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Performance Analysis of an M/M(a,d,b)/(2,1) Queuing System with RWV policy and Breakdown

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Abstract. This study investigates the complex queuing system M/M(a,d,b)/(2,1) in which two servers are involved and a server will always be available in the system for delivering service while the other server is on working vacation. In this system, the servers follow the repeated working vacation (RWV) policy; that is, they undergo vacations repeatedly and shift to working vacation in the vacation epoch as soon as they discover that a sufficient number of customers are waiting in line. Working vacation demonstrates that the service providers serve the customer in the vacation epoch but at a slower rate. Late arrivals to the service facility are also allowed to join the ongoing service if they are below the threshold limit 'd'. Here, the arrival and service completion process adheres to Poisson and exponential distributions. The aspect of service interruption is also analyzed in this queuing system framework. The steady-state solutions and the performance metrics of the proposed queuing system are computed from the standpoint of service interruption. Analytical results are evaluated with a numerical outcome.

Key Words and Phrases: Markovian process, batch service, repeated working vacation, breakdown.

2010 Mathematics Subject Classifications: 60K25, 68M20, 90B22

1. Introduction

On receiving service, a customer might encounter situations where the service provider is unavailable because the provider may leave the service station for an unpredictable amount of time, either to perform another task or to take a break. This period of the service provider's unavailability is called a "vacation". So far, the research done in queuing theory, studying queuing models with vacation policy, has been an important area of investigation for many researchers. This study also deals with the servers' vacation policy, which is taken up by the server

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whenever they are unavailable with the minimum accessible batch size.

The study on queuing models with vacation policies appeared in the literature during late 20th century.A survey on these vacation queuing models was conducted by B.T. Doshi [1] and Upadhyaya [19] as it is an important existing real-life situation that has been encountered in our daily life routine and industrial management. S. Sindhu et al. [17] and Zhang [21] have worked on these vacation policies. Collective arrival and departure in the system involving a main and a standby service provider were studied by R.K. Srivastava et al. [18]. The multilevel threshold policy for vacation was studied in the application systems, like manufacturing systems or call centres, by modelling it as a Markovian queue. Wu and Ke [20] dealt with this system by developing a cost model. There may also exist situations where the servers undergo random withdrawal from the system multiple times in order to avoid staying unengaged while not having an accessible batch size in the system. Palaniammal and Nadarajan [11] analyzed a queuing model providing service in batches with the above notable vacation policy RV (Repeated Vacation). The significant feature of the system is that it was assisted by at least one service provider for granting service between two service providers. This model was also analyzed by Parimala and Baskar [13, 1] with repeated and delayed vacation policy.

On account of minimizing the waiting time in the line during the period of vacation, it is preferable if the service facility has a few more service providers to grant service so that customers can be served at a moment's notice. Chakravarthy and Kulshrestha [3] discussed a model in which the service process was assisted by a standby server continuously irrespective of failure, repair, and vacation. This analysis focuses on interpreting a queuing system with two servers starting to provide service whenever they are available with the minimum number of customers, aiming to reduce the waiting time even though they are on vacation. The service providers work comparatively at a slower rate than the usual rate of service in the vacation epoch. This can be termed as a working vacation. The transient behavior of the bulk arrival and service queuing model with limited buffer area was analyzed numerically by Kumar et al. [9]. The service providers work comparatively slower than the usual rate of service in the vacation epoch. Earlier the single-server queuing system was studied by Gupta and Kumar [7] under the retrial policy in relation to the Bernoulli schedule working vacation, breakdown and balking for cost optimization. The computations are carried out based on probability generating functions. Later the same model was analyzed by Seenivasan and Chandiraleka [15]under the servers' multiple working vacation policy and system failure using the matrix geometric method. Again, the same model was analyzed by Seenivasan et al. [14] by incorporating the aspects of working vacation and balking by introducing a standby server in the model. The bulk arrival queuing model was discussed by Jain and Kumar [8] from the perspective of balking, working vacation and vacation interruption. Panda and Goswami [12] studied the single-server queuing system by incorporating the phenomenon of working vacation policy to determine the equilibrium joining strategies among customers who have positive and negative intentions to join the system. Melikov et al. [6] also analysed the single-server system with working vacations and customers equilibrium behavior of retrial policy. Recently a survey on working vacation models was also contributed by Divya and Indhira [5]. In general, it is noticeable that queuing systems have many applications in the industrial sectors. From the aspect of its tremendous applications, authors have also worked on machine models by incorporating the working vacation policy. Deora et al. [4] and Sharma et al. [16] analyzed the machine repair problem with the aspects of multiple working vacations. A multi-server unreliable machine model was studied by Bouchentouf et al. [2] to optimise the cost. Generally, it is possible for the occurrence of breakdown in the machine. Thus, a multi-server queuing system, M/M (a,d,b)/(2,1), is analyzed in this paper with the notion of service interruption along with the repeated working vacation policy (RWV). The steady-state equations and solutions are computed using probability generating functions, and the performance metrics of the system M/M (a,d,b)/(2,1)/RWV with service interruption are also evaluated, enabling us to find the behavior of the system. A numerical illustration is made to verify the validity of the outcomes.

2. Mathematical Model Description for M/M(a,d,b)/(2,1)/RWV - Breakdown

This model involves two service facilities (servers). If there are insufficient customers in a queue to start service, one among the service providers remains inactive, and the other service provider takes a vacation. Upon returning from the vacation, if the service provider discovers that the availability of waiting customers in the queue is fewer than 'a while the other service provider is active or idle, the service provider withdraws from the service station (system) for another random interval of time, which is called a repeated vacation. The service provider will continue doing this till he discovers minimum 'a' customer to begin his service. Here, in this proposed model, the server gets access to serve the customers in his vacation epoch as soon as he discovers minimum 'a' customers in the queue while the other is active. This type of job done by the server can be termed as a working vacation. In addition, the system has access to the customers up to the threshold limit 'd' as late arrivals where ($a \leq d \leq b$), which makes the system more resilient and $l \geq d$ are not allowed to join the system if the service provider begins the service with minimum 'a' customers. The term $(2, 1)$ stands for the involvement of two servers in providing service, in which 1 represents the availability of at least one service provider in the system for delivering service at any time. Moreover, the two service providers will not switch over the system at the same time.

Meanwhile, if any kind of interruption occurs while providing service, it will affect both the customers waiting in the line and those who are currently receiving service. It led to an increase in the customers' sojourn period. This investigation delivers the methodological way to interrupt the scenario and provides a solution to this aspect that affects the service process in the system, M/M(a,d,b)/(2,1)/RWV. The notable notations used in this study are:

λ - Poisson arrival rate
β_k - Breakdown rate (exponentially distributed)
θ - Vacation rate (exponentially distributed)
μ - service rate of active servers in batches
μ_{rwv}- service rate of servers during working vacation in batches

The phenomenal rule followed while serving the customers are:

1. Service gets started with minimum 'a' number of customers.
2. Accessible limit less than the threshold 'd' are allowed to join the ongoing service process.
3. The maximum customers the service providers can hold is 'b'
4. If $l \geq b$, the number of customers greater than the maximum threshold then the first 'b' customers are taken up by the service provider for service.

Now, for the states (s, l) , $s = 0, 1, 2; l \geq 0$, queuing behavior follows Markov process. Here $l \geq 0$ denotes the number of customers waiting in the line and 's' denotes the stage of the service provider in the system.

1. $(0, l)$ - state with one service provider is idle and the other service provider is on working vacation, where $0 \leq l \leq d - 1$.
2. $(1, l)$ - state with one service provider is active and the other service provider is on a working vacation, where $l \geq 0$.
3. $(2, l)$ - state with both the service providers are active where $l \geq 0$.

Let us define, $P_{s\,l}(t) = P_{rb}\{\ (s,\ l) s = 0, 1, 2, ; l \geq 0$, at time t $\}$ and considering that the steady state probability values $P_{0\,l} = \lim\limits_{t\to\infty} p_{0\,l}(t)$,$P_{1\,l} = \lim\limits_{t\to\infty} p_{1\,l}(t)$ and

$P_{2\,l} = \lim_{t\to\infty} p_{2\,l}(t)$.
The balanced equations of the steady-state are

$$\lambda p_{0\,0} = \mu p_{1\,0} \tag{1}$$

$$\lambda p_{0\,l} = \lambda p_{0\,l-1} + \mu p_{1\,l} \quad (1 \le l \le a-1) \tag{2}$$

$$(\lambda + \beta_k + \mu) p_{1\,0} = \lambda \sum_{l=a}^{d} p_{0\,l-1} + 2\mu p_{2\,0} + (\mu + \mu_{rwv}) \sum_{l=a}^{d-1} p_{1\,l} \tag{3}$$

$$(\lambda + \beta_k + \mu) p_{1\,l} = (\lambda + \beta_k)\, p_{1\,l-1} + 2\mu p_{2\,l} + (\mu + \mu_{rwv}) p_{1\,l+b} \quad (1 \le l \le a-1) \tag{4}$$

$$(\lambda + \beta_k + \mu + \mu_{rwv} + \theta) p_{1l} = (\lambda + \beta_k)\, p_{1\,l-1} \quad (l \ge a) \tag{5}$$

$$(\lambda + \beta_k + 2\mu) p_{2\,0} = \theta \sum_{l=d}^{b} p_{1\,l} + 2\mu \sum_{l=d}^{b} p_{2\,l} + (\mu + \mu_{rwv}) \sum_{l=d}^{b} p_{1\,l} \tag{6}$$

$$(\lambda + \beta_k + 2\mu) p_{2\,l} = (\lambda + \beta_k)\, p_{2\,l-1} + (\mu + \mu_{rwv})\, p_{1\,l+b} + \theta p_{1\,l+b} + 2\mu p_{2\,l+b} \quad (l \ge 1) \tag{7}$$

$$\lambda p_{0\,l} = \theta p_{1\,l} + 2\mu p_{2\,l} \quad (a \le l \le d-1) \tag{8}$$

The above equations are defined to analyze the solutions of the steady-state for the described model.

3. Solutions of the Steady-State

To find the steady-state solution, let us consider E where E is the forward shifting operator defined by $E(p_{1\,l}) = p_{1\,l+1}$. Then equation 5 implies

$$\Big[(\lambda + \beta_k + \mu + \mu_{rwv} + \theta)E - (\lambda + \beta_k)\Big] p_{1\,l} = 0. \qquad (l \ge a).$$

The characteristic equation corresponding to the above equation is

$$(\lambda + \beta_k + \mu + \mu_{rwv} + \theta) Z - (\lambda + \beta_k) = 0. \tag{9}$$

It is obvious that the characteristic equation has a real root lies within the unit circle $|z| = 1$ when $\rho_1 = \dfrac{\lambda + \beta_k + \theta}{b(\mu + \mu_{rwv})} < 1$. Let it be r_0 where $r_0 = \dfrac{\lambda + \beta_k}{\lambda + \beta_k + \mu_{rwv} + \theta}$ which also satisfies $|r_0| < 1$.
Thus, the solution to the homogeneous difference equation 9 takes of the form.

$$p_{1\,l} = A r_0^l \qquad (l \ge a-1)$$

So, we get

$$p_{1\,l} = r_0^{l-a+1} p_{1\,a-1} \quad (l \geq a) \tag{10}$$

Using equation 7 we get

$$\Big[2\mu E^{b+1} - (\lambda + \beta_k + 2\mu)E + (\lambda + \beta_k)\Big] p_{2\,l} = -(\theta + \mu + \mu_{rwv}) p_{1\,l+b+1} \quad (l \geq 1)$$

which yields the characteristic equation,

$$2\mu Z^{b+1} - (\lambda + \beta_k + 2\mu)Z + (\lambda + \beta_k) = 0 \tag{11}$$

Let r_1 be the root of the above equation 11 which exists $\rho_2 = \left(\dfrac{\lambda + \beta_k}{2b\mu}\right) < 1$ with $|r_1| < 1$. As a result, the general solution to the non-homogeneous difference equation 11 takes the form

$$p_{2\,l} = (A_1 r_1^l + K r_0^l) p_{1\,a-1} \quad (l \geq 0) \tag{12}$$

where A_1 is constant and $K = \dfrac{-\,(\theta + \mu + \mu_{rwv})\, r_0^{b-a+1}}{\mu(2r_0^b - 1) + \theta + \mu_{rwv}}$

Using equation 4 and substituting for $p_{2\,l+1}$ and $p_{1\,l+b+1}$ we have after simplification.

$$p_{1\,l} = \Big[A_2 r_2^l + B_1(r_0) r_0^l + B_2(r_1) r_1^l\Big] p_{1\,a-1} \quad (0 \leq l \leq a-1) \tag{13}$$

where $r_2 = \dfrac{\lambda + \beta_k}{\lambda + \beta_k + \mu}$, $\quad B_1(r_0) = \dfrac{2\mu K r_0}{(\lambda + \beta_k + \mu)r_0 - (\lambda + \beta_k)}$ and $B_2(r_1) = \dfrac{2\mu A_1 r_1}{(\lambda + \beta_k + \mu)r_1 - (\lambda + \beta_k)}$

Now, add equation 2 over $k = 1$ to l and replace the value for $p_{1\,k}$ from equation 13 we get

$$p_{0\,l} = \frac{\mu}{\lambda}\left[A_2 \frac{1 - r_2^{l+1}}{1 - r_2} + B_1(r_0)\frac{1 - r_0^{l+1}}{1 - r_0} + B_2(r_1)\frac{1 - r_1^{l+1}}{1 - r_1}\right] p_{1\,a-1} \quad (0 \leq l \leq a-1) \tag{14}$$

From equation 8 we obtain

$$p_{0\,l} = \frac{2\mu}{\lambda}\left[\left(\frac{\theta r_0^{1-a}}{2\mu} + K\right) r_0^l + A_1 r_1^l\right] p_{1\,a-1} \quad (a \leq l \leq d-1) \tag{15}$$

Using equation 10 and 12 in 6 then we obtain

$$A_1 = \frac{E(r_0)\Big[(\theta + \mu + \mu_{rwv})\, r_0^{1-a} + 2\mu K\Big] - (\lambda + \beta_k + 2\mu)}{(\lambda + \beta_k + 2\mu) - 2\mu E(r_1)} \tag{16}$$

where $E(w) = \dfrac{w^d - w^{b+1}}{1-w}$

Further, In equation 14 replacing $l = a-1$ yields the value of A_2 as

$$A_2 = \frac{1}{r_2^{a-1}}\left[1 - B_1(r_0)r_0^{a-1} - B_2(r_1)r_1^{a-1}\right] \tag{17}$$

The normalizing condition for this queuing system is

$$\sum_{l=0}^{\infty} p_{2\,l} + \sum_{l=a}^{d-1} p_{0\,l} + \sum_{l=a}^{\infty} p_{1\,l} + \sum_{l=0}^{a-1}(p_{0\,l} + p_{1\,l}) = 1 \tag{18}$$

and substituting for $p_{2\,l}$, $p_{1\,l}$ and $p_{0\,l}$ and simplifying we get $p_{1\,a-1}$ as,

$$p_{1\,a-1}^{-1} = A_1U(r_2) + B_1(r_0)U(r_0) + B_2(r_1)U(r_1) + \left(\frac{\theta r_0^{1-a}}{2\mu} + K\right)V(r_0) + A_2V(r_1) + \frac{A_1}{1-r_1} + \frac{K}{1-r_0} + \frac{r_0}{1-r_0} \tag{19}$$

where $U(w) = \dfrac{1-w^a}{1-w} + \dfrac{\mu}{\lambda}\left(\dfrac{a}{1-w} - \dfrac{w}{1-w}\dfrac{1-w^a}{1-w}\right)$ and $V(w) = \dfrac{2\mu}{\lambda}\left(\dfrac{w^a. - w^{d+2}}{1-w}\right)$ the values of $R, B_1(r_0), B_2(r_1)$ are obtained from equation 13.

4. Performance Metrics of M/M(a,d,b)/(2,1)/RWV - Breakdown

The following are the performance metrics in relation to the queuing system M/M(a,d,b)/(2,1), which accounts for both repeated working vacations of service providers and service interruptions during operation.

1. For the proposed model, the expected length of the queue will be

$$L_q = \sum_{l=1}^{\infty} lp_{2\,l} + \sum_{l=a}^{\infty} lp_{1\,l} + \sum_{l=a}^{d-1} lp_{0\,l} + \sum_{l=1}^{a-1} l\,(p_{0\,l} + p_{1\,l})$$

Using equations 9 to 15 and simplifying, we have

$$L_q = \left[A_2I(r_2) + B_1(r_0)I(r_0) + B_2(r_1)I(r_1) + \left(\frac{\theta r_0^{1-a}}{2\mu} + K\right)J(r_0) + A_2J(r_1) + \frac{ar_0}{1-r_0} + \frac{r_0^2}{(1-r_0)^2} + \frac{A_1r_1}{(1-r_1)^2} + \frac{Kr_0}{(1-r_0)^2}\right]p_{1\,a-1}$$

where $I(w) = \left(\frac{w^{a+1} - w - aw^a(1-w)}{(1-w)^2}\right)\left(1 - \frac{\mu w}{\lambda(1-w)}\right) + \frac{\mu a(a-1)}{2\lambda(1-w)}$
and $J(w) = \frac{2\mu}{\lambda}\left[aw^a\left(\frac{1-w^{d-a}}{1-w}\right) + w^{a+1}\left(\frac{w^{d-a} - 1 - (d-a)w^{d-a-1}(1-w)}{(1-w)^2}\right)\right]$

2. Let $P_{2\,B}$ denote the probability that both service providers are active then

$$P_{2\,B} = \sum_{l=0}^{\infty} p_{2\,l} = \left(\frac{A_1}{1-r_1} + \frac{K}{1-r_0}\right) p_{1\,a-1}$$

3. Let $P_{1\,B}$ denote the probability that one of the service provider is active and the other is on working vacation, then

$$P_{1\,B} = \sum_{l=0}^{\infty} p_{1\,l} = \sum_{l=0}^{a-1} p_{1\,l} + \sum_{l=a}^{\infty} p_{1\,l} = \left(A_2 \frac{1-r_2^a}{1-r_2} + B_1(r_0)\frac{1-r_0^a}{1-r_0} + B_2(r_1)\frac{1-r_1^a}{1-r_1} + \frac{r_0}{1-r_0}\right) p_{1\,a-1}$$

4. Let $P_{0\,B}$ denote the probability that one of the service provider is idle and the other is on vacation, then

$$P_{0\,B} = \sum_{l=0}^{d-1} p_{0\,l} = \sum_{l=0}^{a-1} p_{0\,l} + \sum_{l=a}^{d-1} p_{0\,l} = \Big[A_2 G(r_2) + B_1(r_0)G(r_0) + B_2(r_1)G(r_1) + \left(\frac{\theta r_0^{1-a}}{2\mu} + K\right) X(r_0) + A_1 X(r_1)\Big] p_{1\,a-1}$$

where $G(w) = \frac{\mu}{\lambda}\left(\frac{a}{1-w} + \frac{w(1-w^a)}{(1-w)^2}\right)$, $X(w) = \frac{w^a - w^d}{1-w}$ and the values of $r_2, B_1(r_0), B_2(r_1)$ are obtained from equation 13.

5. Particular Cases

5.1. Case I

When $\mu_{rwv} = 0, d = b$ the result coincides with the classical model M/M(a,b)/(2,1)/RV with breakdown studied by K. J. R. Mary [10]. The expected length of the queue be

$$L_q = \left[A_2 I(r_2) + B_1(r_0) I(r_0) + B_2(r_1) I(r_1) + \frac{a r_0}{1 - r_0} + \frac{r_0^2}{(1 - r_0)^2} + \frac{A_1 r_1}{(1 - r_1)^2} \right.$$
$$\left. + \frac{K r_0}{(1 - r_0)^2} \right] p_{1\,a-1}$$

where $I(w) = \left(\frac{1 - w^a - a w^{a-1} (1 - w)}{(1 - w)^2} \right) \left(w - \frac{w^2 \mu}{\lambda(1 - w)} \right) + \frac{\mu a(a-1)}{2\lambda(1 - w)}$ and the values of $B_1(r_0)$, $B_2(r_0)$, A_1 and K take the form

$$B_1(r_0) = \frac{\mu k (\lambda + \beta_k)(1 - r_0)}{\theta[(\lambda + \beta_k + \mu) r_0 - (\lambda + \beta_k)]}, \quad B_2(r_1) = \frac{2\mu A_1 r_1}{(\lambda + \mu + \beta_k) r_1 - (\lambda + \beta_k)}$$

and

$$K = \frac{-\theta r_0^{b-a+2}}{(\lambda + \beta_k + 2\theta) r_0 - (\lambda + \beta_k)} \text{ and } A_1 = \frac{(1 - r_1)}{(1 - r_1^a)(1 - r_0)} \left[\frac{r_0 \theta}{2\mu} - K (1 - r_0^a) \right]$$

5.2. Case II

When $\mu_{rwv} = 0$, $d = b$ and $\beta_k = 0$, the result coincides with the classical queuing model studied by Palaniammal [11]. The expected length of the queue be

$$L_q = \left[A_2 S(r_2) + B_1(r_0) S(r_0) + B_2(r_1) S(r_1) + \frac{a r_0}{1 - r_0} + \frac{r_0^2}{(1 - r_0)^2} + \frac{A_1 r_1}{(1 - r_1)^2} \right.$$
$$\left. + \frac{K r_0}{(1 - r_0)^2} \right] p_{1\,a-1}$$

where $I(w) = \left(\frac{1 - w^a - a w^{a-1} (1 - w)}{(1 - w)^2} \right) \left(w - \frac{w^2 \mu}{\lambda(1 - w)} \right) + \frac{\mu a(a-1)}{2\lambda(1 - w)}$ and the values of $B_1(r_0)$, $B_2(r_0)$, A_1 and K take the form

$$B_1(r_0) = \frac{\mu k \lambda (1 - r_0)}{\theta[(\lambda + \mu) r_0 - \lambda]}, \quad B_2(r_1) = \frac{2\mu A_1 r_1}{(\lambda + \mu) r_1 - \lambda}, \quad K = \frac{-\theta r_0^{b-a+2}}{(\lambda + 2\theta) r_0 - \lambda}$$

and

$$A_1 = \frac{(1 - r_1)}{(1 - r_1^a)(1 - r_0)} \left[\frac{r_0 \theta}{2\mu} - K (1 - r_0^a) \right]$$

6. Sensitivity Analysis on M/M(a,d,b)/(2,1)/RWV - Breakdown

The proposed queuing model serves the best for the customers seeking service, as the system is continuously assessed by the servers as to whether they are available in the system or in the vacation epoch. Though it pretends to be an obstacle for the service facility when the system undergoes a catastrophe which ceases the ongoing service process, it may be due to several reasons. Estimating the performance metrics is valuable in achieving the customers' satisfaction and the short-term system managing goals. In view of it, a numerical outcome is presented in this section with synthetic data, and the methodological computations are verified.

For different batch sizes $a = 5, d = 10, b = 25$, $a = 10, d = 15, b = 30$ and $a = 15, d = 20, b = 40$ the length of the queue is obtained for the servers providing service at the rate of $\mu = 1$ in the usual active state and at the rate of $\mu_{rwv} = 0.3, 0.4, 0.5, 0.6$ in the vacation period which is comparatively slower to the active period. To be consider that the system also undergoes breakdown at the rate of $\beta_k = 0.25, 0.35, 0.45, 0.65$.

Table 1: L_q with respect to β_k and μ_{rwv}

μ_{rwv}	β_k	$a = 5$, $d = 10 \,\&\, b = 25$	$a = 10$, $d = 15 \,\&\, b = 30$	$a = 15$, $d = 20 \,\&\, b = 40$
0.3	0.25	8.9932	9.8497	10.9108
	0.35	9.07141	9.9405	11.0039
	0.45	9.1464	10.0328	11.0973
	0.65	9.3051	10.2158	11.2845
0.4	0.25	8.6104	9.5412	10.6770
	0.35	8.6868	9.6264	10.7667
	0.45	8.8005	9.7171	10.8575
	0.65	8.9354	9.8891	11.0379
0.5	0.25	8.3524	9.249	10.4674
	0.35	8.4243	9.3379	10.5563
	0.45	8.5019	9.4208	10.6363
	0.65	8.6458	9.5965	10.8142
0.6	0.25	8.0869	9.0087	10.2904
	0.35	8.1575	9.0921	10.3753
	0.45	8.2318	9.1736	10.4586
	0.65	8.3730	9.3412	10.6292

On observing table below 1, it is notable that the expected queue length increases in relation to the breakdown rate and decreases in relation to the working vacation rate. The queue length decreases from 8.9932 to 8.0869 for the values of μ_{rwv}, (working vacation) increases from 0.3 to 0.6 and for the breakdown rate 0.25. The queue length increases from 8.9932 to 9.3051 for the increase in breakdown rate from 0.25 to 0.65. This validates that serving customers during the vacation reduces the congestion on the system significantly, and the system congestion increases while the service is ceased by impersistent breakdown rate.

From the standpoint of varying batch sizes, significant changes can also be noted from Table 1, the length of the queue increases with an increase in minimum and maximum limit and also with the accessible threshold limit 'd' taken up for the service. This helps optimise the selection of a suitable batch size to minimize the waiting time. The graphical representation is also shown below in the following figures, Fig 1 and Fig 2.

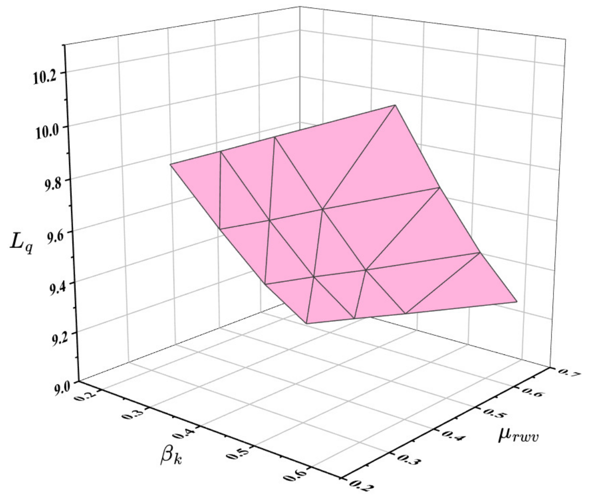

Figure 1: L_q with respect to β_k and μ_{rwv}

In the same way, the expected size of the queue is estimated for the queuing system assessing the customers with the limit of $a = 10, d = 20$ and $b = 40$. The probability of state of idleness (P_{0B}), working vacations (P_{1B}) and active periods (P_{2B}) are also estimated and tabulated in Table 2.

By considering the vacation parameter $\theta = 0.2$, and the rate of customers entering the system for getting service as $\lambda = 12, 20, 28, 36$, the expected length of the queue increases from 8.1223 to 25.2591 with the rate of occurrence of breakdown $\beta_k = 0.2$,which is tabulated in the following Table 2. In addition, the queue length also increases from 8.1223 to 8.4347 as the breakdown rate increases from $\beta_k = 0.2, 0.4, 0.6$. This result is interpreted graphically in the Figure 3.

In the following table 3, the behavior of queue is tabulated for the batch size of $a = 15$ and $b = 35$ in relation to various accessible threshold limits $d = 20, 25, 30$.

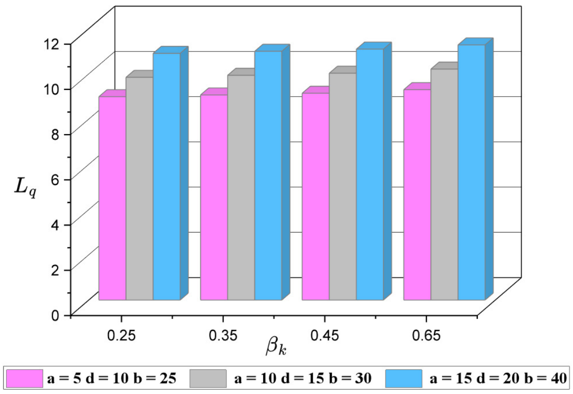

Figure 2: L_q with respect to β_k and various batch sizes

Table 2: L_q, $P_{0\,B}$, $P_{1\,B}$ and $P_{2\,B}$ with respect to β_k and λ

β_k	λ	L_q	$P_{0\,B}$	$P_{1\,B}$	$P_{2\,B}$
0.2		8.1223	0.2635	0.7102	0.04867
0.4	12	8.27856	0.2638	0.71059	0.05043
0.6		8.4347	0.2649	0.7109	0.0522
0.2		12.6465	0.1594	0.7512	0.1358
0.4	20	12.8143	0.1599	0.74993	0.1379
0.6		12.9824	0.1616	0.7486	0.1401
0.2		18.2532	0.1360	0.7065	0.2315
0.4	28	18.4400	0.1355	0.7049	0.2335
0.6		18.6362	0.1418	0.7028	0.2360
0.2		25.2591	0.1181	0.6352	0.3275
0.4	36	25.4898	0.1135	0.6332	0.3297
0.6		25.7338	0.1342	0.6308	0.3324

A significant decrease is observed from the tabulated values in terms of queue length. The queue reduces from 11.0299 to 10.7499 with $\beta_k = 0.25$ and $\mu_{rwv} = 0.3$ in relation to the accessible limit 'd'.

The following Figure 4 shows the comparison between the queue length of the system M/M(a,d,b)/(2,1)/RWV experiencing breakdown and not experiencing such an aspect. Moreover, it is evident from Figure 4, the queue length is significantly less compared to the queue length of the system experiencing breakdown.

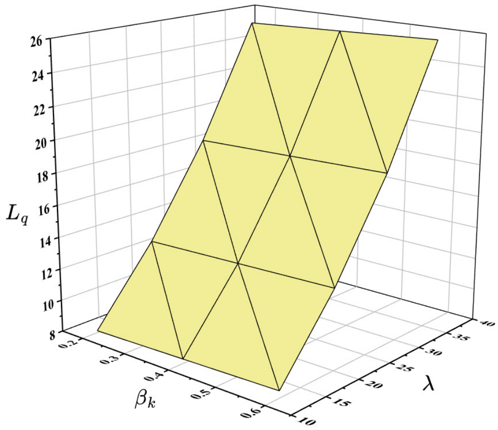

Figure 3: L_q with respect to λ and β_k

Table 3: L_q with respect to β_k and μ_{rwv} for various 'd'

β_k	μ_{rwv}	$a = 15$, $d = 20 \,\&\, b = 35$	$a = 15$ $d = 25 \,\&\, b = 35$	$a = 15$ $d = 30 \,\&\, b = 35$
0.25	0.3	11.0299	10.8332	10.7499
0.35		11.1149	10.9249	10.8376
0.45		11.1994	11.0166	10.9253
0.65		11.3958	11.2018	11.1023
0.25	0.4	10.7965	10.5877	10.5107
0.35		10.8497	10.6754	10.5943
0.45		10.9028	10.76288	10.6777
0.65		11.0859	10.9406	10.8472
0.25	0.5	10.4912	10.3703	10.2998
0.35		10.7051	10.4540	10.3795
0.45		10.9167	10.5392	10.4604
0.65		11.1042	10.7053	10.6184
0.25	0.6	10.3495	10.1838	10.1199
0.35		10.4695	10.2648	10.1968
0.45		10.5894	10.3462	10.2741
0.65		10.7697	10.5085	10.4281

The comparative analysis gives us practical insights on how operational disturbances influence the congestion dynamics of the system. Also, it offers insights for system designers and decision-makers in capacity planning, maintenance scheduling, and resource allocation. By estimating the deviation in queue performance caused by breakdowns, it aids in identifying tolerance thresholds for service reliability and optimal repair policies to overcome adverse effects. Overall, this comparative analysis strengthens the model's applicability to real-world systems

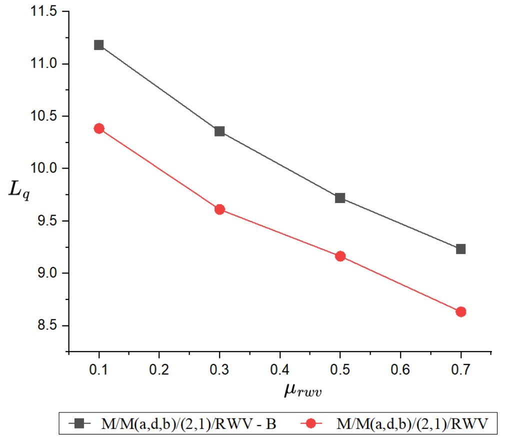

Figure 4: $M/M(a, d, b)/(2, 1)/RWV$ Vs $M/M(a, d, b)/(2, 1)/RwV - B$

where both service degradation and recovery play critical roles in determining operational efficiency.

7. Conclusion

This investigation has presented a deterministic analysis on M/M(a,d,b)/(2,1)/RWV queuing model based on the aspect of breakdown in the service facility. The parameter 'd' representing the accessible limit also facilitates the system to optimize the work-flow by clearing the queue faster. The policy of server's repeated working vacation is also interpreted in this study to capture the realistic behavior of service providers between active and idle state without ceasing the service. Performance metrics of the system such as expected queue length and probability measures are computed. A numerical result has been provided to validate the analytical conclusions to illustrate the impact of service failure, and the outcomes highlight the model's applicability in practical environments. This work can be further investigated under the circumstances of system congestion to reflect the complex real world systems.

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