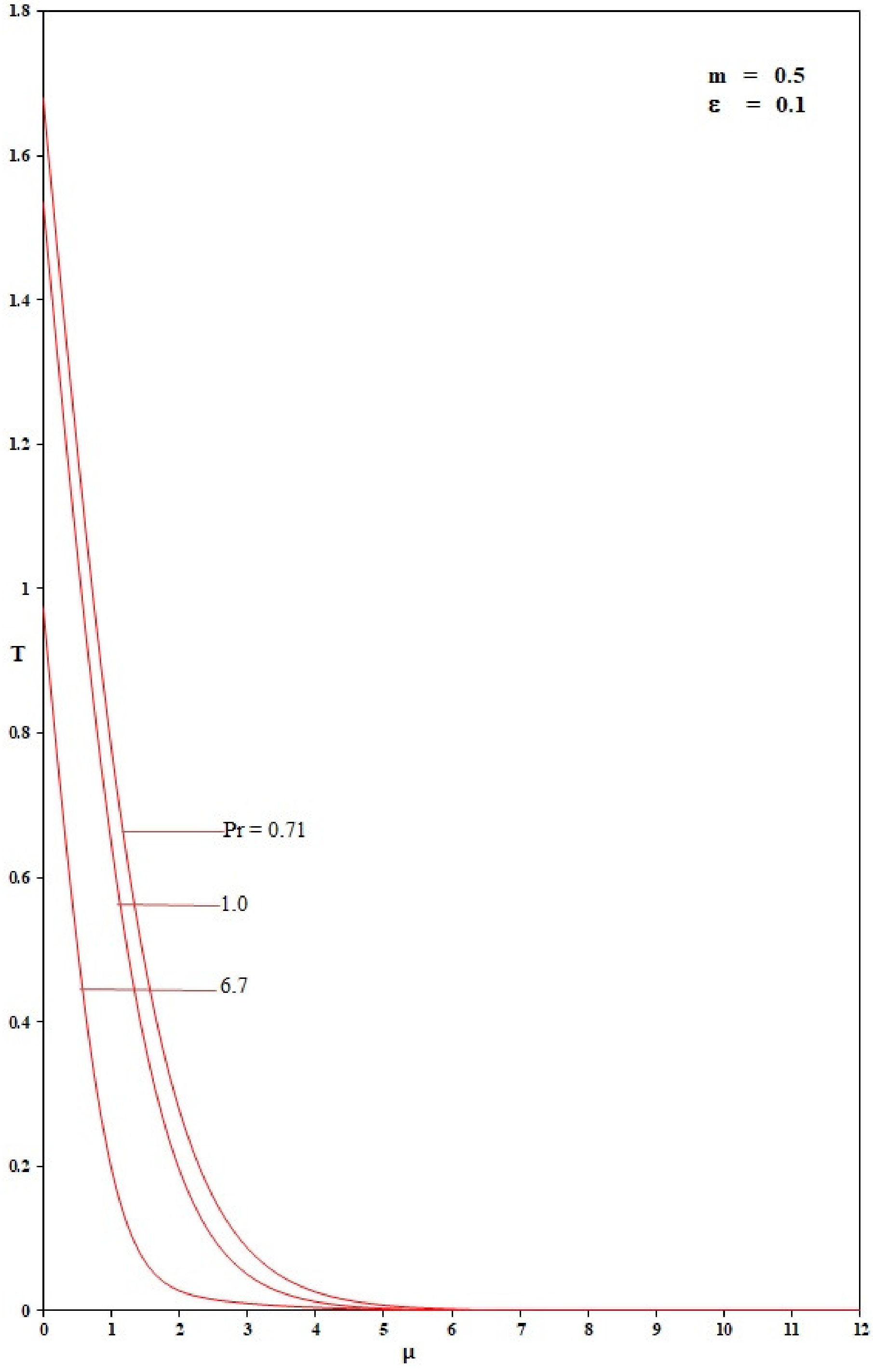

Figure 6: Temperature profile for various Pr values.

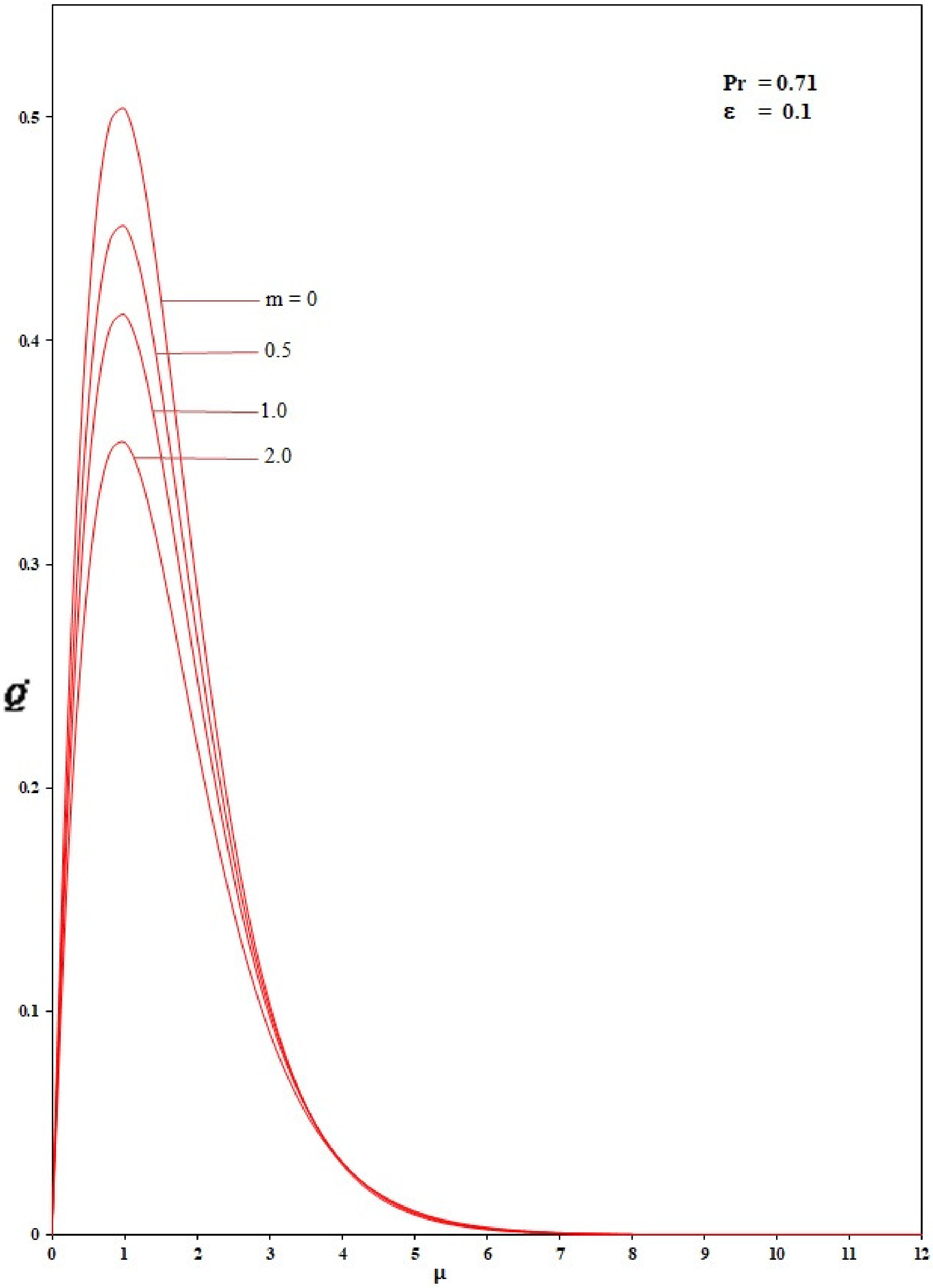

Figure 7: Velocity profile for various m values.

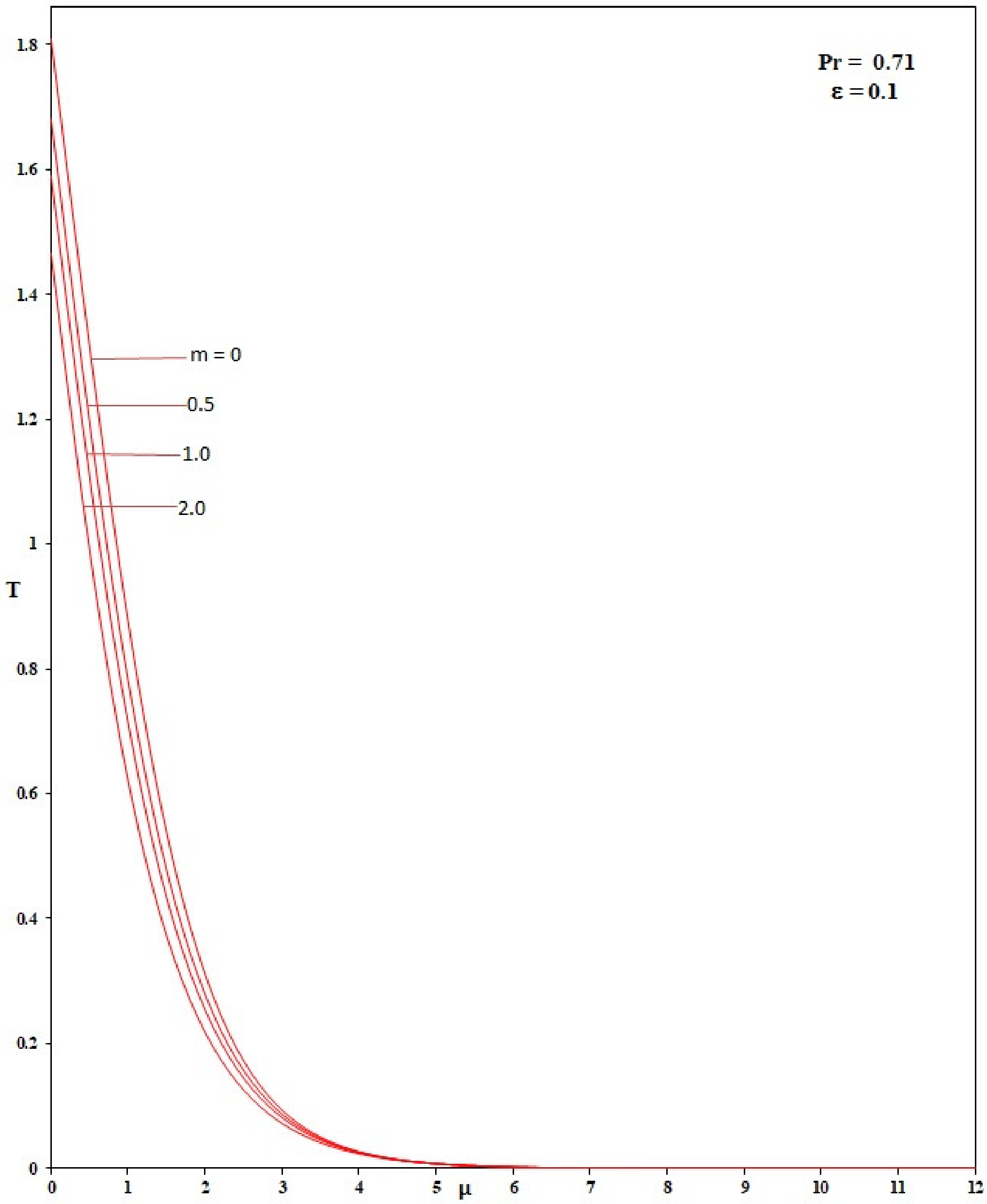

Figure 8: Temperature profile for various m values.

9. Conclusions

This work applied the group method of transformations in combination with the Runge-Kutta shooting scheme to examine the non-uniform surface heat flux impact on the continuous laminar free convection of a dissipative fluid across a vertical cone geometry. The approach successfully reduced the governing PDEs to a tractable system of ODEs and offered accurate numerical solutions. The results highlight that viscous dissipation acts as an internal heat source, thereby enhancing buoyancy and increasing velocity and temperature. The surface heating parameter modifies the distribution of buoyancy along the cone, producing notable changes in flow gradients.
The following findings were reached:

- The velocity and temperature intensities increase with greater viscous dissipation parameter, and decrease when the parameters Pr and m are greater.
- Arise in m thickens the thermal and momentum boundary layer.
- The Nu_x rises for accelerating values of Pr and m, whereas lowers for the values of ϵ.

The novelty of this study lies in demonstrating the applicability of the group method to a cone geometry with dissipation effects under non-uniform heat flux. Future extensions of this work may include the effects of radiation, porous media, or hybrid nanofluids, which would broaden the engineering relevance to energy systems, electronic cooling, and advanced manufacturing processes.

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Bapuji Pullepu
Department of Mathematics, SRM Institute of Science & Technology, Kattankulathur-603 203, Tamil Nadu, INDIA
E-mail: bapujip@yahoo.com

Thirupathy Maheshwaran
Department of Statistics, Dwaraka Doss Goverdhan Doss Vaishnav College, Chennai-600 106, Tamil Nadu, INDIA
E-mail: mage_maths@yahoo.co.in

Shyam Sundar Santra*
Department of Mathematics, JIS College of Engineering, Nadia, West Bengal, 741235, INDIA
* *Corresponding Author E-mail:* shyam01.math@gmail.com

Dumitru Baleanu
Department of Computer Science and Mathematics, Lebanese American University, Beirut 11022801, Lebanon
E-mail: dumitru.baleanu@gmail.com

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