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Geometric Properties of Analytic Functions Defined by the Miller–Ross-Type Poisson Distribution Series

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Abstract. In this paper, we introduce and investigate a new subclass of uniformly convex functions with negative coefficients defined by means of the Miller–Ross-type Poisson distribution series. By employing analytic techniques from geometric function theory, we derive necessary and sufficient coefficient conditions for functions to belong to this class. We establish sharp coefficient bounds and determine the extreme points representation as well as closure properties under convex combinations. Furthermore, we obtain the radii of starlikeness and convexity associated with the class. In addition, we study the behavior of partial sums and prove neighborhood inclusion results. A Fekete–Szegö type inequality is derived, and its sharpness is discussed. By specializing the defining parameters, the introduced class reduces to various well-known subclasses of uniformly convex, starlike, and convex functions with negative coefficients. Consequently, the coefficient bounds, Fekete–Szegö inequality, and radius results established here generalize and unify several earlier results in the literature.

Key Words and Phrases: analytic, starlike; convex; neighborhood; partial sums

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1. Introduction

Geometric Function Theory is one of the most significant branches of complex analysis, focusing on the geometric properties of analytic and univalent functions. Complex analysis itself plays a fundamental role in both pure and applied mathematics. Over the years, numerous researchers have investigated various geometric aspects of special subclasses of analytic and univalent functions defined in the open unit disk. In particular, considerable attention has been devoted to

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studying properties such as univalence, starlikeness, and convexity for functions associated with special functions and distribution series. These investigations continue to enrich the development of geometric function theory and its applications. In recent years, probability distributions of random variables have received considerable attention due to their wide applicability in statistics and applied sciences. Probability density functions play a central role in the study of statistical models and probabilistic structures. Several classical distributions arise naturally in real-world situations, including the binomial, Poisson, and hypergeometric distributions.

Within the framework of geometric function theory, various distribution series such as the Pascal, Poisson, logarithmic, binomial, and beta negative binomial distributions have been investigated from a theoretical perspective (see [2, 7, 19, 20, 30]). Moreover, analytic function classes associated with two-parameter Mittag?Leffler-type probability distributions have also been studied extensively (see [8, 28, 17]). These investigations reveal significant connections between probability theory and geometric properties of analytic functions. Let us now recall some known definitions and results in Geometric Function Theory.

Let $\mathcal{A}$ denote the class of all functions $u(z)$ of the form

$$u(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

in the open unit disc $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. Let S be the subclass of $\mathcal{A}$ consisting of univalent functions and satisfy the following usual normalization condition $u(0) = u'(0) - 1 = 0$. We denote by S the subclass of $\mathcal{A}$ consisting of functions $u(z)$ which are all univalent in $\mathbb{U}$. A function $u \in \mathcal{A}$ is a starlike function of the order $\xi,\ 0 \leq \xi < 1$, if it satisfies

$$\Re\left\{\frac{zu'(z)}{u(z)}\right\} > \xi, \quad z \in \mathbb{U}. \tag{2}$$

We denote this class with $S^*(\xi)$. A function $u \in \mathcal{A}$ is a convex function of the order $\xi,\ \ 0 \leq \xi < 1$, if it satisfies

$$\Re\left\{1 + \frac{zu''(z)}{u'(z)}\right\} > \xi, \quad z \in \mathbb{U}. \tag{3}$$

We denote this class with $K(\xi)$. Note that $S^*(0) = S^*$ and $K(0) = K$ are the usual classes of starlike and convex functions in $\mathbb{U}$, respectively. For $u \in \mathcal{A}$ given by (1) and $g(z)$ given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \tag{4}$$

their convolution (or Hadamard product), denoted by $(u * g)$, is defined as

$$(u * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (g * u)(z), \quad (z \in \mathbb{U}). \tag{5}$$

Note that $u * g \in \mathcal{A}$.
Let T denotes the class of functions analytic in $\mathbb{U}$ that are of the form

$$u(z) = z - \sum_{n=2}^{\infty} a_n z^n, \quad a_n \geq 0 \ \ (z \in \mathbb{U}) \tag{6}$$

and let $T^*(\xi) = T \cap S^*(\xi)$, $C(\xi) = T \cap K(\xi)$. The class $T^*(\xi)$ and allied classes possess some interesting properties and have been extensively studied by Silverman [25].

Miller and Ross proposed the following special function in their monograph (p. 314, [16]), which is now called the Miller-Ross function, defined as

$$E_{\nu,c}(z) = z^{\nu} e^{cz} \gamma^*(\nu, cz),$$

where γ^* is the incomplete gamma function. Using the properties of the incomplete gamma functions the Miller Ross function can easily be written as

$$E_{\nu,\ c}(z) = z^{\nu} \sum_{n=0}^{\infty} \frac{(cz)^n}{\Gamma(n+\nu+1)}; \quad z, c, \nu \in \mathbb{C}. \tag{7}$$

In this paper, we shall restrict our attention to the case of real-valued $c > 0$ and $z \in \mathbb{U}$. It is clear that the Miller Ross function $E_{\nu,\ c}(z)$ does not belong to the family $\mathcal{A}$. Thus, it is natural to consider the following normalization of Miller Ross function [5]:

$$\begin{aligned} E_{\nu,c}(z) &= z^{1-\nu}\Gamma(\nu+1)E_{\nu,\ c}(z) \\ &= z + \sum_{n=2}^{\infty} \frac{c^{n-1}\Gamma(\nu+1)}{\Gamma(n+\nu)} z^n \end{aligned} \tag{8}$$

For $c, \nu \in \mathbb{C}$, we can write the following

$$E_{\nu,c}(1) - 1 = \sum_{n=2}^{\infty} \frac{c^{n-1}\Gamma(\nu+1)}{\Gamma(n+\nu)},$$

$$E'_{\nu,c}(1) - 1 = \sum_{n=2}^{\infty} \frac{n\ c^{n-1}\Gamma(\nu+1)}{\Gamma(n+\nu)},$$
$$E''_{\nu,c}(1) = \sum_{n=2}^{\infty} \frac{n(n-1)c^{n-1}\Gamma(\nu+1)}{\Gamma(n+\nu)}.$$

In recent years, a large literature has evolved on the use of distribution series such as Poisson, Pascal, Borel, etc., in geometric function theory. Many researchers have examined some important features in the field of geometric function theory, such as coefficient estimates, inclusion relations, and conditions of being in some known classes, using different probability distributions, see for example [10 -15]. We now recall that a discrete random variable X whose probability mass function is given by

$$P[X = i] = \frac{e^{-m}m^i}{i!}, \quad i = 0, 1, 2\cdots, \quad m > 0$$

is said to have a Poisson distribution with parameter m.

Recently, Porwal and Dixit [21] introduced Mittag Leffler-type Poisson distribution and obtained moments, moment generating function. Bajpai [3] introduced Mittag Leffler-type Poisson distribution series. Lately, Srivastava et al.[29] introduced the Poisson distribution, a two-parameter Mittag Leffler-type Poisson distribution. Motivated by results on connections between various subclasses of analytic univalent functions using special functions and distribution series [10, 28, 23, 24, 11] we obtain coefficient inequalities, distortion theorem, radii of starlike, convex, convex linear combination and convolution property for the Miller Ross-type Poisson distribution series to be in classes. First, we recall the definition of the Miller Ross-type distribution.
Gives the probability mass function of the Miller Ross-type Poisson distribution

$$P_{\nu,c}(m,n) = \frac{m^{\nu}(cm)^n}{E_{\nu,c}(m)\Gamma(n+\nu+1)}, \quad n = 0, 1, 2, \cdots, \tag{9}$$

where $\nu > -1, c > 0$ and $E_{\nu,\ c}(z)$ is Miller-Ross function given in (7).
The Miller-Ross-type Poisson distribution series is defined by

$$\mathbb{F}^m_{\nu,c}(z) = z + \sum_{n=2}^{\infty} \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)E_{\nu,c}(m)} z^n, \quad z \in \mathbb{U}. \tag{10}$$

(see [18], see also [22]). Furthermore, using the convolution (or Hadamard product), we define

$$\mathbb{F}^m_{\nu,c}u(z) = \mathbb{F}^m_{\nu,c}(z) * u(z)$$

$$= z + \sum_{n=2}^{\infty} \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)E_{\nu,c}(m)} a_n z^n$$

$$= z + \sum_{n=2}^{\infty} \Phi_c^{\nu}(n,m) a_n z^n, \tag{11}$$

where

$$\Phi_c^{\nu}(n,m) = \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)E_{\nu,c}(m)}. \tag{12}$$

Inspired by the work of [1, 6, 9, 12, 15, 18], we introduce the new subclass involving Miller -Ross -type Poisson distribution series $\mathbb{F}_{\nu,c}^{m}u(z)$, as below:

Definition 1. *For $-1 \leq \upsilon < 1$, and $0 \leq \sigma < 1$, we let $S_{\nu,c}^{m}(\sigma,\upsilon)$ be the subclass of $\mathcal{A}$ consisting of functions of the form* (1) *and satisfying the analytic criterion*

$$\Re\left\{ \frac{z(\mathbb{F}_{\nu,c}^{m}u(z))' + \sigma z^2(\mathbb{F}_{\nu,c}^{m}u(z))''}{(1-\sigma)\mathbb{F}_{\nu,c}^{m}u(z) + \sigma z(\mathbb{F}_{\nu,c}^{m}u(z))'} - \upsilon \right\} \geq \left| \frac{z(\mathbb{F}_{\nu,c}^{m}u(z))' + \sigma z^2(\mathbb{F}_{\nu,c}^{m}u(z))''}{(1-\sigma)\mathbb{F}_{\nu,c}^{m}u(z) + \sigma z(\mathbb{F}_{\nu,c}^{m}u(z))'} - 1 \right|, \tag{13}$$

for $z \in \mathbb{U}$.

We also, let $TS_{\nu,c}^{m}(\sigma,\upsilon) = T \cap S_{\nu,c}^{m}(\sigma,\upsilon)$.

The main objective of this paper is to investigate several fundamental properties in geometric function theory for the newly introduced class. In particular, we establish sharp coefficient bounds, determine the extreme points and closure properties, and obtain the radii of starlikeness and convexity. Furthermore, we analyze the behavior of partial sums and derive neighborhood inclusion results for functions belonging to the class. In addition, a Fekete?Szeg? type inequality is obtained, and its sharpness is discussed. These results collectively provide a comprehensive geometric characterization of the class under consideration.

2. Coefficient Bounds

Throughout this section, we assume that $m > 0$, $c > 0$, and $\nu > -1$. Then, for $n \geq 2$,

$$\Phi_c^{\nu}(n,m) = \frac{m^{\nu}(cm)^{n-1}}{\Gamma(n+\nu)\,E_{\nu,c}(m)} > 0.$$

Hence, all series considered below have non-negative coefficients. In this section, we derive necessary and sufficient conditions for a function $u(z)$ to belong to the classes $S_{\nu,c}^{m}(\sigma,\upsilon)$ and $TS_{\nu,c}^{m}(\sigma,\upsilon)$.

Theorem 1. *Let u be defined by* (1). *Then $u \in S^m_{\nu,c}(\sigma, \upsilon)$ if*

$$\sum_{n=2}^{\infty}[1+\sigma(n-1)](2n-\upsilon-1)\Phi^{\nu}_{c}(n,m)a_n \leq 1-\upsilon, \tag{14}$$

where $-1 \leq \upsilon < 1$, $0 \leq \sigma \leq 1$, *and*

$$\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi^{\nu}_{c}(n,m)a_n < 1. \tag{15}$$

Proof. Let

$$W(z) = \frac{z(\mathbb{F}^m_{\nu,c}u(z))' + \sigma z^2(\mathbb{F}^m_{\nu,c}u(z))''}{(1-\sigma)\mathbb{F}^m_{\nu,c}u(z) + \sigma z(\mathbb{F}^m_{\nu,c}u(z))'}.$$

It suffices to prove that

$$|W(z)-1| - \Re\{W(z)-1\} \leq 1-\upsilon.$$

Using the inequality $|w| - \Re(w) \leq 2|w|$, we obtain

$$|W(z)-1| - \Re\{W(z)-1\} \leq 2|W(z)-1|.$$

From the series representation, we have

$$W(z) - 1 = \frac{\sum_{n=2}^{\infty}(n-1)[1+\sigma(n-1)]\Phi^{\nu}_{c}(n,m)a_n z^{n-1}}{1-\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi^{\nu}_{c}(n,m)a_n z^{n-1}}.$$

By condition , we have

$$\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi^{\nu}_{c}(n,m)a_n < 1.$$

Hence, for $|z| < 1$,

$$\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi^{\nu}_{c}(n,m)a_n z^{n-1} \leq \sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi^{\nu}_{c}(n,m)a_n < 1,$$

which implies

$$1-\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi^{\nu}_{c}(n,m)a_n z^{n-1} > 0.$$

Thus, the denominator is strictly positive. Taking modulus and applying the triangle inequality, we obtain

$$|W(z)-1| \leq \frac{\sum_{n=2}^{\infty}(n-1)[1+\sigma(n-1)]\Phi_c^{\nu}(n,m)a_n}{1-\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi_c^{\nu}(n,m)a_n}.$$

Thus,

$$|W(z)-1| - \Re\{W(z)-1\} \leq 2\frac{\sum_{n=2}^{\infty}(n-1)[1+\sigma(n-1)]\Phi_c^{\nu}(n,m)a_n}{1-\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi_c^{\nu}(n,m)a_n}.$$

Since $-1 \leq \upsilon < 1$, we have

$$2(n-1) \leq (2n-\upsilon-1),$$

and hence

$$2(n-1)[1+\sigma(n-1)] \leq [1+\sigma(n-1)](2n-\upsilon-1).$$

Therefore,

$$2\sum_{n=2}^{\infty}(n-1)[1+\sigma(n-1)]\Phi_c^{\nu}(n,m)a_n \leq \sum_{n=2}^{\infty}[1+\sigma(n-1)](2n-\upsilon-1)\Phi_c^{\nu}(n,m)a_n.$$

Using condition (14), we obtain

$$|W(z)-1| - \Re\{W(z)-1\} \leq 1-\upsilon.$$

Hence $u \in S_{\nu,c}^{m}(\sigma,\upsilon)$.

Theorem 2. *A necessary and sufficient condition for $u(z)$ of the form (6) to belong to the class $TS_{\nu,c}^{m}(\sigma,\upsilon)$ is that*

$$\sum_{n=2}^{\infty}[1+\sigma(n-1)](2n-\upsilon-1)\Phi_c^{\nu}(n,m)a_n \leq 1-\upsilon, \tag{16}$$

where $-1 \leq \upsilon < 1$ and $0 \leq \sigma \leq 1$, together with

$$\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi_c^{\nu}(n,m)a_n < 1. \tag{17}$$

Proof. The sufficiency follows from Theorem 1. For necessity, assume that $u \in TS^m_{\nu,c}(\sigma, \upsilon)$. Since $a_n \geq 0$, we have

$$\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi^{\nu}_c(n,m)a_n z^{n-1} \leq \sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi^{\nu}_c(n,m)a_n < 1,$$

for $z \in (0,1)$. Hence,

$$1-\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi^{\nu}_c(n,m)a_n z^{n-1} > 0,$$

and the denominator is strictly positive. We write

$$W(z)-1 = \frac{\sum_{n=2}^{\infty}(n-1)[1+\sigma(n-1)]\Phi^{\nu}_c(n,m)a_n z^{n-1}}{1-\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi^{\nu}_c(n,m)a_n z^{n-1}}.$$

Since all quantities are real, the class condition implies

$$W(z)-1 \geq -(1-\upsilon).$$

Rearranging, we obtain

$$\sum_{n=2}^{\infty}[1+\sigma(n-1)](2n-\upsilon-1)\Phi^{\nu}_c(n,m)a_n z^{n-1} \leq 1-\upsilon.$$

Letting $z \to 1^-$, we obtain (16).

Remark 1. *All estimates are valid under the condition*

$$\sum_{n=2}^{\infty}[1+\sigma(n-1)]\Phi^{\nu}_c(n,m)a_n < 1,$$

which guarantees that the denominator does not vanish.

Remark 2. *The above results are valid under the assumptions $m > 0$, $c > 0$, and $\nu > -1$, which ensure that $\Phi^{\nu}_c(n,m) > 0$ for all $n \geq 2$. Consequently, all terms in* (16) *are non-negative. Moreover, the growth of $\Phi^{\nu}_c(n,m)$ with respect to m is controlled by the denominator $E_{\nu,c}(m)$, which increases rapidly for large m. Therefore, the series in* (16) *remains non-negative and bounded for admissible parameter values. For example, in the special case $c = 1$, $\nu = 0$, and $a_2 = 1$, we obtain*

$$\Phi^{0}_1(2,m) = \frac{m}{E_{0,1}(m)} > 0,$$

and hence the left-hand side of (16) *remains positive for all $m > 0$. Thus, no contradiction arises for large values of m, and the inequalities hold within the stated parameter range.*

Theorem 3. *Let $u(z)$ defined by (6) and $g(z) = z - \sum_{n=2}^{\infty} b_n z^n$ be in the class $TS_{\nu,c}^{m}(\sigma, \upsilon)$. Then the function $h(z)$ defined by*

$$h(z) = (1-\zeta)u(z) + \zeta g(z) = z - \sum_{n=2}^{\infty} c_n z^n,$$

where $c_n = (1-\zeta)a_n + \zeta b_n, 0 \leq \zeta < 1$ is also in the class $TS_{\nu,c}^{m}(\sigma, \upsilon)$.

Proof. Let the function

$$u_j = z - \sum_{n=2}^{\infty} a_{n,j} z^n, \ a_{n,j} \geq 0, \ j = 1, 2, \tag{18}$$

be in the class $TS_{\nu,c}^{m}(\sigma, \upsilon)$. It is sufficient to show that the function $g(z)$ defined by

$$g(z) = \zeta u_1(z) + (1-\zeta)u_2(z), \quad 0 \leq \zeta \leq 1,$$

is in the class $TS_{\nu,c}^{m}(\sigma, \upsilon)$. Since

$$g(z) = z - \sum_{n=2}^{\infty} [\zeta a_{n,1} + (1-\zeta)a_{n,2}] z^n,$$

an easy computation with the aid of Theorem 2 gives,

$$\begin{aligned}
&\sum_{n=2}^{\infty} [1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^{\nu}(n,m)\zeta a_{n,1} \\
&\quad + \sum_{n=2}^{\infty} [1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^{\nu}(n,m)(1-\zeta)a_{n,2} \\
&\leq \zeta(1-\upsilon) + (1-\zeta)(1-\upsilon) \\
&\leq 1-\upsilon,
\end{aligned}$$

which implies that $g \in TS_{\nu,c}^{m}(\sigma, \upsilon)$.

Hence $TS_{\nu,c}^{m}(\sigma, \upsilon)$ is convex.

3. Extreme points

The proof of Theorem 4, follows on lines similar to the proof of the theorem on extreme points given in Silverman [25].

Theorem 4. *Let* $u_1(z) = z$ *and*

$$u_n(z) = z - \frac{1-\upsilon}{[1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^\nu(n,m)} z^n, \tag{19}$$

for $n = 2, 3, \cdots$. *Then* $u(z) \in TS_{\nu,c}^m(\sigma, \upsilon)$ *if and only if* $u(z)$ *can be expressed in the form* $u(z) = \sum\limits_{n=2}^{\infty} \zeta_n u_n(z)$, *where* $\zeta_n \geq 0$ *and* $\sum\limits_{n=1}^{\infty} \zeta_n = 1$.

Next, we prove the following closure theorem.

4. Closure theorem

Theorem 5. *Let the function* $u_j(z), j = 1, 2, \cdots, l$ *defined by* (18) *be in the classes* $TS_{\nu,c}^m(\sigma, \upsilon_j)$, $j = 1, 2, \cdots, l$ *respectively. Then the function* $h(z)$ *defined by*

$$h(z) = z - \frac{1}{l} \sum_{n=2}^{\infty} \left(\sum_{j=1}^{l} a_{n,j} \right) z^n$$

is in the class $TS_{\nu,c}^m(\sigma, \upsilon)$, *where* $\upsilon = \min\limits_{1 \leq j \leq l} \{\upsilon_j\}$, *where* $-1 \leq \upsilon_j \leq 1$.

Proof. Since $u_j(z) \in TS_{\nu,c}^m(\sigma, \upsilon_j)$, $j = 1, 2, \cdots, l$ by applying Theorem 2 to (18), we observe that

$$\begin{aligned}
&\sum_{n=2}^{\infty} [1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^\nu(n,m) \left(\frac{1}{l} \sum_{j=1}^{l} a_{n,j} \right) \\
=&\frac{1}{l} \sum_{j=1}^{l} a_{n,j} \left(\sum_{n=2}^{\infty} [1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^\nu(n,m) a_{n,j} \right) \\
\leq&\frac{1}{l} \sum_{j=1}^{l} (1-\upsilon_j) \\
\leq&1-\upsilon
\end{aligned}$$

which in view of Theorem 2, again implies that $h(z) \in TS_{\nu,c}^m(\sigma, \upsilon)$ and so the proof is complete.

Theorem 6. *Let* $u \in TS_{\nu,c}^m(\sigma, \upsilon)$. *Then*

(1). u is starlike of order δ, $0 \le \delta < 1$, *in the disc* $|z| < r_1$ *i.e.,* $\Re\left\{\frac{zu'(z)}{u(z)}\right\} > \delta$, $|z| < r_1$, *where*

$$r_1 = \inf_{n\ge 2}\left\{\left(\frac{1-\delta}{n-\delta}\right)\frac{[1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^{\nu}(n,m)}{1-\upsilon}\right\}^{\frac{1}{n-1}}.$$

(2). u is convex of order δ, $0 \le \delta < 1$, *in the disc* $|z| < r_1$ *i.e.,* $\Re\left\{1+\frac{zu''(z)}{u'(z)}\right\} > \delta$, $|z| < r_2$, *where*

$$r_2 = \inf_{n\ge 2}\left\{\left(\frac{1-\delta}{n(n-\delta)}\right)\frac{[1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^{\nu}(n,m)}{1-\upsilon}\right\}^{\frac{1}{n-1}}.$$

Each result is sharp for the extremal function $u(z)$ *given by* (19)*.*

Proof. Given $u \in \mathcal{A}$ and u is starlike of order δ, we have

$$\left|\frac{zu'(z)}{u(z)} - 1\right| < 1-\delta. \tag{20}$$

For the left hand side (20), we have

$$\left|\frac{zu'(z)}{u(z)} - 1\right| \le \frac{\sum\limits_{n=2}^{\infty}(n-1)a_n|z|^{n-1}}{1-\sum\limits_{n=2}^{\infty}a_n|z|^{n-1}}.$$

The last expression is less than $1-\delta$ if

$$\sum_{n=2}^{\infty}\frac{n-\delta}{1-\delta}a_n|z|^{n-1} < 1.$$

Using the fact, that $u \in TS_{\nu,c}^{m}(\sigma,\upsilon)$ if and only if

$$\sum_{n=2}^{\infty}\frac{[1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^{\nu}(n,m)}{1-\upsilon}a_n < 1.$$

We can say (20) is true if

$$\frac{n-\delta}{1-\delta}|z|^{n-1} < \frac{[1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^{\nu}(n,m)}{1-\upsilon}$$

Or equivalently,

$$|z|^{n-1} < \frac{(1-\delta)[1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^{\nu}(n,m)}{(n-\delta)(1-\upsilon)}$$

which yields the star likeness of the family.

(2). Using the fact that u is convex if and only if zu' is starlike, we can prove (2), on lines similar to the proof of (1).

5. Partial Sums

Following the earlier works of Silverman [26] and Silvia [27] on partial sums of analytic functions, we consider in this section the partial sums of functions belonging to the class $S_{\nu,c}^{m}(\sigma,\upsilon)$ and obtain sharp lower bounds for the real parts of the ratios $\frac{u(z)}{u_q(z)}$, $\frac{u_q(z)}{u(z)}$, $\frac{u'(z)}{u_q'(z)}$ and $\frac{u_q'(z)}{u'(z)}$.

Theorem 7. *Let*

$$u(z) = z + \sum_{n=2}^{\infty} a_n z^n \in S_{\nu,c}^{m}(\sigma,\upsilon),$$

and define the partial sums by

$$u_q(z) = z + \sum_{n=2}^{q} a_n z^n, \qquad q \in \mathbb{N}.$$

Suppose that

$$\sum_{n=2}^{\infty} d_n |a_n| \leq 1,$$

where

$$d_n = \frac{[1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^{\nu}(n,m)}{1-\upsilon}.$$

Then

$$\Re\left(\frac{u(z)}{u_q(z)}\right) > 1 - \frac{1}{d_{q+1}}, \tag{21}$$

$$\Re\left(\frac{u_q(z)}{u(z)}\right) > \frac{d_{q+1}}{1+d_{q+1}}, \tag{22}$$

for $z \in \mathbb{U}$. Both bounds are sharp.

Proof. Since $0 \le \sigma < 1$, $-1 \le \upsilon < 1$ and $\Phi_c^\nu(n,m) > 0$, it follows that d_n is strictly increasing and

$$d_{n+1} > d_n > 1, \qquad n \ge 2.$$

Hence

$$\sum_{n=2}^{q} |a_n| + d_{q+1} \sum_{n=q+1}^{\infty} |a_n| \le \sum_{n=2}^{\infty} d_n |a_n| \le 1.$$

Now,

$$\frac{u(z)}{u_q(z)} = 1 + \frac{\sum_{n=q+1}^{\infty} a_n z^n}{z + \sum_{n=2}^{q} a_n z^n}.$$

Define

$$g_1(z) = d_{q+1}\left[\frac{u(z)}{u_q(z)} - \left(1 - \frac{1}{d_{q+1}}\right)\right].$$

Then

$$g_1(z) = 1 + \frac{d_{q+1}\sum_{n=q+1}^{\infty} a_n z^{n-1}}{1 + \sum_{n=2}^{q} a_n z^{n-1}}.$$

Thus

$$\left|\frac{g_1(z)-1}{g_1(z)+1}\right| \le \frac{d_{q+1}\sum_{n=q+1}^{\infty} |a_n|}{2 - 2\sum_{n=2}^{q} |a_n| - d_{q+1}\sum_{n=q+1}^{\infty} |a_n|} \le 1,$$

which implies $\Re g_1(z) \ge 0$ and hence

$$\Re\left(\frac{u(z)}{u_q(z)}\right) > 1 - \frac{1}{d_{q+1}}.$$

The second inequality follows similarly by defining

$$g_2(z) = (1 + d_{q+1})\left[\frac{u_q(z)}{u(z)} - \frac{d_{q+1}}{1 + d_{q+1}}\right],$$

and proceeding analogously.

Sharpness follows for the extremal function

$$u(z) = z + \frac{z^{q+1}}{d_{q+1}}.$$

Theorem 8. *Under the same hypotheses,*

$$\Re\left(\frac{u'(z)}{u_q'(z)}\right) > 1 - \frac{q+1}{d_{q+1}}, \qquad z \in \mathbb{U}.$$

The result is sharp.

Proof. We have

$$u'(z) = 1 + \sum_{n=2}^{\infty} n a_n z^{n-1}, \qquad u_q'(z) = 1 + \sum_{n=2}^{q} n a_n z^{n-1}.$$

Hence

$$\frac{u'(z)}{u_q'(z)} = 1 + \frac{\sum_{n=q+1}^{\infty} n a_n z^{n-1}}{1 + \sum_{n=2}^{q} n a_n z^{n-1}}.$$

Define

$$g(z) = \frac{d_{q+1}}{q+1}\left[\frac{u'(z)}{u_q'(z)} - \left(1 - \frac{q+1}{d_{q+1}}\right)\right].$$

Then

$$g(z) = 1 + \frac{\frac{d_{q+1}}{q+1}\sum_{n=q+1}^{\infty} n a_n z^{n-1}}{1 + \sum_{n=2}^{q} n a_n z^{n-1}}.$$

Since $d_n \geq n$ for all $n \geq 2$, we obtain

$$\sum_{n=2}^{q} n|a_n| + \frac{d_{q+1}}{q+1}\sum_{n=q+1}^{\infty} n|a_n| \leq \sum_{n=2}^{\infty} d_n|a_n| \leq 1.$$

Hence

$$\left|\frac{g(z)-1}{g(z)+1}\right| \leq 1,$$

which implies $\Re g(z) \geq 0$ and gives the desired result.

Sharpness follows for

$$u(z) = z + \frac{z^{q+1}}{d_{q+1}}.$$

Theorem 9. *Under the same hypotheses,*

$$\Re\left(\frac{u_q'(z)}{u'(z)}\right) > \frac{d_{q+1}}{q+1+d_{q+1}}, \qquad z \in \mathbb{U}.$$

The result is sharp.

Proof. We write

$$\frac{u_q'(z)}{u'(z)} = 1 - \frac{\sum_{n=q+1}^{\infty} n a_n z^{n-1}}{1 + \sum_{n=2}^{\infty} n a_n z^{n-1}}.$$

Define

$$g(z) = (q+1+d_{q+1}) \left[\frac{u_q'(z)}{u'(z)} - \frac{d_{q+1}}{q+1+d_{q+1}} \right].$$

Then

$$g(z) = 1 - \frac{\left(1 + \frac{d_{q+1}}{q+1}\right) \sum_{n=q+1}^{\infty} n a_n z^{n-1}}{1 + \sum_{n=2}^{\infty} n a_n z^{n-1}}.$$

Using again that $d_n \geq n$ and

$$\sum_{n=2}^{\infty} d_n |a_n| \leq 1,$$

we obtain

$$\left| \frac{g(z)-1}{g(z)+1} \right| \leq 1,$$

which yields the required result.

Sharpness follows for

$$u(z) = z + \frac{z^{q+1}}{d_{q+1}}.$$

6. Neighbourhoods for the class $S_{\nu,c}^{m,\xi}(\sigma, \upsilon)$

In this section, we determine neighbourhood properties of the class $S_{\nu,c}^{m}(\sigma, \upsilon)$.

Definition 2. *A function $u \in \mathcal{A}$ is said to belong to the class $S_{\nu,c}^{m;\xi}(\sigma, \upsilon)$ if there exists a function $g \in S_{\nu,c}^{m}(\sigma, \upsilon)$ such that*

$$\left| \frac{u(z)}{g(z)} - 1 \right| < 1 - \xi, \qquad (z \in \mathbb{U},\ 0 \leq \xi < 1). \tag{23}$$

For a function $u(z) = z + \sum_{n=2}^{\infty} a_n z^n \in \mathcal{A}$ and $\delta \geq 0$, we define the δ neighbourhood of u by

$$N_\delta(u) = \left\{ g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{A} : \sum_{n=2}^{\infty} n|a_n - b_n| \leq \delta \right\}. \tag{24}$$

The concept of neighbourhoods was introduced by Goodman [13] and later generalized by Ruscheweyh [22].

Theorem 10. *Let* $g \in S^{m}_{\nu,c}(\sigma, \upsilon)$ *and suppose that*

$$d_n = \frac{[1+\sigma(n-1)][2n-\upsilon-1]\Phi^{\nu}_{c}(n,m)}{1-\upsilon}, \qquad n \geq 2.$$

If

$$\xi = 1 - \frac{\delta(1-\upsilon)}{2\Big[(1-\upsilon)-(1+\sigma)(3-\upsilon)\Phi^{\nu}_{c}(2,m)\Big]},$$

and

$$(1-\upsilon) > (1+\sigma)(3-\upsilon)\Phi^{\nu}_{c}(2,m),$$

then

$$N_{\delta}(g) \subset S^{m,\xi}_{\nu,c}(\sigma,\upsilon).$$

Proof. Let $u \in N_{\delta}(g)$. Then from (24) we have

$$\sum_{n=2}^{\infty} n|a_n - b_n| \leq \delta. \tag{25}$$

Since $n \geq 2$, it follows that

$$\sum_{n=2}^{\infty} |a_n - b_n| \leq \frac{\delta}{2}. \tag{26}$$

Now, since $g \in S^{m}_{\nu,c}(\sigma,\upsilon)$, the main coefficient theorem gives

$$\sum_{n=2}^{\infty} d_n|b_n| \leq 1.$$

Because d_n is increasing for $n \geq 2$, we have $d_n \geq d_2$ and therefore

$$\sum_{n=2}^{\infty} |b_n| \leq \frac{1}{d_2}.$$

Since

$$d_2 = \frac{(1+\sigma)(3-\upsilon)\Phi^{\nu}_{c}(2,m)}{1-\upsilon},$$

we obtain

$$\sum_{n=2}^{\infty} |b_n| \leq \frac{1-\upsilon}{(1+\sigma)(3-\upsilon)\Phi^{\nu}_{c}(2,m)}. \tag{27}$$

Next,

$$\left|\frac{u(z)}{g(z)}-1\right|=\left|\frac{u(z)-g(z)}{g(z)}\right|\leq\frac{\sum_{n=2}^{\infty}|a_n-b_n|}{1-\sum_{n=2}^{\infty}|b_n|}.$$

Using (26) and (27), we obtain

$$\left|\frac{u(z)}{g(z)}-1\right|\leq\frac{\delta/2}{1-\frac{1-\upsilon}{(1+\sigma)(3-\upsilon)\Phi_c^{\nu}(2,m)}}.$$

After simplification, this becomes

$$\left|\frac{u(z)}{g(z)}-1\right|=\frac{\delta(1-\upsilon)}{2\Big[(1-\upsilon)-(1+\sigma)(3-\upsilon)\Phi_c^{\nu}(2,m)\Big]}.$$

Thus,

$$\left|\frac{u(z)}{g(z)}-1\right|<1-\xi,$$

provided ξ has the stated form. This completes the proof.

7. Fekete–Szegö Inequality

In this section, we obtain the Fekete–Szegö functional estimate for the class $S_{\nu,c}^{m}(\sigma,\upsilon)$.

Theorem 11. *Let*

$$u(z)=z+\sum_{n=2}^{\infty}a_nz^n\in S_{\nu,c}^{m}(\sigma,\upsilon).$$

Then for any real number μ*,*

$$|a_3-\mu a_2^2|\leq\max\left\{\frac{1}{d_3},\frac{|\mu|}{d_2^2}\right\},$$

where

$$d_n=\frac{[1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^{\nu}(n,m)}{1-\upsilon}.$$

The result is sharp.

Proof.
Since

$$\sum_{n=2}^{\infty}d_n|a_n|\leq1,$$

we immediately obtain the individual bounds

$$|a_2| \le \frac{1}{d_2}, \qquad |a_3| \le \frac{1}{d_3}.$$

Now consider the functional

$$|a_3 - \mu a_2^2|.$$

Using the triangle inequality,

$$|a_3 - \mu a_2^2| \le |a_3| + |\mu||a_2|^2.$$

Thus,

$$|a_3 - \mu a_2^2| \le \frac{1}{d_3} + \frac{|\mu|}{d_2^2}.$$

However, sharper estimates follow by considering extremal cases.

Case 1: $a_2 = 0$.

Then

$$|a_3 - \mu a_2^2| = |a_3| \le \frac{1}{d_3}.$$

Equality is attained for

$$u(z) = z + \frac{z^3}{d_3}.$$

Case 2: $a_3 = 0$.

Then

$$|a_3 - \mu a_2^2| = |\mu|\,|a_2|^2 \le \frac{|\mu|}{d_2^2}.$$

Equality is attained for

$$u(z) = z + \frac{z^2}{d_2}.$$

Since the coefficient region is convex, the maximum of $|a_3 - \mu a_2^2|$ is attained at these extremal points. Therefore,

$$|a_3 - \mu a_2^2| \le \max\left\{\frac{1}{d_3}, \frac{|\mu|}{d_2^2}\right\}.$$

The result is sharp.

Sharpness. Equality is attained for the functions

$$u_1(z) = z + \frac{z^3}{d_3} \quad \text{and} \quad \nu_2(z) = z + \frac{z^2}{d_2},$$

which satisfy the coefficient condition

$$\sum_{n=2}^{\infty} d_n|a_n| = 1.$$

For ν_1,

$$|a_3 - \mu a_2^2| = \frac{1}{d_3},$$

while for ν_2,

$$|a_3 - \mu a_2^2| = \frac{|\mu|}{d_2^2}.$$

Hence the bound cannot be improved.

8. Fekete–Szegö Inequality (Carathéodory Method)

Theorem 12. *Let*

$$u(z) = z + \sum_{n=2}^{\infty} a_n z^n \in S_{\nu,c}^{m}(\sigma, \upsilon),$$

and let $\mu \in \mathbb{R}$. *Then*

$$|a_3 - \mu a_2^2| \leq \frac{1-\upsilon}{d_3} \max\{1, |2\Theta - 1|\},$$

where

$$\Theta = \frac{1}{2}\left(1 - \frac{d_3}{d_2^2}\right) - \mu\frac{(1-\upsilon)d_3}{2d_2^2},$$

and

$$d_n = \frac{[1+\sigma(n-1)][2n-\upsilon-1]\Phi_c^{\nu}(n,m)}{1-\upsilon}.$$

The result is sharp.

Proof.
From the defining condition of the class $S_{\nu,c}^{m}(\sigma, \upsilon)$, the function

$$H(z) = \frac{z(\mathbb{L}_{\nu,c}^{m}u(z))' + \sigma z^2(\mathbb{L}_{\nu,c}^{m}u(z))''}{(1-\sigma)\mathbb{L}_{\nu,c}^{m}u(z) + \sigma z(\mathbb{L}_{\nu,c}^{m}u(z))'}$$

satisfies

$$\Re\{H(z) - \upsilon\} \geq |H(z) - 1|.$$

This region is a disk contained in the right half?plane. Hence there exists a Carathéodory function

$$p(z) = 1 + c_1 z + c_2 z^2 + \cdots, \qquad \Re p(z) > 0,$$

such that

$$H(z) = \frac{1 + (1-\upsilon)p(z)}{2}.$$

Expanding $H(z)$ and comparing coefficients yields

$$a_2 = \frac{1-\upsilon}{2d_2} c_1,$$

and

$$a_3 = \frac{1-\upsilon}{2d_3}\left(c_2 + \lambda c_1^2\right),$$

where

$$\lambda = \frac{1}{2}\left(1 - \frac{d_3}{d_2^2}\right).$$

We compute

$$a_3 - \mu a_2^2.$$

Substituting the above expressions gives

$$a_3 - \mu a_2^2 = \frac{1-\upsilon}{2d_3}\left[c_2 + \left(\lambda - \mu\frac{(1-\upsilon)d_3}{2d_2^2}\right)c_1^2\right].$$

Set

$$\Theta = \lambda - \mu\frac{(1-\upsilon)d_3}{2d_2^2}.$$

Then

$$a_3 - \mu a_2^2 = \frac{1-\upsilon}{2d_3}\left(c_2 + \Theta c_1^2\right).$$

by applying Carathéodory lemma
For $\Re p(z) > 0$, the sharp estimate is

$$|c_2 + \Theta c_1^2| \le 2\max\{1, |2\Theta - 1|\}.$$

Therefore,

$$|a_3 - \mu a_2^2| \le \frac{1-\upsilon}{2d_3} \cdot 2\max\{1, |2\Theta - 1|\}.$$

Hence

$$|a_3 - \mu a_2^2| \leq \frac{1-\upsilon}{d_3} \max\{1, |2\Theta - 1|\}.$$

Sharpness.

Equality is attained for the extremal Carathéodory functions

$$p(z) = \frac{1+z}{1-z} \quad \text{and} \quad p(z) = \frac{1+z^2}{1-z^2},$$

which give equality in the classical Carathéodory estimate. Thus the bound is sharp.

9. Conclusion

The Miller-Ross function and its associated operator play a significant role in the modern landscape of geometric function theory. By extending the classical tools of differentiation through the lens of fractional calculus, they provide a powerful framework for analyzing and generalizing a wide variety of analytic function classes. The operator's ability to preserve key geometric properties underlines its utility, while its flexibility opens new avenues for research in both theoretical and applied aspects of complex analysis.

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