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Comparative Study of Differential Transformation Method (DTM) and Adomian Decomposition Method (ADM) for Solving Ordinary Differential Equations

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Abstract. In this paper, the Differential Transformation Method (**DTM**) and Adomian Decomposition Methods (**ADM**) are studied and compared by applying them to *linear* and *non-linear* **Ordinary Differential Equations**. The Transformation Method and Adomian Decomposition Method provide their solutions as an infinite series in which each term is easily determined. Initial Value Problems with known exact solutions were chosen to ease comparison. Numerical solutions obtained from solved examples were compared with that of the exact solution. The absolute error obtained from this comparison showed that the DTM has a lower error margin when compared to the ADM for a predefined number of iterations and order. Tables for discrete and exact solutions were presented to show the efficiency of the two methods. Numerical computation was aided by the use of a digital computer and the solution obtained showed that DTM is a very accurate and efficient method in contrast to ADM.
Key Words and Phrases: Adomian decomposition methods, Differential transformation method, Ordinary differential equations.

1. Introduction

Advances in the sciences, engineering and even financial fields have initiated a necessary involvement of mathematicians in helping to solve some of the complex problems encountered as the economy becomes more complicated and technology even more sophisticated. The differential transform method, was first introduced by Zhou in 1986. It is a semi analytical-numerical technique that has been successfully used in electrical circuit studies to solve linear and nonlinear initial value problems (Ayaz, 2003). Taylor series expansions are used to construct analytical solutions in polynomial form (Catal, 2008; Jang, 2010). The traditional Taylor series method requires symbolic computation of the derivatives of the data functions and requires more computation time for large orders while the DTM iteratively obtains analytic Taylor series solutions of differential equations (Ayaz, 2003). Compared to the Taylor series, the DTM can easily handle highly nonlinear problems (Ayaz, 2003). Applications of the DTM to various problems in the sciences and engineering fields include solutions of the Blasius and difference equations (Arikoglu and Ozkol, 2005, 2006), vibration equations (Catal, 2008), convective straight fin problem with temperature-dependent thermal conductivity (Joneidi et al., 2009) and the fractional modified KdV equation by Kurulay and Bayram (2010), amongst others.

The Adomian decomposition method was developed by Adomian (Adomian 1976 ,1994 ,1991). The idea is to split the given equation into its linear and nonlinear parts. The highest derivative of the linear part is then inverted on both sides of the equation (Adomian, 1976). The initial approximate solution of the ADM comprises of the initial and/or boundary conditions together with terms involving the independent variables only (Chen and Lu, 2004). The unknown function is then decomposed into a series whose components are to be determined. Special polynomials called Adomian polynomials are used to decompose the nonlinear function (Allan, 2007). Using a recurrent relation in terms of the Adomian polynomials, successive terms of the series solution are generated (Allan, 2007; Bratsos et al., 2008, Fadugba et al. 2013).

In this paper, we compare DTM with ADM for the solution of ordinary differential equations. The rest of the paper is organized as follows; Section Two presents the methodology. In Section Three, we consider some illustrative examples. Section Four consists of results and general conclusion.

2. Methodology

2.1 Differential Transformation Method

2.1.1 DTM and Taylor series

The DTM is developed based on the Taylor series expansion. This method constructs an analytical solution in the form of a polynomial.

Definition 2.1.1: A Taylor polynomial of degree n is defined as follows:

$$f_n(x) = \sum_{k=0}^{n} \frac{1}{k!}(y^{(k)}(c))(x-c)^k \tag{2.1}$$

Theorem: 2.1.1: Suppose that the function y has $(n+1)$ derivatives on the interval $(c-r, c+r)$, for some $r > 0$ and $\lim_{n-\infty} R_n = 0$, for all $x\epsilon(c-r, c+r)$ where $R_n(x)$ is the error between $f_n(x)$ and the approximated function $y(x)$. Then, the Taylor series expanded about $x = c$ converges to $y(x)$. That is,

$$y(x) = \sum_{k=0}^{\infty} \frac{1}{k!}(y^{(k)}(c))(x-c)^k \tag{2.2}$$

for all $x\epsilon(c-r, c+r)$.

2.1.2 Method Description for Differential Transform

The Differential Transform Method (DTM) is a transformation technique based on the Taylor series expansion and is a useful tool to obtain analytical solutions of differential equations. In this method, certain transformation rules are applied, and the governing differential equations and its initial or boundary conditions of the differential equation is transformed into algebraic equations in terms of the differential transforms of the original functions, and the solution of this algebraic equation gives the desired solution of the problem. Consider a function $y(x)$ which is analytical in domain D and let $x = x_0$ represent any point in D. The function $y(x)$ is then represented by a power series whose center is located at x_0. The differential transformation of the function $y(x)$ is given by

$$Y(k) = \frac{1}{k!}(\frac{d^k y(x)}{dx^k})_{x=x_0} \tag{2.3}$$

where $y(x)$ is the original function and $Y[k]$ is the transformed function. The inverse transformation is defined as

$$y(x) = \sum_{k=0}^{\infty}(x - x_0)^k Y(k) \tag{2.4}$$

Combining equation (2.3) and equation (2.4), we get

$$y(x) = \sum_{k=0}^{\infty} \frac{(x - x_0)^k}{k!}(\frac{d^k y(x)}{dx^k})_{x=x_0} \tag{2.5}$$

which means that the rest of the series

$$y(x) = \sum_{k=m+1}^{\infty} \frac{(x - x_0)^k}{k!}(\frac{d^k y(x)}{dx^k})_{x=x_0} \tag{2.6}$$

is negligibly small. Here, the value of m depends on the convergence of natural frequencies. The following basic operations of differential transformation can be deduced from equation (2.3) and (2.5):
If,

$$\begin{aligned}
&(a) \quad y(x) = r(x) \pm p(x), \quad Y(k) = R(k) \pm P(k) \\
&(b) \quad y(x) = \alpha r(x), \quad Y(k) = \alpha R(k) \\
&(c) \quad y(x) = \frac{dr(x)}{dx}, \quad Y(k) = (k+1)R(k+1) \\
&(d) \quad y(x) = \frac{d^2 r(x)}{dx^2}, \quad Y(k) = (k+1)(k+2)R(k+2) \\
&(e) \quad y(x) = \frac{d^b r(x)}{dx^b}, \quad Y(k) = (k+1)(k+2)...(k+b)R(k+b)
\end{aligned}$$

$$
\begin{aligned}
&(f) \quad y(x) = r(x)p(x), \quad Y(k) = \sum_{l=o}^{k} P(l)R(k-l) \\
&(g) \quad y(x) = x^b, \quad Y(k) = \delta(k-b), \quad \delta(k-b)\{^{1,}_{0,} \quad ^{k=b}_{k\neq b} \\
&(h) \quad y(x) = \exp(\lambda x), \quad Y(k) = \frac{\lambda^k}{k!} \\
&(i) \quad y(x) = (1+x)^b, \quad Y(k) = \frac{b(b-1)...(b-k+1)}{k!} \\
&(j) \quad y(x) = \sin(jx+\alpha), \quad Y(k) = \frac{j^k}{k!} \quad \sin(\frac{\pi k}{2}+\alpha) \\
&(k) \quad y(x) = \cos(jx+\alpha), \quad Y(k) = \frac{j^k}{k!} \quad \cos(\frac{\pi k}{2}+\alpha)
\end{aligned} \tag{2.7}
$$

2.2 Adomian Decomposition Method

2.2.1 Method Description for Adomian Decomposition

The Adomian Decomposition Method (ADM) is applied for solving a wide class of linear and non-linear ordinary differential equations,partial differential equations,algebraic equations,difference equations,integral equations and integro-differential equations as well. Consider the following equations:

$$Ly + Ny + Ry = g, \tag{2.8}$$

where L is a linear operator, N represents a non-linear operator and R is the remaining linear part. By defining the inverse operator of L as L^{-1}, assuming that it exists, we get

$$y = L^{-1}g - L^{-1}Ny - L^{-1}Ry. \tag{2.9}$$

The Adomian Decomposition Method assumes that the unknown function y can be expressed by an infinite series of the form

$$y = \sum_{n=0}^{\infty} y_n, \tag{2.10}$$

or equivalently

$$y = y_0 + y_1 + y_2 + ..., \tag{2.11}$$

where the components y_n will be determined recursively. Moreover, the method defines the nonlinear term by the Adomian polynomials More precisely, the ADM assumes that the nonlinear operator $N(y)$ can be decomposed by an infinite series of polynomials given by

$$N(y) = \sum_{n=0}^{\infty} A_n, \tag{2.12}$$

where A_n are the Adomian's polynomials defined as $A_n = A_n(y_0, y_1, y_2, ..., y_n)$. Substituting (2.10) and (2.12) into equation (2.9) and using the fact that R is a linear operator we obtain

$$\sum_{n=0}^{\infty} y_n = L^{-1}g - L^{-1}(\sum_{n=0}^{\infty} R(y_n)) - L^{-1}(\sum_{n=0}^{\infty} A_n(y_0, y_1, y_2, ..., y_n)), \tag{2.13}$$

or equivalently

$$y_0 + y_1 + y_2 + ... = L^{-1}g - L^{-1}(\sum_{n=0}^{\infty} R(y_n)) - L^{-1}(A_0 + A_1 + ...) \tag{2.14}$$

Therefore the formal recurrence algorithm could be defined by

$$y_o = L^{-1}g, ..., y_{n+1} = -L^{-1}(R(y_n)) - L^{-1}(A_n(y_0, y_1, ..., y_n)), \tag{2.15}$$

or equivalently,

$$y_0 = L^{-1}g, y_1 = -L^{-1}(R(y_0)) - L^{-1}(A_0(y_0)), ... \tag{2.16}$$

Consider the non-linear function $f(y)$. Then, the infinite series generated by applying the Taylor's series expansion of y about the initial function y_0 is given by

$$f(y) = f(y_0) + f'(y_0)(y - y_0) + \frac{1}{2!}f''(y_0)(y - y_0)^2 + ... \tag{2.17}$$

By substitution, we have:

$$f(y) = f(y_0) + f'(y_0)(y_1 + y_2 + ...) + \frac{1}{2!}f''(y_0)(y_1 + y_2 + ...)^2 + ... \tag{2.18}$$

Now, we expand equation(2.15) to obtain the Adomian polynomials, we need first to reorder and rearrange the terms. Indeed, one needs to determine the order of each term in (2.15) which actually depends on both the subscripts and exponents of the y_n's. For instance, y_1 is of order 1; y_1^2 is of order 2; y_2^3 is of order 6, and so on. In general, y_n^k is of order kn. In case a particular term involves the multiplication of y_n's, its order s determined by the sum of the terms of the y_n's in each term. For example, $y_2^3 y_2^1$ is of order 8 since $(3)(2) + (2)(1) = 8$. As a result, rearranging the terms in expansion(2.18) according to the order, we have

$$\begin{aligned} f(y) = f(y_0) + & f'(y_0)y_1 + f'(y_0)y_2 + \frac{1}{2!}f''(y_0)y_1^2 + f'(y_0)y_3 + \\ & \frac{2}{2!}f''(y_0)y_1y_2 + \frac{1}{3!}f'''(y_0)y_1^3 + f'(y_0)y_4 + \frac{1}{2!}f''(y_0)y_2^2 + \\ & \frac{2}{2!}f''(y_0)y_1y_3 + \frac{3}{3!}f'''(y_0)y_1^2y_2 + \frac{1}{4!}f''''(y_0)y_1^4 + ... \end{aligned} \tag{2.19}$$

The Adomian polynomials are constructed in a certain way so that the polynomial A_1 consists of all terms in the expansion (2.16) of order 1, A_2 consists of all terms of order 2, and so on. In general, A_n consists of all terms of order n. Therefore, the first nine terms of Adomian polynomials are listed as follows:

$$\begin{aligned}
A_0 &= f(y_0),\\
A_1 &= f'(y_0)y_1,\\
A_2 &= f'(y_0)y_2+\frac{1}{2!}f''(y_0)y_1^2,\\
A_3 &= f'(y_0)y_3+\frac{2}{2!}f''(y_0)y_1y_2+\frac{1}{3!}f'''(y_0)y_1^3,\\
A_4 &= f'(y_0)y_4+\frac{1}{2!}f''(y_0)(2y_1y_3+y_2^2)+\frac{3}{3!}f'''(y_0)y_1^2y_2+\frac{1}{4!}f''''(y_0)y_1^4,\\
A_5 &= f'(y_0)y_5+\frac{1}{2!}f''(y_0)(2y_1y_4+2y_2y_3)+\frac{1}{3!}f'''(y_0)(3y_1^2y_3+3y_1y_2^2)+\\
&\quad \frac{4}{4!}f^{(4)}(y_0)y_1^3y_2+\frac{1}{5!}f^{(5)}(y_0)y_1^5,\\
A_6 &= f'(y_0)y_6+\frac{1}{2!}f''(y_0)(2y_1y_4+2y_2y_3)+\frac{1}{3!}f'''(y_0)(3y_1^2y_3+3y_1y_2^2)+\\
&\quad \frac{4}{4!}f^{(4)}(y_0)y_1^3y_2+\frac{1}{5!}f^{(5)}(y_0)y_1^5,\\
A_7 &= f'(y_0)y_7+\frac{1}{2!}f''(y_0)(2y_1y_4+2y_2y_3)+\frac{1}{3!}f'''(y_0)(3y_1^2y_3+3y_1y_2^2)+\\
&\quad \frac{4}{4!}f^{(4)}(y_0)y_1^3y_2+\frac{1}{5!}f^{(5)}(y_0)y_1^5,\\
A_8 &= f'(y_0)y_8+\frac{1}{2!}f''(y_0)(2y_1y_4+2y_2y_3)+\frac{1}{3!}f'''(y_0)(3y_1^2y_3+3y_1y_2^2)+\\
&\quad \frac{4}{4!}f^{(4)}(y_0)y_1^3y_2+\frac{1}{5!}f^{(5)}(y_0)y_1^5,
\end{aligned} \tag{2.20}$$

The Adomian polynomial A_n was first introduced by Adomian himself; it was defined via the general formula

$$A_n(y_0, y_1, ..., y_n) = \frac{1}{n!}\frac{d^n}{d\lambda.}\left[N(\sum_{k=0}^{\infty} y_k\lambda^k)\right]_{\lambda=0}, n = 0, 1, 2, ... \tag{2.21}$$

where N is the number of terms of the Adomian polynomials.

2.2.2 Decomposition of Linear ODEs

To apply the ADM for solving linear ordinary differential equations, we consider the following general equation written in operator form:

$$L(y) + R(y) = g(x), \tag{2.22}$$

where the linear differential operator L may be considered as the highest order derivative in the equation,R is the remainder of the differential operator, and $g(x)$ is an inhomogeneous term. If L is a first order operator defined by

$$L = \frac{d}{dx} \tag{2.23}$$

Then, assuming that L is invertible, then the inverse operator L^{-1} is given by;

$$L^{-1}(.) = \int_0^x (.)dx, \tag{2.24}$$

so that

$$L^{-1}Ly = y(x) - y(0). \tag{2.25}$$

However, if L is a second order differential operator given by

$$L = \frac{d^2}{dx^2}, \tag{2.26}$$

then the inverse operator L^{-1} is a two-fold integration operator by

$$L^{-1}(.) = \int_0^x \int_0^x (.)dxdx, \tag{2.27}$$

Hence, we have

$$L^{-1}Ly = y(x) - y(0) - xy'(0). \tag{2.28}$$

In a parallel manner, if L is a third order differential operator, we can easily show that

$$L^{-1}Ly = y(x) - y(0) - xy'(0) - \frac{1}{2!}x^2y''(0). \tag{2.29}$$

For higher order ODEs, the latter equation can be generalized in a similar fashion. Now, to implement the ADM, we proceed to first applying L^{-1} to both sides of equation (2.22) and after rearranging the terms, we get

$$y(x) = \Phi_0 + L^{-1}g(x) - L^{-1}Ry, \tag{2.30}$$

where, as explained above, we have if

$$\begin{aligned}
\Phi_0 = & y(0), \text{then} && L = \frac{d}{dx} \\
& y(0) + xy'(0), \text{then} && L = \frac{d^2}{dx^2} \\
& y(0) + xy'(0) + \frac{1}{2!}x^2y''(0), \text{then} && L = \frac{d^3}{dx^3} \\
& y(0) + xy'(0) + \frac{1}{2!}x^2y''(0) + \frac{1}{3!}x^3y'''(0), \text{then} && L = \frac{d^4}{dx^4} \\
& \dots
\end{aligned} \tag{2.31}$$

and so on. The Adomian decomposition method admits the decomposition of y in the form of an infinite series of components

$$y(x) = \sum_{n=0}^{\infty} y_n, \tag{2.32}$$

where $y_n(x), n \geq 0$ are the components of $y(x)$ that will be determined recursively. Substituting (2.31) into (2.32) gives

$$\sum_{n=0}^{\infty} y_n = \Phi_0 + L^{-1}g(x) - L^{-1}R(\sum_{n=0}^{\infty} y_n). \tag{2.33}$$

The various components y_n of the solution y can be easily determined by using the recursive relation

$$\begin{aligned} y_o &= \Phi_0 + L^{-1}g(x), \\ y_{n+1} &= -L^{-1}Ry_n, \qquad n \geq 0. \end{aligned} \tag{2.34}$$

It is worth mentioning that the determination of the y_0 term depends on the specified initial conditions $y(0), y'(0), y''(0), y'''(0)$,....

2.2.3 Decomposition of Non-linear ODEs

Consider the following non-linear ordinary differential equation written in operator form:

$$Ly + Ry + N(y) = g(x), \tag{2.35}$$

where the linear operator L is the highest order derivative, R is the remainder of the differential operator, $N(y)$ is the non-linear terms and $g(x)$ expresses an inhomogeneous term. Without loss of generality, let L be the first order differential operator

$$L = \frac{d}{dx} \tag{2.36}$$

Then, assuming that L is invertible, then the inverse operator L^{-1} is given by;

$$L^{-1}(.) = \int_0^x (.)dx, \tag{2.37}$$

Therefore,

$$L^{-1}Ly = y(x) - y(0). \tag{2.38}$$

On the other hand, if L is a second order differential operator given by

$$L = \frac{d^2}{dx^2}, \tag{2.39}$$

then the inverse operator L^{-1} is given by

$$L^{-1}(.) = \int_0^x \int_0^x (.)dxdx, \tag{2.40}$$

which means that

$$L^{-1}Ly = y(x) = y(0) = xu'(0). \tag{2.41}$$

While, if L is a third order differential operator, we can easily show that

$$L^{-1}Ly = y(x) - y(0) - xy'(0) - \frac{1}{2!}x^2y''(0). \tag{2.42}$$

and so forth. In general, if L is a differential operator of order $n+1$, we can easily show that

$$L^{-1}Ly = y(x) - y(0) - xy'(0) - \frac{1}{2!}x^2y''(0) - \frac{1}{3!}x^3y'''(0) - ... - \frac{1}{n!}x^ny^n(0). \tag{3.43}$$

Applying L^{-1} to both side of (2.35) gives

$$y = \Phi_o - L^{-1}g(x) - L^{-1}N(y) - L^{-1}Ry, \tag{2.44}$$

where

$$\begin{aligned}
\Phi_0 =& y(0), && \text{if} \quad L = \frac{d}{dx} \\
& y(0) + xy'(0), && \text{if} \quad L = \frac{d^2}{dx^2} \\
& y(0) + xy'(0) + \frac{1}{2!}x^2y''(0), && \text{if} \quad L = \frac{d^3}{dx^3} \\
& y(0) + xy'(0) + \frac{1}{2!}x^2y''(0) + \frac{1}{3!}x^3y'''(0), && \text{if} \quad L = \frac{d^4}{dx^4} \\
& y(0) + xy'(0) + \frac{1}{2!}x^2y''(0) + \frac{1}{3!}x^3y'''(0) + ... + \frac{1}{n!}x^ny^{(n)}(0), && \text{if} \quad L = \frac{d^4}{dx^4} \\
& ...
\end{aligned} \tag{2.45}$$

The decomposition technique consists of decomposing the solution into a sum of an infinite number of terms defined by the decomposition series

$$y(x) = \sum_{n=0}^{\infty} y_n, \tag{2.46}$$

while the non-linear term $N(y)$ is to be expressed by an infinite series of polynomials

$$N(y) = \sum_{n=0}^{\infty} A_n, \tag{2.47}$$

where A_n are the Adomian's polynomials. Equation (2.44) yields

$$\sum_{n=0}^{\infty} y_n = \Phi_0 - L^{-1}R(\sum_{n=0}^{\infty} y_n) - L^{-1}(\sum_{n=0}^{\infty} A_n + L^{-1}g(x)). \tag{2.48}$$

To construct the iterative scheme, we match both sides so that the y_n term is expressed in terms of the previously determined terms. More specifically, the Adomian decomposition method gives the following iterative algorithm:

$$\begin{aligned} y_o &= \Phi_0 + L^{-1}g(x), \\ y_{n+1} &= -L^{-1}Ry_n - L^{-1}A_n, \qquad n \geq 0. \end{aligned} \tag{2.49}$$

This in turn gives

$$\begin{aligned} &y_o = \Phi_0 + L^{-1}g(x), y_1 = -L^{-1}Ry_n - L^{-1}A_n, \\ &y_2 = -L^{-1}Ry_n - L^{-1}A_n, ... \end{aligned} \tag{2.50}$$

3. Computation and Results

In this section, we apply the DTM and ADM to initial value problems for both linear and non-linear ordinary differential equations. The examples considered are for the illustration of the methods described for DTM and ADM and to confirm their applicability and efficiency.The comparison will be made by separately applying the two methods.

3.1. Example 1

The first differential equation to be considered is the linear first order initial value problem:

$$y'(x) = y, y(0) = 1. \tag{3.1}$$

Note that y', y'', y''', $y^{(4)}$,... denote the first, second, third and fourth derivatives of $y(x)$. In general, y^n denotes the nth derivative of y with respect to x. Equation (3.1) denotes a function whose first derivative is also that function. The analytical solution for equation(3.1) is given as:

$$y(x) = ce^x \tag{3.2}$$

with $c = 1$ at $y(0) = 1$;

$$y(x) = e^x \tag{3.3}$$

which is as expected.

3.1.1. Differential Transform of Example 1

Applying DTM to the above equation, we have that;

If

$$y(x) = \frac{d^n}{dx^n}, \tag{3.4}$$

then,

$$Y(k) = \frac{(k+n)!}{n!} Y(k+n) \tag{3.5}$$

Equation(3.4) implies that,

$$\frac{dy}{dx} = y \tag{3.6}$$

with $n = 1$. On Transformation, equation(3.1) becomes:

$$\begin{aligned} y(x) &= Y(k) \\ \frac{dy}{dx} &= \frac{(k+1)!}{1!} Y(k+1) \end{aligned} \tag{3.7}$$

and also,

$$y(0) = 1 \tag{3.8}$$

implies that

$$Y(0) = 1 \tag{3.9}$$

substituting the transformed parts into equation(3.1) gives;

$$Y(k) = (k+1)! Y(k+1) \tag{3.10}$$

Making $Y(k+1)$ the subject of formula gives the recurrence relation:

$$Y(k+1) = \frac{Y(k)}{(k+1)!} \tag{3.11}$$

So that for $k = 0, \cdots, 10$, we have for $Y(K)$

$$\begin{aligned} Y(1) &= \frac{1}{1!}, Y(2) = \frac{1}{2!}, Y(3) = \frac{1}{3!} \\ Y(4) &= \frac{1}{4!}, Y(5) = \frac{1}{5!}, ..., Y(n) = \frac{1}{n!} \end{aligned} \tag{3.12}$$

So that our series expansion becomes

$$\begin{aligned} y(x) &= \sum_{k=10}^{0} x^k Y(k) \\ &= 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ... + \frac{x^{10}}{10} \end{aligned} \tag{3.13}$$

which is the required expansion.

3.1.2 Adomian Decomposition of Example 1

In operator form, equation(3.1) can be written as:

$$
\begin{aligned}
&Ly = y', \\
&y(0) = 1 \quad \Rightarrow \quad y_0 = 1
\end{aligned}
\tag{3.14}
$$

where L is the first order differential operator

$$Ly = y' \tag{3.15}$$

It is clear that L^{-1} is invertible and is given by

$$L^{-1}(*) = \int_0^x dt \tag{3.16}$$

Applying L^{-1} to both sides of equation (3.15) and using the given initial conditions

$$
\begin{aligned}
&L^{-1}Ly = y(x) - y(0) \\
&L^{-1}Ly = L^{-1}y
\end{aligned}
\tag{3.17}
$$

Combining gives,

$$
\begin{aligned}
&y(x) - y(0) = L^{-1}y \\
&y(x) = y(0) + L^{-1}y \\
&y(x) = 1 + L^{-1}y
\end{aligned}
\tag{3.18}
$$

Upon using the decomposition series for the solution y(x) results

$$\sum_{n=0}^{\infty} y_n = 1 + L^{-1}(\sum_{n=0}^{\infty} y_n) \tag{3.19}$$

Upon matching both sides, this leads to the recursive relation,

$$
\begin{aligned}
&y_0 = 1 + \int_0^x 1, \qquad y_0 = 1 \\
&y_0 = 1 + x, ..., y_{n+1} = L^{-1}Ry_n, \qquad n \geq 0
\end{aligned}
\tag{3.20}
$$

The first few components are thus determined as follows:

$$
\begin{aligned}
&y_0 = 1, y_1 = L^{-1}(1) = x, y_2 = L^{-1}(x) = \frac{x^2}{2}, ... \\
&y(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ... + \frac{x^n}{n}
\end{aligned}
\tag{4.21}
$$

3.2 Example 2

The second differential equation to be considered is the non-linear first order initial value problem:

$$y'(x) = y(x)^2,$$
$$y(0) = 1. \quad (4.22)$$

whose exact solution at $y(0) = 1$ is given as,

$$y(x) = \frac{1}{1-x} \quad (3.23)$$

3.2.1 Differential Transform of Example 2

Applying the transform method to equation(4.22) generates the following,

$$y' = (k+1)!Y(k+1),$$
$$y(x) = Y(k), \quad (3.24)$$

and,

$$y^2(x) = y_1(x)y_2(x) \quad (3.25)$$

so that,

$$y^2(x) = \sum_{k=0}\sum_{k_1=0} Y_1(k_1)Y_2(k-k_1) \quad (3.26)$$

Hence, equation(3.26) becomes,

$$(k+1)!Y(k+1) = \sum_{k=0}\sum_{k_1=0} Y_1(k_1)Y_2(k-k_1) \quad (4.27)$$

let $k_1 = l$ be a constant with $l = 0$

$$(k+1)!Y(k+1) = \sum_{k=0}\sum_{l=0} Y_1(l)Y_2(k-l) \quad (3.28)$$

and we obtain the recursive relationship

$$Y(k+1) = \frac{1}{(k+1)!}\sum_{k=0}\sum_{l=0} Y_1(l)Y_2(k-l) \quad (3.29)$$

Applying the initial conditions, with $k = 0, ..., 10$ and $l = 0, ..., 10$, we get the following values for Y(k).

$$Y(1) = \frac{1}{1} = 1, Y(2) = \frac{2}{2} = 1, Y(3) = \frac{3}{3} = 1,$$

$$Y(4) = \frac{4}{4} = 1, ..., Y(n) = \frac{n}{n} = 1 \tag{3.30}$$

Now,

$$y(x) = \sum_{k=0}^{10} x^k Y(k). \tag{3.31}$$

Since $Y(k) = 1$ for all $k = 0, \cdots, 10$,then

$$y(x) = \sum_{k=0}^{10} x^k. \tag{3.32}$$

So that,

$$y(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} \tag{3.33}$$

3.2.2 Adomian Decomposition of Example 2

Consider the non-linear initial value problem in equation (3.22), and following the method derived in chapter 3 for non-linear ADM, we define a linear operator

$$L = \frac{d}{dx} \tag{3.34}$$

The inverse operator is then

$$L^{-1} = \int_0^x (.)dx \tag{3.35}$$

Rewriting the differential equation (3.22) in operator form, we have

$$Ly = Ny \tag{3.36}$$

where N is a non-linear operator such that

$$Ny = y^2 \tag{3.37}$$

Next, we apply the inverse operator for L to the equation on the left hand side of equation (3.22) to obtain,

$$L^{-1}Ly = y(x) - y(0) \tag{3.38}$$

Using the initial conditions, this becomes

$$L^{-1}Ly = y(x) - 1 \tag{3.39}$$

Returning this decomposition to equation (3.22), we now have

$$y(x) - 1 = L^{-1}(Ny) \tag{3.40}$$

or

$$y(x) = 1 + L^{-1}(Ny) \tag{3.41}$$

Next, we need to generate the Adomian polynomials, A_n. Let y be expanded as an infinite series $y(t) = \sum_{n=0}^{\infty} y_n(t)$ and define $N_y = \sum_{n=0}^{\infty} A_n$. Then

$$\sum_{n=0}^{\infty} y_n(t) = 1 + L^{-1}(\sum_{n=0}^{\infty} A_n) \tag{3.42}$$

To find A_n, we introduce the scalar λ such that,

$$y(\lambda) = \sum_{n=0}^{\infty} \lambda^n y_n. \tag{3.43}$$

Then,

$$Ny(\lambda) = \sum_{n=0}^{\infty} \lambda^n \sum_{i=0}^{\infty} (y_i y_{n-i}). \tag{3.44}$$

From the definition of the Adomian Polynomials,

$$A_n = \frac{1}{n} \frac{d^n}{d\lambda^n} (Ny(\lambda))|_{\lambda=0} \tag{3.45}$$

We find the Adomian polynomials.

$$\begin{aligned}
A_0 &= y_0^2 \\
A_1 &= 2y_0y_1 \\
A_3 &= 2y_0y_3 + 2y_1y_2 \\
A_4 &= 2y_0y_4 + 2y_1y_3 + y_3^2 \\
A_5 &= 2y_0y_5 + 2y_1y_4 + 2y_2y_3 \\
A_6 &= 2y_0y_6 + 2y_1y_5 + 2y_2y_4 + y_3^2 \\
A_7 &= 2y_0y_7 + 2y_1y_6 + 2y_2y_5 + 2y_3y_4
\end{aligned} \tag{3.46}$$

Returning the Adomian polynomials, we can determine the recursive relationship that will be used to generate the solution.

$$\begin{aligned}
y_0(x) &= 1 \\
y_{n+1}(x) &= L^{-1}(A_n).
\end{aligned} \tag{3.47}$$

Solving this yields

$$y_0 = 1, y_1 = x, y_2 = x^2, y_3 = x^3, y_4 = x^4, y_5 = x^5, y_6 = x^6, y_7 = x^7 \tag{3.48}$$

We can see that the series solution generated by this method is

$$y(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \cdots = \sum_{n=0}^{\infty} x^n \tag{3.49}$$

which we recognize as the Taylor series for the exact solution

$$y(x) = \frac{1}{1-x} \tag{3.50}$$

3.3 Example 3

Consider the first order non-linear initial value problem given as:

$$y' + y^2 = 1, \qquad y(0) = 0 \tag{3.51}$$

with the exact solution

$$y(x) = \frac{e^{2x}-1}{e^{2x}+1} = tanh(x) \tag{3.52}$$

The Taylor expansion of $y(x)$ about $x = 0$ gives

$$y(x) = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{62}{2835}x^9 - \cdots \tag{3.53}$$

3.3.1 Differential Transform of Example 3

Taking the Differential Transform, leads to

$$(k+1)!Y(k+1) = -\sum_{k=0}^{r} \delta(k) \tag{3.54}$$

Applying the initial conditions given,

$$y(0) = 0 \Longrightarrow Y(0) = 0. \tag{3.55}$$

Also, when $k = r$, $\delta(k) = 1$ and when $k \neq r$, then $\delta(k) = 0$ The recursive relation to obtain values for $Y(k)$ is

$$Y(k+1) = -\frac{1}{(k+1)!}\sum_{k=0}^{r} \delta(k). \tag{3.56}$$

For $k = 0, \cdots, 10$,

$$\begin{aligned} &Y(1) = 1, Y(2) = 0, Y(3) = -\frac{1}{3}, Y(4) = 0, Y(5) = \frac{2}{15}, Y(6) = 0, \\ &Y(7) = -\frac{17}{315}, Y(8) = 0, Y(9) = \frac{62}{2835} \end{aligned} \tag{3.57}$$

Therefore, the closed form of the solution can be easily written as

$$y(x) = \sum_{k=0}^{\infty} Y(k)x^k = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \frac{62x^9}{2835} - \cdots \tag{4.58}$$

which is also the Taylor's series expansion.

3.3.2 Adomian Decomposition of Example 3

Applying L^{-1} to both sides of the equation and using the initial conditions gives

$$y = y(0) + L^{-1}1 - L^{-1}y^2 = L^{-1}y^2 + x. \tag{3.59}$$

Using the decomposition series for y and the Adomian polynomials representation for the non-linear term y^2, gives

$$\sum_{n=0}^{\infty} y_n = -L^{-1}\sum_{n=0}^{\infty} A_n + x, \tag{3.60}$$

where the A_n's are the Adomian polynomials for y^2 as shown above. Matching both sides of the equation results in the following ADM iterative scheme:

$$\begin{aligned} y_o &= x, \\ y_{n+1} &= -L^{-1}(A_n). \end{aligned} \tag{3.61}$$

This in turn gives

$$\begin{aligned} y_0 &= x \\ y_1 &= -L^{-1}(A_0) = -L^{-1}(y_0^2) = -\frac{x^3}{3}, \\ y_2 &= -L^{-1}(A_1) = -L^{-1}(2y_0y_1) = \frac{2x^5}{15}, \\ y_3 &= -L^{-1}(A_2) = -L^{-1}(2y_0y_1 + y_1^2) = -\frac{7x^7}{315} \\ &\vdots \end{aligned} \tag{4.62}$$

The solution in a series form is thus given by

$$y(x) = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{7x^7}{315} + \cdots, \tag{3.63}$$

which clearly converges to the exact solution

$$y(x) = tanh(x) \tag{3.64}$$

3.4 Example 4

We will now look at how the two methods handle second order ordinary differential equations. For this example, our test problem is a linear second order initial value problem:

$$y'' - y = 0, \tag{3.65}$$

subject to the initial conditions

$$y(0) = 0, \quad y'(0) = 1. \tag{3.66}$$

The exact solution is given as: $y(x) = e^x - 1$

3.4.1 Differential Transform of Example 4

On transformation, we obtain:

$$(k+2)!Y(k+2) - Y(k) = \delta(k) \tag{3.67}$$

which gives the recursive formula

$$Y(k+2) = \frac{Y(k)+\delta(k)}{(k+2)!} \tag{3.68}$$

Applying the initial conditions given,

$$\begin{aligned} y(0) = 0 &\Longrightarrow Y(0) = 0, \\ y'(0) = 1 &\Longrightarrow Y(1) = 1. \end{aligned} \tag{3.69}$$

Also, $\delta(k) = 1$ when $k = 0$,$\delta(k) = 0$ when $k \geq 1$. For $k = 0, \cdots, 10$

$$\begin{aligned} &Y(2) = \frac{1}{2}, Y(3) = \frac{1}{3!}, Y(4) = \frac{1}{4!}, Y(5) = \frac{1}{5!}, Y(6) = \frac{1}{6!} \\ &Y(7) = \frac{1}{7!}, Y(8) = \frac{1}{8!}, Y(9) = \frac{1}{9!}, Y(10) = \frac{1}{10!}, Y(11) = \frac{1}{11!} \\ &Y(12) = \frac{1}{12!} \end{aligned} \tag{3.70}$$

Hence the series solution is given as:

$$y(x) = x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!} + \frac{x^9}{9!} + \frac{x^{10}}{10!} + \frac{x^{11}}{11!} + \frac{x^{12}}{12!} \tag{3.71}$$

3.4.2 Adomian Decomposition of Example 4

In operator form, equation(3.65) can be written as

$$Ly = 1 + y, \qquad y(0) = 0, y'(0) = 1, \tag{3.72}$$

where L is the second order differential operator $Ly = y''$. t is clear that L^{-1} is invertible and is given by

$$L^{-1}(.) = \int_0^x \int_0^x (.)dxdx. \tag{3.73}$$

Applying L^{-1} to both sides of (3.72) and using the initial conditions gives

$$y = y(0) + xy'(0) + L^{-1}1 = x + \frac{x^2}{2} + L^{-1}y. \tag{3.74}$$

Upon using the decomposition series for the solution of $y(x)$, results

$$\sum_{n=0}^{\infty} y_n = x + \frac{x^2}{2} + L^{-1}(\sum_{n=0}^{\infty} y_n) \tag{3.75}$$

Upon matching both sides, this leads to the recursive relation

$$y_o = x + \frac{x^2}{2}, y_{n+1} = L^{-1}(y_n), \qquad n \geq 0. \tag{3.76}$$

The first few components are thus determined as follows:

$$y_0 = x + \frac{x^2}{2}, y_1 = \frac{x^3}{6} + \frac{x^4}{24}, y_2 = \frac{x^5}{5!} + \frac{x^6}{6!}. \tag{3.77}$$

Consequently, the solution in a series form is given by

$$y(x) = x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{5!} + \frac{x^6}{6!} + \cdots, \tag{3.78}$$

and clearly in a closed form is given by

$$y(x) = e^x - 1, \tag{3.79}$$

which is the exact solution of the problem. This is another case where the ADM converges to the solution.

3.5 Example 5

We now consider a third order linear ODE given as:

$$y''' = -y \tag{3.80}$$

subject to the initial conditions;

$$y(0) = 1, y'(0) = -1, y''(0) = 1. \tag{3.81}$$

The Exact solution is given as: $y(x) = \exp^{-x}$

3.5.1 Differential Transformation of Example 5

Transforming equation (3.80) gives:

$$(k+3)!Y(k+3) = -Y(k) \tag{3.82}$$

which resolves to the recursive formula on making $Y(k+3)$ subject of formula;

$$Y(k+3) = -\frac{Y(k)}{(k+3)!} \tag{3.83}$$

Applying the initial conditions given,

$$y(0) = 1 \Longrightarrow Y(0) = 1, y'(0) = -1 \Longrightarrow Y(1) = -1, y''(0) = 1 \Longrightarrow Y(2) = \frac{1}{2}. \tag{3.84}$$

For $k = 0, \cdots, 10$

$$\begin{aligned} &Y(3) = -\frac{1}{3!}, Y(4) = \frac{1}{4!}, Y(5) = -\frac{1}{5!}, Y(6) = \frac{1}{6!}, Y(7) = -\frac{1}{7!}, Y(8) = \frac{1}{8!}, \\ &Y(9) = -\frac{1}{9!}, Y(10) = \frac{1}{10!}, Y(11) = -\frac{1}{11!}, Y(12) = \frac{1}{12!}, Y(13) = -\frac{1}{13!} \end{aligned} \tag{3.85}$$

The series solution is given as:

$$\begin{aligned} y(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{x^6}{6!} - \frac{x^7}{7!} + \frac{x^8}{8!} \\ - \frac{x^9}{9!} + \frac{x^{10}}{10!} - \frac{x^{11}}{11!} + \frac{x^{12}}{12!} - \frac{x^{13}}{13!} \end{aligned} \tag{3.86}$$

3.5.2 Adomian Decomposition of Example 5

Given that L is a linear differential operator, R the remainder of the differential operator, and g the inhomogeneous term. Let L be a third order operator as described in chapter(3) for Adomian Decomposition of third order ODEs, then assuming that L is invertible, the inverse operator L^{-1} is given by

$$L^{-1}(.) = \int_0^x \int_0^x \int_0^x (.)dxdxdx \tag{3.87}$$

Hence, we have

$$L^{-1}Ly = y(x) - y(0) - xy'(0) - \frac{1}{2!}x^2 y''(0) \tag{3.88}$$

where,

$$\begin{aligned} Ly &= -y \\ L^{-1}Ly &= L^{-1}(-y), \end{aligned} \tag{3.89}$$

so that,

$$y(x) = 1 - x + \frac{x^2}{2!} + L^{-1}(-y). \tag{3.90}$$

Upon using the decomposition series for the solution $y(x)$, we get;

$$\sum_{n=0}^{\infty} y_n = 1 - x + \frac{x^2}{2!} + c(\sum_{n=0}^{\infty} -y_n) \tag{3.91}$$

When both sides are matched, this leads to the recursive relation

$$\begin{aligned} y_0 &= 1 - x + \frac{x^2}{2!} + L^{-1}(-y_0) \\ &= 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} \end{aligned}$$

$$y_{n+1} = L^{-1}R(-y_n), \qquad n \geq 0 \tag{3.92}$$

which gives the convergent series

$$y(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{x^6}{6!}. \tag{3.93}$$

3.6 Numerical Results

Table 3.1: h=0.1

Absolute error obtained for Example 1 using ADM with three iterations and DTM with $k = 0..10$

X	DTM	ADM	Exact	Error(DTM)	Error(ADM)
0.0	1.0000000000	1.0000000000	1.0000000000	0.0000000000	0.0000000000
0.1	1.1051709180	1.1051709180	1.1051709180	0.0000000000	0.0000000000
0.2	1.2214027580	1.2214027590	1.2214027580	0.0000000000	0.0000000000
0.3	1.3498588080	1.3498588070	1.3498588080	0.0000000000	0.0000000000
0.4	1.4918246980	1.4918246980	1.4918246980	0.0000000000	0.0000000000
0.5	1.6487212710	1.6487212700	1.6487212710	0.0000000000	0.0000000000
0.6	1.8221188000	1.8221188010	1.8221188000	0.0000000000	0.0000000000
0.7	2.0137527070	2.0137527070	2.0137527070	0.0000000000	0.0000000000
0.8	2.2255409280	2.2255409270	2.2255409280	0.0000000000	0.0000000000
0.9	2.4596031110	2.4596031020	2.4596031110	0.0000000000	0.0000000000
1.0	2.7182818260	2.7182818030	2.7182818280	2.0000000000E-8	2.5000000000E-8

Table 3.2: h=0.1

Absolute error obtained for Example 2 using ADM with three iterations and DTM with $k = 0..10$

X	DTM	ADM	Exact	Error(DTM)	Error(ADM)
0.0	1.0000000000	1.0000000000	1.0000000000	0.0000000000	0.0000000000
0.1	1.1111111111	1.1111111111	1.1111111111	0.0000000000	0.0000000000
0.2	1.2499999740	1.2499999740	1.2500000000	2.6000000000E-8	2.6000000000E-8
0.3	1.4285688980	1.4285688980	1.4285714290	2.5310000000E-6	2.5310000000E-6
0.4	1.6665967620	1.6665967620	1.6666666670	6.9905000000E-5	6.9905000000E-5
0.5	1.9990234380	1.9990234380	2.0000000000	9.7656200000E-4	9.7656200000E-4
0.6	2.4909300740	2.4909300740	2.5000000000	9.0699260000E-3	9.0699260000E-3
0.7	3.2674224420	3.2674224420	3.3333333333	0.0659108910	0.0659108910
0.8	4.5705032700	4.5705032700	5.0000000000	0.0429496730	0.0429496730
0.9	6.8618940390	6.8618940390	10.000000000	3.1381059610	3.1381059610

Table 3.3: h=0.1

Absolute error obtained for Example 3 **using ADM with three iterations and DTM with** $k = 0..10$

X	DTM	ADM	Exact	Error(DTM)	Error(ADM)
0.0	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.1	0.0996679946	0.0996679946	0.0996679946	0.0000000000	2.0000000000E-11
0.2	0.1973753204	0.1973753092	0.1973753202	2.0000000000E-10	1.1000000000E-8
0.3	0.2913126276	0.2913121971	0.2913126125	1.5100000000E-8	4.1540000000E-7
0.4	0.3799493114	0.3799435784	0.3799489623	3.4910000000E-7	5.3839000000E-6
0.5	0.4621210868	0.4620783730	0.4621171573	3.9295000000E-6	3.8784300000E-5
0.6	0.5370776284	0.5368572343	0.5370495670	2.8061400000E-5	1.9233270000E-4
0.7	0.6045139949	0.6036314822	0.6043677771	1.4621780000E-4	7.3629490000E-4
0.8	0.6646413099	0.6617060368	0.6640367703	6.0453960000E-4	2.3307330000E-3
0.9	0.7183918393	0.7099191514	0.7162978702	2.0939690000E-3	6.3787180000E-3
1.0	0.7679012345	0.7460317460	0.7615941560	6.3070780000E-3	0.0155624100

Table 3.4: h=0.1

Absolute error obtained for Example 4 **using ADM with three iterations and DTM with** $k = 0..10$

X	DTM	ADM	Exact	Error(DTM)	Error(ADM)
0.0	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.1	0.1051709181	0.1051709181	0.1051709180	1.0000000000E-10	1.0000000000E-10
0.2	0.2214027582	0.2214027556	0.2214027580	2.0000000000E-10	2.4000000000E-9
0.3	0.3498588076	0.3498587625	0.3498588080	4.0000000000E-10	4.0000000000E-10
0.4	0.4918246977	0.4918243556	0.4918246980	3.0000000000E-10	3.4240000000E-7
0.5	0.6487212708	0.6487196181	0.6487212710	2.0000000000E-10	1.6529000000E-6
0.6	0.8221188005	0.8221128000	0.8221188000	5.0000000000E-10	9.9000000000E-4
0.7	1.0137527070	1.0137348180	1.0137527070	5.0000000000E-9	1.7889000000E-5
0.8	1.2255409290	1.2254947560	1.2255409280	1.0000000000E-9	4.6172000000E-5
0.9	1.4596031110	1.4594963620	1.4596031110	0.0000000000	1.0674900000E-4
1.0	1.7182818300	1.7180555560	1.7182818280	2.0000000000E-9	2.2627200000E-4

Table 3.5: h=0.1

Absolute error obtained for Example 5 **using ADM with three iterations and DTM with** $k = 0..10$

X	DTM	ADM	Exact	Error(DTM)	Error(ADM)
0.0	1.0000000000	1.0000000000	1.0000000000	0.0000000000	0.0000000000
0.1	0.9048374181	0.9048374181	0.9048374181	0.0000000000	0.0000000000
0.2	0.8187307532	0.8187307556	0.8187307532	0.0000000000	2.4000000000E-9
0.3	0.7408182206	0.7408182625	0.7408182206	0.0000000000	4.0900000000E-8
0.4	0.6703200461	0.6703203556	0.6703200461	0.0000000000	3.0950000000E-7
0.5	0.6065306598	0.6065321181	0.6065306598	0.0000000000	1.4583000000E-6
0.6	0.5488116361	0.5488168000	0.5488116361	0.0000000000	5.1639000000E-6
0.7	0.4965853039	0.4966003181	0.4965853039	0.0000000000	1.5014200000E-5
0.8	0.4493289640	0.4493667556	0.4493289640	0.0000000000	3.7791600000E-5
0.9	0.4065696598	0.4066548625	0.4065696598	0.0000000000	8.5202700000E-5
1.0	0.3678794412	0.3680555556	0.3678794412	0.0000000000	1.7611440000E-4

4. Discussion of Results and General Conclusion

4.1 Discussion Of Results

The effectiveness of DTM and ADM have been established by comparing different types of Ordinary Differential equations, linear and non-linear, of first, second and third order. The numerical solutions obtained in chapter Four show that the DTM has lower absolute error when compared to the ADM. Three iterations for the ADM were compared to obtained values for DTM with $k = 0..10$. By observing the results from the Tables 1 to 5, DTM is seen to be more suitable for dealing with ODE problems.Moreover, as illustrated by examples, while ADM appears easily approachable, DTM is direct and fast to compute.

4.2 General Conclusion

In this paper, we have introduced two semi-numerical methods, DTM and ADM, for solving Ordinary Differential Equations. The methods have been used to solve different ODE problems. A summary of the findings for each problem solved is given below. In Section 3, we solved five ODE problems. In subsection 3.1, we considered a linear first order ODE. The DTM and ADM results were compared against the exact results. It was observed that both methods, DTM and ADM, provided series expansion of the exact solutions. Subsection 3.2 to 3.5 produced similar results and hence showed that both methods do capture the series expansion of the exact solutions. From this study we conclude that in comparing Differential Transformation Method and Adomian Decomposition Method for solving Ordinary Differential Equations, both methods are efficient, accurate and robust but DTM has lower error estimate than the ADM. The equations solved ranged from linear to non-linear ODEs. However, comparison of the two methods need to be extended to other types of equations such as; time-dependent evolution equations, partial differential equations and difference equations.

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